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THE

COMPLETE ARITHMETIC.

ORAL AND WRITTEN.

BY DANIEL W. FISH, A.M.,

EDITOR OF ROBINSON'S SERIES OF PROGRESSIVE ARITHMETICS.

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PREFACE

THE design of the author, in the preparation of this work, has been to furnish a text-book on the subject of arithmetic, complete not only as a treatise, but as a comprehensive manual for the class-room, and, therefore, embodying every necessary form of illustration and exercise, both *oral* and *written*. Usually, this subject has been treated in such a way as to form the contents of *three or more graded text-books*, the oral exercises being placed in a separate volume. In the present treatise, however, the *whole subject* is presented in all its different grades; and the oral, or mental, arithmetic, so called, has been inserted, where it logically and properly belongs, either as introductory to the enunciation of principles or to the statement of practical rules—the treatment of every topic from the beginning to the end of the book being thoroughly *inductive*.

In this way, and by carefully constructed *analyses*, applied to all the various processes of mental arithmetic, the pupil's mind cannot fail to become thoroughly imbued with clear and accurate ideas in respect to each particular topic before he is required to learn, or apply to written examples, any set rule whatever. The intellect of the pupil is thus addressed at every step; and every part of the instruction is made the means of effecting that *mental development* which constitutes the highest aim, as well as the most important result, of every branch of education.

This mode of treatment has not only the advantage of logically training the pupil's mind, and cultivating his powers of calculation, but must also prove a *source* of *economy*, both of *time* and *money*, inasmuch as it is the means of substituting a *single volume* for an entire *series of text-books*.

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As the time of many pupils will not permit them to pursue this study through all of its departments, the work is issued in *two parts,* as well as in a *single volume.* This will, it is thought, be also convenient for graded schools, in supplying a separate book for classes of the higher and lower grades respectively, without requiring any unnecessary repetition or review.

The author feels assured that, on examination, this work will commend itself to teachers and others, by the careful and progressive *grading of its topics,* the clearness and conciseness of its *definitions* and *rules,* its improved methods of *analysis* and *operation,* and the great number and variety of its progressively arranged examples, both *oral* and *written,* embodying and elucidating all the ordinary business transactions. The use of *equations* as a form of expression in these examples will be found to possess many advantages, not only as an arithmetical drill, but also in familiarizing the pupil with the use of algebraic symbols.

All obsolete terms and discarded usages have been studiously ignored, and many novel features introduced, favorable to clearness as well as brevity. The work has been carefully adapted, in other respects, to the present time, recognizing and explaining all the recent changes in *Custom-house Business, Exchange,* etc., and presenting, in connection with the examples for practice under each topic, information not only *fresh* but *important.*

Attention is especially called to the manner in which *United States Money* is introduced in connection with the elementary rules; to the comprehensive treatment of all the various departments of *Percentage,* so essential at the present time in commercial transactions; to the articles on *Measurements* and *Mensuration,* and the vast amount of valuable information given in connection with this part of the subject. In these respects this part of the work will be found to be particularly adapted to the wants of High Schools and Academies, as well as of Mercantile and Commercial Colleges.

The *Reviews* interspersed throughout the book will be found to be just what is needed by the student to make his progress sure at each step, and to give him comprehensive ideas of the subject as he advances. Carefully constructed *Synopses* have also been inserted, with the view to afford to both teacher and pupil a ready means of drill and examination, as well as to present, in a clear, concise, and logical manner, the relations of all the different departments of the subject, with their respective *sub-topics, definitions, principles*, and *rules*.

Great pains have also been taken to make this work superior to all others in its typographical arrangement and finish, and in the general tastefulness of its mechanical execution.

The author takes pleasure in acknowledging his indebtedness for many valuable suggestions received from teachers of experience and others interested in the work of education; and to Henry Kiddle, A. M., late Superintendent of Schools in the city of New York, for valuable assistance, especially in the higher departments of Percentage, and for important suggestions in relation to other parts of the work.

How nearly the author has accomplished his purpose, to give to the public, in *one volume*, a clear, scientific, and *complete* treatise on this subject, combining and systematizing many *real improvements* of practical value and importance to the business man and the student, the intelligent and experienced educator must decide.

D. W. F.

SUGGESTIONS TO TEACHERS

IN order to teach any subject with the best success, the instructor should not only fully understand it, in all its principles and details, but should also clearly perceive what particular faculties of the mind are concerned in its acquisition and use.

Arithmetic is pre-eminently a subject of practical value; that is, it is one to be constantly applied to the practical affairs of life. But this is true only in a limited sense. Very few ever need to apply to any of the purposes of business more than a small part of the principles and rules of calculation taught in the text-books. Every branch of business has its own requirements in this respect, and these are all confined within very narrow limits.

The teaching of arithmetic must, therefore, to a great extent, be considered as *disciplinary*,—as training and developing certain faculties of the mind, and thus enabling it to perform its functions with accuracy and dispatch. The following *suggestions*, having reference to this twofold object of arithmetical instruction are presented to the teacher, as a partial guide, not only in the use of this text-book, but in the treatment of the subject as a branch of education.

Seek to cultivate in the pupil the habit of *self-reliance*. Avoid doing for him anything which, either with or without assistance, he should be able to do for himself. Encourage and stimulate his exertions, but do not supersede them.

Never permit him to accept any statement as true which he does not understand. Let him learn not by authority but by demonstration addressed to his own intelligence. Encourage him to ask questions and to interpose objections. Thus he will acquire that most important of all mental habits, that of *thinking for himself*.

Carefully discriminate, in the instruction and exercises, as to which faculty is addressed,—whether that of *analysis* or *reasoning*, or that of *calculation*. Each of these requires peculiar culture, and each has its appropriate period of development. In the *first stage* of arithmetical instruction, *calculation* should be chiefly addressed, and analysis or reasoning employed only after some progress has been made, and then very slowly and progressively. A young child will perform many operations in calculation which are far beyond its powers of analysis to explain thoroughly.

In the exercise of the calculating faculty, the examples should be rapidly performed, without pause for explanation or analysis; and they should have very great variety, and be carefully arranged so as to advance from the simple and rudimental to the complicated and difficult.

In the exercise of the analytic faculty, great care should be taken that the processes do not degenerate into the mere repetition of *formulæ*. These forms of expression should be as simple and concise as possible, and should be, as far as practicable, expressed in the pupil's own language. Certain necessary points being attended to, the precise form of expression is of no more consequence than any particular letters or diagrams in the demonstration of geometrical theorems. Of course, the teacher should carefully criticise the logic or reasoning, not so as to discourage, but still insisting upon *perfect accuracy* from the first.

The oral or mental arithmetic should go hand in hand with the written. The pupil should be made to perceive that, except for the difficulty in retaining long processes in the mind, all arithmetic ought to be oral, and that the slate is only to be called into requisition to aid the mind in retaining intermediate processes and results. The arrangement of this text-book is particularly favorable for this purpose.

Definitions and principles should be carefully committed to memory. No slovenliness in this respect should be permitted. A definition is a basis for thought and reasoning, and every word which it contains is necessary to its integrity. A child should not be expected to frame a good definition. Of course, the pupil should be required to examine and criticise the definitions given, since this will conduce to a better understanding of their full meaning.

In conducting recitations, the teacher should use every means that will tend to *awaken thought*. Hence, there should be great variety in the examples, both as to their construction and phraseology, so as to prevent all mechanical ciphering according to fixed methods and rules.

The *Rules* and *Formulæ* given in this book are to be regarded as *summaries* to enable the pupil to retain processes previously analyzed and demonstrated. They need not be committed to memory, since the pupil will have acquired a sufficient knowledge of the principles involved to be able, at any time, to construct rules, if he has properly learned what precedes them.

In the higher department of arithmetic, the chief difficulty consists in giving the pupil a clear idea of the *nature* of the business transactions involved. The teacher should, therefore, strive by careful elucidation, to impart clear ideas of these transactions before requiring any arithmetical examples involving them to be performed. When the exact nature of the transaction is understood, the pupil's knowledge of abstract arithmetic will often be sufficient to enable him to solve the problem without any special rule.

The teacher should be careful not to advance too rapidly. The mind needs time to grasp and hold firmly every new case, and then additional time to bring its new acquisition into relation with those preceding it. Hence the need of frequent *reviews*, in order to give the pupil a comprehensive as well as an accurate and permanent knowledge of this subject.

The *Synopses for Review* interspersed throughout this work are designed for this purpose. The whole or a part of a Synopsis, embracing one or more topics, may be placed upon the blackboard, and the pupil required to give briefly but accurately the *subdivisions*, *definitions*, *principles*, etc., involved in each. By this means, if further tested by questions, a thorough and well classified knowledge of the whole subject will be permanently impressed upon his mind.

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N. B.—Editions of this book are bound *with*, and *without*, the answers. The edition *with* answers will be supplied unless otherwise ordered.

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COMPLETE ARITHMETIC

PRELIMINARY DEFINITIONS

ARTICLE 1. Arithmetic is the Science of Numbers, and the Art of Computation.

As a science, Arithmetic treats of the nature and properties of numbers. As an art, it teaches how to apply a knowledge of numbers to practical and business purposes.

2. A ***Unit*** is one, or a single thing; as *one, one* boy, *one* year, *one* dozen.

3. A ***Number*** is a unit, or a collection of units; as *one, three, five* boys; it answers the question, How many?

4. An ***Integral Number*** or ***Integer*** is a number which expresses whole things; as *seven, four* days.

5. The ***Unit of a Number*** is *one* of the collection of units which constitute the number. Thus, the unit of twelve is *one,* of twenty dollars is *one* dollar.

6. A ***Concrete Number*** is a number that is applied to a particular kind of object, or quantity; as three *houses,* four *dollars,* five *minutes.*

7. An ***Abstract Number*** is a number that is not applied to any object; as *four, seven, eight.*

8. ***Like Numbers*** are such as have the same kind of unit, or express the same kind of quantity. They may be either concrete or abstract; as *eight* and *nine*, six *days* and ten *days*, two *rods* five *feet*, and five *rods* three *feet*.

9. ***Unlike Numbers*** are such as have different kinds of units, or express different kinds of quantity; as ten *months* and eight *miles*; seven *dollars* and five *barrels*.

10. A ***Scale*** in Arithmetic, is a succession of units, increasing and decreasing according to a certain law, or rule. Scales are *uniform* or *varying*.

11. A ***Uniform Scale*** is one in which the law of increase and decrease is the same throughout the entire succession of units.

12. A ***Varying Scale*** is one in which the law of increase and decrease is not the same throughout the entire succession of units.

13. A ***Decimal Scale*** is one in which the law of increase and decrease is *uniformly* *TEN.*

EXERCISES.

14. 1. How many units in two? In five cents? In six dollars? In seven acres?

2. What is the unit of six cents? Of nine books?

3. Are two trees and five trees like or unlike numbers?

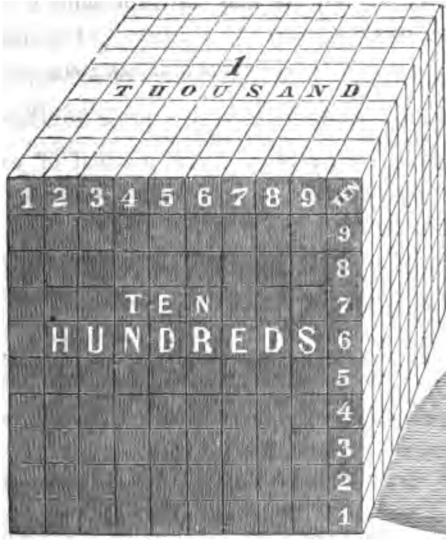
4. Are they concrete or abstract? Why?

5. What kind of numbers are seven and nine? Are five acres and seven cords? Are four coats and six coats?

6. Name two numbers that are *like* and *abstract*.

7. Name two numbers that are *like* and *concrete*.

8. Name three numbers that are *unlike* and *concrete*.



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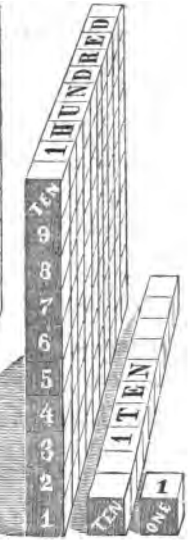


ONE HUNDRED AND ELEVEN.

NOTATION AND NUMERATION.

15. In representing numbers, objects are regarded as arranged in *groups* of *tens;* hence we have single things, or *units;* next, groups containing *ten* units, or *ten;* next, groups containing *ten* tens, or *one hundred;* and again, groups containing *ten* hundreds, or *one thousand,* etc.

16. This method of grouping is called the *Decimal System,* from the Latin word *decem,* which signifies *ten.*

17. ***Notation*** is a method of *writing,* or representing numbers by characters.

18. ***Numeration*** is a method of *reading* numbers represented by characters.

19. The number of objects may be represented by *words*, or by *characters.*

20. The characters may be either *figures* or *letters.*

21. ***Figures*** are characters used to express numbers.

22. The ***Arabic Notation*** is the method of expressing numbers by *figures.* It is so called because it was invented by the Arabs.

23. This method employs *ten* different characters, or figures, to represent numbers, viz.:

Figures.	0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Names.	Naught,	One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine.

24. The first character, or *cipher,* is called *Naught,* or *Zero,* and when standing alone, has no value.

The other nine are called *significant* figures, because each has a value of its own. They are also called *digits.*

These ten characters, when combined according to certain principles, can be made to express any number.

25. The *first nine* numbers are each represented by a *single figure,* and are called *units* of the *first order.*

26. By grouping ten ones, or *units* of the *first* order into a larger collection, there is formed a unit of the *second* order, called *ten,* which is represented by writing the figure 1 with a cipher after it; thus, 10.

27. In the same manner are represented

Two tens, or Twenty,	by	20	Six tens, or Sixty,	by	60
Three tens, or Thirty,	"	30	Seven tens, or Seventy,	"	70
Four tens, or Forty,	"	40	Eight tens, or Eighty,	"	80
Five tens, or Fifty,	"	50	Nine tens, or Ninety,	"	90

28. The numbers between *ten* and *twenty* are represented by writing 1 in the *second* place, and the units in the *first* place. Thus,

Eleven	11	Fourteen	14	Seventeen	17
Twelve	12	Fifteen	15	Eighteen	18
Thirteen	13	Sixteen	16	Nineteen	19

29. In like manner, the numbers between 20 and 30 are represented, thus, 11 1 5

Twenty-one	21	Twenty-four	24	Twenty-seven	27
Twenty-two	22	Twenty-five	25	Twenty-eight	28
Twenty-three	23	Twenty-six	26	Twenty-nine	29

30. The greatest number that can be expressed by *two* figures is 99. 31

31. By grouping ten units of the *second order,* or *ten tens,* into a larger collection, there is formed a unit of the *third* order, called a *hundred,* represented by writing the figure 1 with two ciphers after it; thus, 100. 7 10

32. In like manner are represented

Two hundred	by 200	Six hundred	by 600
Three hundred	" 300	Seven hundred	" 700
Four hundred	" 400	Eight hundred	" 800
Five hundred	" 500	Nine hundred	" 900

33. The numbers from one hundred, to nine hundred and ninety-nine, are represented by writing the *hundreds* in the *third* place, the *tens* in the *second* place, and the *units* in the *first* place.

34. The greatest number that can be expressed by *three* figures is 999.

35. ***Orders of Units*** are denoted by the *position* of the figures used in expressing a number.

Thus, 532 represents 2 units of the *first order,* 3 units of the *second order,* or 3 *tens,* and 5 units of the *third order,* or 5 *hundreds,* and is read five hundred and thirty-two.

36. PRINCIPLES.—1. *Ten units of any order in a number make one unit of the next higher order.*

2. *When any order of units in a number is vacant, the place is filled with a cipher.*

EXERCISES.

37. Express the following numbers by figures

1. One hundred twenty.
2. Four hundred eighty.
3. Seven hundred six.
4. Five hundred seven.
5. Seven hundred.
6. Three hundred eight.
7. Six hundred ninety.
8. Eight hundred five.
9. Seven hundred ten.
10. Six hundred eleven.
11. Nine hundred seven.
12. Two hundred sixty.

38. Copy and read the following, and name the number of *hundreds, tens,* and *units* in each :

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
67	321	190	840	592	219
85	406	761	269	904	807
77	289	345	793	531	395
98	672	402	503	762	608

39. By grouping ten units of the *third* order, or *ten hundreds,* into a larger collection, there is formed a unit of the *fourth* order, called a *thousand,* represented by writing the figure 1 with three ciphers after it ; thus, 1000.

40. In like manner are represented

Two thousand	by 2000	Six thousand	by 6000
Three thousand	" 3000	Seven thousand	" 7000
Four thousand	" 4000	Eight thousand	" 8000
Five thousand	" 5000	Nine thousand	" 9000

41. The numbers from one thousand, to nine thousand nine hundred and ninety-nine, are represented by writing *thousands* in the *fourth* place, *hundreds* in the *third* place, *tens* in the *second* place, and *units* in the *first* place.

Thus, 5304 represents 4 units of the *first* order, 0 units of the *second* order, or *tens*, 3 units of the *third* order, or *hundreds*, and 5 units of the *fourth* order, or *thousands*, and is read five thousand three hundred and four.

42. The greatest number that can be expressed by four figures is 9999.

43. In the same manner, other *new* orders are formed to represent larger numbers, by grouping *ten* units of the *fourth* order to form the *fifth* order, or *tens* of thousands ; and *ten* units of the *fifth* order, to form the *sixth* order, or *hundreds* of thousands, etc.

Thus, 432076 represents 6 units of the *first* order, 7 units of the *second* order, 0 units of the *third* order, 2 units of the *fourth* order, 3 units of the *fifth* order, and 4 units of the *sixth* order, and is read four hundred thirty-two thousand and seventy-six.

From the preceding illustrations it is obvious, that

44. Moving a figure one place to the *left*, *increases* its *representative* value *tenfold ;* and,

45. Moving a figure one place to the *right*, *diminishes* its *representative* value *tenfold.*

EXERCISES.

46. Write in figures and read :

1. Two units of the third order, four units of the second order, and three units of the first order.

2. Five units of the fourth order, six units of the third order, and two units of the second order.

3. Seven units of the fourth order, eight of the second order, and three of the first.

4. One unit of the third order and four of the second.

5. Three units of the fifth order, two of the third, and one of the first.

6. Eight units of the fourth order, and five of the second.

7. Two units of the sixth order, nine of the fifth, fou of the third, one of the second, and seven of the first.

47. Express the following numbers by figures :

1. Thirty-seven thousand.
2. Sixteen thousand one hundred.
3. Twelve thousand five hundred fifty.
4. Forty-nine thousand five hundred twenty-seven.
5. Fifteen thousand two hundred six.
6. Seventeen thousand twenty-four.
7. Sixty thousand six hundred eight.
8. Seven hundred twenty thousand.
9. Two hundred forty thousand five hundred.

48. Copy, and read the following, naming the number of units of *each order :*

(1.)	(2.)	(3.)	(4.)	(5.)
1542	1020	32507	76387	528031
3473	1256	53106	627324	600320

49. This method of numeration groups the successive orders into ***periods*** of *three* figures each. The periods are commonly separated by commas, each period taking the name of its lowest order, as shown in the following

NUMERATION TABLE.

PERIODS.	6th.	5th.	4th.	3d.	2d.	1st.
NAME.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
ORDERS OF UNITS IN THE PERIODS.	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units
NUMBER.	3 0,	2 9 1,	0 4 0,	0 2 7,	3 0 6,	4 0 0

The number is read 30 *quadrillion*, 291 *trillion*, 40 *billion*, 27 *million*, 306 *thousand*, 400.

1. In *reading* numbers, the name of the units period is omitted.
2. Every period except the highest must contain *three* figures.

50. The names of the periods above Quadrillions are :

PERIODS.	NAMES.	PERIODS.	NAMES.
7th	Quintillions.	15th	Tredecillions.
8th	Sextillions.	16th	Quatuordecillions.
9th	Septillions.	17th	Quindecillions.
10th	Octillions.	18th	Sexdecillions.
11th	Nonillions.	19th	Septendecillions.
12th	Decillions.	20th	Octodecillions.
13th	Undecillions.	21st	Novendecillions.
14th	Duodecillions.	22d	Vigintillions.

51. The pupil may be required to prepare and arrange on the slate or on paper, exercises similar to the following.

Bill.			Mill.			Thou.			Units.		
H.	T.	U.	H.	T.	U.	H.	T.	U.	H.	T.	U.
									3	4	1
						1	2	5	0	0	4
					4	0	4	4	0	3	4
				3	3	3	0	0	3	3	0
			2	0	0	2	2	0	0	2	2
		5	0	0	5	5	0	0	0	5	5

The first *example* is read, 341.

The second, is read, 125 *thousand* and 4.

The third, is read, 4 *million* 44 *thousand* and 34.

The fourth, is read, 33 *million* 300 *thousand* 330.

The diagram may be prepared at first for only two or three periods, and may be gradually enlarged to five or six periods.

Each pupil may be allowed to dictate an example, to be written and read by the whole class.

52. RULE FOR NOTATION.—*Begin at the left, and write the hundreds, tens, and units of each period in their proper order, filling all vacant places and periods with ciphers.*

53. RULE FOR NUMERATION.—I. *Begin at the right, and separate the number into periods of three figures each.*

II. *Begin at the left, and read each period as if it were units, giving its name.*

EXERCISES IN NOTATION AND NUMERATION.

54. Write in figures and read:

1. Six units of the 3d order, five of the 2d, and four of the first.

2. Five units of the 4th order, seven of the 2d, and six of the 1

3. Eight units of the 4th order and four of the 2d.

4. Five units of the 4th order and eight of the 2d.

5. Three units of the 5th order, six of the 4th, four of the 3d, and seven of the 1st.

6. Two units of the 6th order, four of the 5th, nine of the 4th, three of the 3d, and five of the 1st.

7. Three units of the 9th order, eight of the 7th, four of the 6th, six of the 5th, and nine of the 1st.

55. Write and read the following numbers in figures:

1. Twenty-five units in the 2d period, and four hundred ninety-six in the 1st. *Ans.* 25,496.

2. Four hundred thirty-six units in the 4th period, twelve in the 3d, one hundred in the 2d, and three hundred and one in the 1st.

3. Eighty-one units in the 5th period, two hundred and nineteen in the 4th, and fifty-six in the 2d.

4. Nine hundred and forty units in the seventh period, eighteen in the fifth, and one hundred and three in the 3d.

56. Express the following numbers by figures:

1. Twenty-six thousand twenty-six.
2. Fourteen thousand two hundred eighty.
3. One hundred seventy-six thousand.
4. Four hundred fifty thousand thirty-nine.
5. Seven million thirty-six.
6. Five hundred sixty-three thousand four.
7. One million ninety-six thousand.
8. Ten million ten thousand ten hundred ten.
9. Four hundred eighty-three million eight hundred sixteen thousand one hundred forty-nine.
10. Ninety-nine billion thirty-seven thousand four.

Point off and read the following numbers

1. 24835.
2. 2474783.
3. 31628045.
4. 247843112.
5. 100103.
6. 53000008.
7. 406270035.
8. 3730016000.

55. ***Roman Notation*** employs seven capital letters to express numbers. Thus,

Letters.	I,	V,	X,	L,	C,	D,	M.
Values.	1,	5,	10,	50,	100,	500,	1000.

When used *alone,* each letter has its fixed value.

Numbers may be expressed by combining these letters according to the following principles :

1. Repeating a letter repeats its value.

Thus, XX represents 20 ; CCC, 300 ; DD, 1000.

2. When a letter is placed *after* one of greater value, its value is to be *added* to that of the greater.

Thus, VI represents 6 ; XV, 15 ; LXX, 70 ; DC, 600.

3. When a letter is placed *before* one of greater value, its value is *taken from* that of the greater.

Thus, IV represents 4 ; IX, 9 ; XL, 40 ; XC, 90.

4. When a letter of any value is placed *between* two letters, each of greater value, its value is *taken from* the *sum* of the other two.

Thus, XIV represents 14 ; LIX, 59 ; CXL, 140.

5. A bar or dash placed over a letter increases its value *one thousand times.*

Thus, $\overline{\text{X}}$ represents 10000 ; $\overline{\text{XC}}$, 90000 ; $\overline{\text{DL}}$, 550000.

TABLE OF ROMAN NOTATION.

I	=	1	XVI	=	16	C	=	100
II	=	2	XVII	=	17	CXIX	=	119
III	=	3	XVIII	=	18	CC	=	200
IV	=	4	XIX	=	19	CCX	=	210
V	=	5	XX	=	20	D	=	500
VI	=	6	XXI	=	21	DCV	=	605
VII	=	7	XXV	=	25	M	=	1000
VIII	=	8	XXX	=	30	MDL	=	1550
IX	=	9	XXXIV	=	34	MDCLXVI	=	1666
X	=	10	XL	=	40	MXCIX	=	1099
XI	=	11	L	=	50	$\overline{\mathrm{XXV}}$	=	25000
XII	=	12	LX	=	60	$\overline{\mathrm{CXX}}$	=	120000
XIII	=	13	LXX	=	70	$\overline{\mathrm{CLXIV}}$	=	164000
XIV	=	14	LXXX	=	80	$\overline{\mathrm{D}}\overline{\mathrm{L}}\mathrm{CXL}$	=	550140
XV	=	15	XC	=	90	$\overline{\mathrm{M}}\mathrm{DXC}$	=	1000590

MDCCCLXXX = 1880, one thousand eight hundred and eighty.

EXERCISES.

56. Express by Roman notation :

1. Twenty-seven.
2. Forty-nine.
3. Seventy-three.
4. Sixty-eight.
5. Eighty-four.
6. Ninety-seven.
7. One hundred ten.
8. Five hundred fifty.
9. Seven hundred forty.
10. Nine hundred ninety.
11. Sixteen hundred.
12. Fifty thousand five.
13. 318.
14. 796.
15. 1069.
16. 25000.
17. 59300.
18. 87040.

57. Express by Arabic notation :

1. LXVII.
2. XCLXIV.
3. CXXXV.
4. CCXLIX.
5. MXIX.
6. DCLIII.
7. $\overline{\mathrm{C}}\mathrm{XCIX}$.
8. $\overline{\mathrm{V}}\mathrm{DLIX}$.
9. $\overline{\mathrm{DLX}}$.
10. $\overline{\mathrm{XXXI}}\mathrm{D}$.
11. $\overline{\mathrm{LIX}}\mathrm{CCCXLIV}$.
12. $\overline{\mathrm{XV}}\mathrm{DCCXLIX}$.
13. $\overline{\mathrm{M}}\mathrm{MMMXC}$.
14. $\overline{\mathrm{V}}\mathrm{MDCCXLIX}$.
15. $\mathrm{MD}\overline{\mathrm{XXV}}\mathrm{CDLXXXIX}$.

58. SYNOPSIS FOR REVIEW.

PRELIMINARY DEFINITIONS.

1. Arithmetic. 2. A Unit. 3. A Number. 4. An Integral Number. 5. Unit of a Number. 6. A Concrete Number. 7. An Abstract Number. 8. Like Numbers. 9. Unlike Numbers. 10. A Scale. 11. A Uniform Scale. 12. A Varying Scale. 13. A Decimal Scale.

NOTATION AND NUMARATION.

- 1. **Definitions.**
  - 1. Decimal System.
  - 2. Notation.
  - 3. Numeration.
  - 4. Figures.
  - 5. Arabic Notation.
- 2. **Mode of Representing**
  - 1. Units of the first order.
  - 2. " " second "
  - 3. " " third "
  - 4. " " fourth "
  - 5. Numbers between ten and twenty; between twenty and thirty.
  - 6. Other orders and numbers.
- 3. **Order of Units**—how denoted.
- 4. **Principles**, 1 and 2.
- 5. **Value of Figures.**
  - 1. How increased.
  - 2. How diminished.
- 6. **Periods.**
  - 1. Of how many figures.
  - 2. How separated.
  - 3. Names of Periods to Quadrillions.
  - 4. " " " beyond "
- 7. **Rules.**
  - 1. For Notation.
  - 2. For Numeration.
- 8. **Roman Notation.**
  - 1. How expressed.
  - 2. How many letters.
  - 3. Principles 1, 2, 3, 4, and 5.
  - 4. Table.

ADDITION

ORAL EXERCISES.

59. 1. A man gave 5 dollars for a hat, and 9 dollars for a vest. How many dollars did he pay for both?

2. How many miles are 4 miles and 12 miles?

3. How many are 6 men and 14 men? 15 and 7?

4. How many are 14 and 5? 29 and 5? 24 and 5?

5. How many are 15 and 6? 21 and 6? 27 and 6?

6. How many are 7 and 12? 7 and 19? 7 and 26?

7. What kind of numbers are 9 pounds and 14 pounds?

8. Can 9 balls and 12 books be added? Why not?

9. Can 7 rods and 10 rods be added? Why?

10. How many are 4, 5, and 7? 5, 7, and 4? 4, 7, and 5?

11. In a shop are 15 men, 8 boys, and 6 girls at work. How many persons are at work in the shop?

12. I gave 7 cents to one boy, 9 to another, and 6 to another. How many cents did I give to all?

13. Add by 2's from 1 to 31.

Operation.—1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31.

Add

14. By 2's from 4 to 50.
15. By 3's from 1 to 43.
16. By 3's from 6 to 51.
17. By 4's from 1 to 53.
18. By 4's from 5 to 45.
19. By 5's from 1 to 61.
20. By 5's from 7 to 82.
21. By 6's from 0 to 72.
22. By 6's from 2 to 80.
23. By 6's from 10 to 88.

24. Add by 2's and 3's alternately from 1 to 31.

OPERATION.—1, 3, 6, 8, 11, 13, 16, 18, 21, 23, 26, 28, 31.

25. Add by 2's and 4's alternately from 1 to 49.

26. Add by 3's and 4's alternately from 0 to 56.

27. Add by 2's and 5's alternately from 4 to 60.

28. Add by 3's and 5's alternately from 7 to 63.

DEFINITIONS AND PRINCIPLES.

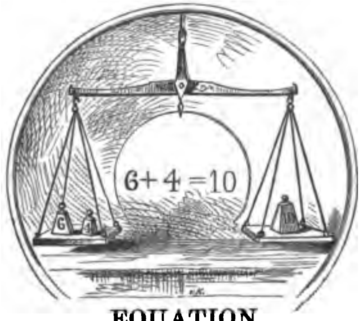
60. ***Addition*** is the process of finding a number equivalent to two or more numbers.

61. The ***Sum*** or ***Amount*** is the number obtained by addition.

62. The ***Sign of Addition*** is +. It is read *plus*, and signifies *more;* thus, 5 + 6 is read, 5 *plus* 6, and means that 5 and 6 are to be *added.*

63. The ***Sign of Equality*** is =. It is read *equals*, or *equal to;* thus, 5 + 6 = 11, is read 5 *plus* 6 *equals* 11. It may be read 5 *and* 6 *are* 11.

64. An ***Equation*** is an expression of *equality* between two numbers or sets of numbers.



EQUATION.

All that is written *before* the sign of *equality* is called the *first member* of the equation, all that is written *after* the sign of equality is called the *second member.*

The numbers in each member are called the *terms* of the equation.

Thus, $6 + 4 = 10$, is an *equation*, and is read 6 *plus* 4 *equals* 10, and means that the sum of 6 and 4 is equal to 10. $6 + 4$ is the *first member* of the equation, and 10 is the *second member*; and 6, 4, and 10 are the *terms* of the equation.

65. Name the *members* and the *terms* of each of the following equations.

1. $9+12=21$
2. $14+10=24$
3. $26+9=35$
4. $44+12=56$
5. $14+40=44+10$
6. $18+20=31+7$

66. PRINCIPLE.—*Only like numbers and units of the same order can be added.*

EXERCISES.

67. The teacher should read the *first* member of the equation, and the pupil be required, as promptly as possible, to give the *second* member.

The expression "$=?$" is read *equals how many,* or *what.*

$6+4+5=?$
$7+0+2=?$
$8+3+1=?$
$9+2+4=?$
$7+6+5=?$
$10+8+3=?$

$3+12+6=?$
$6+4+10=?$
$2+11+5=?$
$8+9+3=?$
$3+12+7=?$
$14+7+6=?$

$12+5+10=?$
$15+2+4=?$
$13+7+8=?$
$16+5+6=?$
$18+0+10=?$
$20+6+8=?$

$5+8+9=?$
$9+8+5=?$
$8+5+9=?$
$10+7+6=?$
$6+10+7=?$
$11+5+8=?$
$15+3+10=?$

$20+7+3=?$
$8+12+10=?$
$11+9+1=?$
$12+7+9=?$
$21+10+7=?$
$24+6+5=?$
$10+25+9=?$

$4+15+10=?$
$8+8+8=?$
$9+9+9=?$
$23+10+6=?$
$7+21+11=?$
$14+6+12=?$
$26+10+9=?$

WRITTEN EXERCISES.

68. When the sum of the units of each order is less than 10.

1. What is the sum of 421, 44, 303, and 230?

OPERATION.

$$\begin{array}{r}421\\44\\303\\230\\\hline 998\end{array}\text{ Sum.}$$

ANALYSIS.—Arrange the numbers so that the units of the same order stand in the same column.

Begin with the lowest order of units, and add each column separately; and instead of saying, 3 units and 4 units are 7 units, and 1 unit are 8 units, pronounce the successive units *only;* thus, 3, 7, 8, the sum of the *units,* which write in the units' place.

Next, 3, 7, 9, the sum of the *tens,* which write in the tens' place. Lastly, 2, 5, 9, the sum of the *hundreds,* which write in the hundreds' place. Hence the *sum* is 998.

PROOF.—Add the columns in the reversed direction. If the two results agree, the work is probably correct.

Copy, add, and prove,

(2.)	(3.)	(4.)	(5.)
204	312	241	403
462	243	520	2052
23	124	27	4324

6. I paid 3104 dollars for a house, 450 dollars for repairs, and 234 dollars for painting. What was the whole cost?

69. The ***Sign of Dollars*** is $. It is read *dollars.* Thus, $35 is read 35 *dollars;* $9 is read 9 *dollars.*

70. When *dollars* and *cents* are written, a period or point (.) is placed before the cents, or between the dollars and cents. Thus, $7.25 is read 7 dollars and 25 cents.

71. Since 100 cents make $1.00, *cents* always occupy *two* places, and never more than two.

72. If the number of cents is less than 10 and expressed by a single figure, a *cipher* must occupy the first place at the right of the point. Thus, 3 dollars 6 cents are written \$3.06; 1 dollar 5 cents are written \$1.05.

73. When cents alone are written, and their number is less than 100, either write the *word* cents *after* the number, or place the dollar sign and the point *before* the number. Thus, 75 cents may be expressed, \$.75.

74. In arranging for addition, dollars should be written under dollars, and cents under cents, in such order that the *points* stand in a *vertical* line.

The *sign* \$, and the *point* (.) should never be omitted.

75. Read the following equations:

1. \$12. + \$8. = \$20.
2. \$25. + \$10. = \$35.
3. \$3.25 + \$6.75 = \$10.
4. \$.75 + \$.20 = \$.95.
5. \$.60 + \$.40 = \$1.00.
6. \$14.08 + \$3.14 = \$17.22.

76. Express the following by proper figures and signs:

1. Nine dollars and thirty cents.
2. Thirty dollars and ten cents.
3. Eighty-four cents.
4. Seventy-eight cents.
5. Six dollars and sixteen cents.
6. 7 dollars and 26 cents.
7. 9 dollars and 5 cents.
8. 19 dollars and 7 cents.
9. 69 cents; 23 cents.
10. 10 cents; 6 cents.

The teacher may exercise the class *orally*, by dictating rapidly, but distinctly, similar examples. Thus, *Sign, five, three?* The prompt response should be, "Fifty-three dollars" (\$53). *Ques. Sign, point, seven, four?* *Ans.* Seventy-four cents (\$.74). *Ques. Sign, point, naught, eight?* *Ans.* Eight cents (\$.08), etc.

Also, the converse; thus, *Ques.* "Forty-five dollars" (\$45)? *Ans. Sign, four, five.* *Ques.* Fifty-six cents (\$.56)? *Ans. Sign, point, five, six.* *Ques.* Nine dollars seven cents (\$9.07)? *Ans. Sign, nine, point, naught, seven,* etc.

77. Copy and add,

(1.)	(2.)	(3.)	(4.)
\$3.04	\$24.12	\$105.	\$200.35
2.21	3.06	32.14	46.41
.53	12.	.73	1.02

5. What is the sum of \$.25, \$3.31, \$14.02, and \$21.

6. What is the sum of ten dollars and twenty cents, four dollars and fifteen cents, forty-three cents, and thirteen dollars?

7. Bought a horse for \$154, and sold him for \$35.75 more than he cost. For how much did I sell him?

8. A lady paid 12 dollars for a scarf, 3 dollars and 25 cents for a fan, two dollars for a pair of gloves, and 42 cents for a collar. How much did she pay for all?

It is not the design to teach here, the *principles* and *reductions* of Decimal Currency, fully taught in another place, but to give a few hints and illustrations, to enable the teacher, by oral instruction and simple written exercises, to make the pupil familiar with the *use* of decimal currency in common business matters.

ORAL EXERCISES.

78. 1. How many are 7 and 6? 13 and 6? 19 and 6? 25 and 6? 31 and 6? 42 and 6?

2. How many are 9 and 7? 16 and 7? 23 and 7?

Add

3. By 7's from 1 to 71.
4. By 7's from 3 to 87.
5. By 8's from 0 to 96.
6. By 8's from 6 to 102.
7. By 9's from 2 to 92.
8. By 9's from 10 to 109.
9. By 10's from 0 to 120.
10. By 10's from 13 to 153.
11. By 11's from 1 to 100.
12. By 11's from 4 to 92.
13. By 12's from 0 to 144.
14. By 12's from 3 to 135.

Add rapidly the following:

15. 4, 6, 5, 3, and 7.
16. 6, 4, 8, 2, and 5.
17. 10, 9, 5, 3, and 6.
18. 7, 3, 10, 9, and 8.
19. 14, 5, 3, 6, and 10.
20. 13, 5, 6, 10, and 3.
21. 12, 10, 2, 0, and 9.
22. 27, 3, 10, 8, and 7.
23. 36, 12, 7, 4, and 10.
24. 11, 12, 10, 9, and 8

Add by repeating the numbers,

25. 2, 3, 4, 2, 3, 4, 2, 3, 4, till the sum = 63.
26. 3, 4, 5, 3, 4, 5, 3, 4, 5, till the sum = 84.
27. 2, 4, 6, 2, 4, 6, 2, 4, 6, till the sum = 96.

Add alternately,

28. 5, 6, 5, 6, 5, 6, 5, 6, till the sum = 88.
29. 6, 4, 6, 4, 6, 4, 6, 4, 6, 4, till the sum = 100.
30. 7, 5, 7, 5, 7, 5, 7, 5, 7, 5, till the sum = 120.
31. 8, 9, 8, 9, 8, 9, 8, 9, 8, 9, till the sum = 119.

32. What is the sum of 46 and 27?

Analysis.—46 is 4 tens and 6 units, and 27 is 2 tens and 7 units; 4 tens and 2 tens are 6 tens, and 6 units and 7 units are 13 units, or 1 ten and 3 units, which added to 6 tens make 7 tens and 3 units, or 73.

2
3
7

33. 36 + 42 = ?
34. 53 + 38 = ?
35. 65 + 40 = ?
36. 54 + 38 = ?
37. 29 + 61 = ?
38. 38 + 37 = ?
39. 44 + 37 = ?
40. 72 + 25 = ?
41. 63 + 54 = ?

42. James earned 44 cents one day, and 52 cents the next. How many cents did he earn in both days? ?

43. Bought a pound of coffee for 35 cents, a pound of butter for 28 cents, and a pound of sugar for 12 cents. What was the cost of the whole? 10 ?

44. A lady bought a silk dress for $28, a shawl for $16, and had $14 left. How much money had she at first?

WRITTEN EXERCISES.

79. When the sum of the units of any order equals or exceeds 10.

1. What is the sum of 467, 536, 84, and 705?

OPERATION.
467
536
84
705
1792 Sum.

ANALYSIS.—Arranging the numbers as before, begin at the right hand and add the column of units; thus, 5, 9, 15, 22 *units*, equal to 2 tens and 2 units. Write the 2 units in the units' place, and reserve the 2 tens to add to the next column.

Next, adding the 2 tens reserved, to the column of tens, say, 2, 10, 13, 19 *tens*, equal to 1 hundred and 9 tens. Write the 9 tens in the tens' place, and reserve the 1 hundred to add to the next column.

Lastly, adding the 1 hundred reserved, say 1, 8, 13, 17 *hundreds*, equal to 1 thousand, and 7 hundreds, which write in hundreds' and thousands' places. Hence the *sum* is 1792.

In like manner, copy, add, and prove,

(2.)	(3.)	(4.)	(5.)
276 miles.	876 feet.	$20.30	$145.24
307 "	94 "	7.56	36.60
638 "	142 "	13.08	105.08
425 "	507 "	25.	.75

6. Find the sum of $370.21, $2.49, $3.07, and $.94.

7. Find the sum of 2008, 1400, 706, 300, and 77.

8. If 4 loads of coal weigh respectively 1922, 1609, 2100, and 1873 pounds, what is the entire weight?

RULE.—I. *Write the numbers so that figures of the same order stand in the same column.*

II. *Beginning at the right, add each column separately, and write the sum, if expressed by one figure, under the column added.*

III. *If the sum of any column consists of two or more figures, write the unit figure under that column, and add the remaining figure or figures to the next column.*

Proof.—*Add each column in the reverse direction. If the results agree, the work is probably correct.*

9. The Duke of Wellington's army at Waterloo consisted of 26661 infantry, 8735 cavalry, 6877 artillery, and 33413 allies. What was the whole number of his army?

10. Napoleon's army at Waterloo was composed of infantry 48950, cavalry 15765, and artillery 7732. What was the whole number of his army?

11. Gave $325 for a horse, $275.50 for a carriage, $75.75 for a harness, and $20.62 for a robe. What was the cost of the whole?

12. Bought a pair of boots for $8.50, an umbrella for $3.62, a pair of gloves for $1.25, some collars for $.75, and a hat for $4. What was the whole cost?

13. A lady gave $48.50 for silk for a dress, $16.75 for the trimmings, and $15.62 for making. What was the cost of the dress?

(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
$99.84	96256	$117.76	98304	1728	$675.84
24.96	6016	29.44	6144	864	168.86
6.24	376	7.36	384	108	10.56
1.56	141	1.84	24576	81	1.32
12.48	188	3.68	3072	5296	.96
.98	1504	58.88	144	3456	2.64
3.12	752	1.38	49152	432	84.48

20. What is the sum of 5736 dollars and 45 cents, 1000 dollars and 80 cents, 405 dollars and 15 cents, 50 dollars and 9 cents, and 79 cents?

21. Find the sum of twenty-five hundred dollars, 420 dollars and 47 cents, $23 and fifty cents, $600, and ten dollars and eight cents.

22. 1 million 400 thousand and 50 + 15 hundred + 25 thousand + 120 thousand 6 hundred and 14 = ?

23. Paid $3456 for a house, $426.75 for painting it, $2809.48 for furniture. What was the cost of the whole?

24. North America has an area of 8825537 square miles, South America 6954131 square miles, and the West Indies 93810 square miles. What is the area of the entire American Continent?

25. A man owns farms valued at $62500, city lots worth $10260, a house worth $21300, and other property to the amount of $10500. What is the total value of his property?

80. SYNOPSIS FOR REVIEW.

ADDITION.

- 1. Definitions.
  - 1. Addition. 2. Sum or Amount. 3. Sign of Addition. 4. Sign of Equality. 5. An Equation. 6. Members and Terms of an Equation.
- 2. Principles, 1 and 2.
- 3. Addition of Dollars and Cents.
  - 1. Sign of Dollars.
  - 2. Use of the Period.
  - 3. Number of places for cents.
  - 4. Mode of expressing cents.
  - 5. How to arrange for Addition.
- 4. Rule, I, II, III.
- 5. Proof.

SUBTRACTION

ORAL EXERCISES.

81. 1. If John is 15 years old and George is 6, what is the difference in their ages?

2. How many are 16 cents less 7 cents?

3. How many are 18 dollars less 5 dollars?

4. How many are 14 less 6? 16 less 4? 12 less 5?

5. How many are 18 less 8? 20 less 6? 21 less 4?

6. Five balls taken from 11 balls leave how many?

7. Six cents from 20 cents leave how many?

8. What kind of numbers are 10 days and 6 pounds?

9. Can 6 miles be taken from 15 acres? Why not?

10. Can 8 dollars be taken from 18 dollars? Why?

11. How many are 7 less 5? 17 less 5? 27 less 5?

12. How many are 9 less 6? 19 less 6? 29 less 6?

13. What number added to 8 will make 12?

14. What number and 9 make 13? 14? 15? 16?

15. Subtract by 2's from 24 to 0.

Operation.—24, 22, 20, 18, 16, 14, 12, 10, 8, 6, 4, 2, 0.

In the same manner, subtract

16. By 2's from 25 to 1.
17. By 2's from 31 to 3.
18. By 3's from 30 to 0.
19. By 3's from 37 to 1.
20. By 3's from 40 to 4.
21. By 4's from 44 to 0.
22. By 4's from 41 to 1.
23. By 4's from 51 to 3.
24. By 5's from 60 to 0.
25. By 5's from 63 to 3.
26. By 6's from 66 to 0.
27. By 6's from 65 to 5.

28. Count by 4's from 2 to 58, and back from 58 to 2.
29. Count by 5's from 1 to 61 and back to 1.
30. Count by 6's from 3 to 69 and back to 3.
31. Count by 4's from 5 to 53 and back to 5.
32. Count by 6's from 7 to 67 and back to 7.

DEFINITIONS.

82. ***Subtraction*** is the process of finding the difference between two numbers.

83. The ***Minuend*** is the greater of the two numbers.

84. The ***Subtrahend*** is the smaller of the two numbers.

85. The ***Difference*** or ***Remainder*** is the result obtained by subtracting.

86. The ***Sign of Subtraction*** is —. It is read *minus*, and signifies *less*.

When placed between two numbers, it indicates that the one *after* it is to be subtracted from the one *before* it. Thus, $12 - 7$ is read 12 *minus* 7, and means that 7 is to be *subtracted* from 12.

87. A ***Parenthesis*** ( ) is used to include within it such numbers as are to be considered together. A ***Vinculum*** $\overline{}$ has the same signification. Thus, $25 - (12 + 7)$, or $25 - \overline{12 + 7}$, signifies that from 25 the sum of 12 and 7 is to be subtracted.

88. PRINCIPLES.—1. *Only like numbers and units of the same order can be subtracted, the one from the other.*

2. *The minuend must be equal to the sum of the subtrahend and remainder.*

EXERCISES.

89. To be given in the same manner as those in Art. **67.**

14—9=?
15—6=?
22—8=?
13—7=?

24— 9=?
11— 6=?
17—12=?
23— 9=?

21—12=?
18—10=?
22— 8=?
19— 7=?

21— 9=?
26— 7=?
20—12=?
25—11=?

16+ 7—10=?
20+ 9— 7=?
23+ 5—12=?
24+ 6—11=?

19—12+11=?
22—11+15=?
26—10+14=?
29— 8+ 6=?

What is the difference between

17 and 4+6?
24 and 9+5?
18 and 7+7?

20 and 6+ 6?
35 and 9+20?
28 and 9+ 9?

12+ 5 and 24+3?
27— 8 and 14+8?
30—10 and 9+8?

Similar dictation exercises may be given by the teacher.

WRITTEN EXERCISES.

90. When each figure of the subtrahend is not greater than the corresponding figure of the minuend.

1. From 798 subtract 563.

OPERATION.
Minuend 7 9 8
Subtrahend 5 6 3
Remainder 2 3 5

ANALYSIS.—Write the less number under the greater, so that units of the same order stand in the same column.

Begin at the right, and subtract each order of units separately; thus, 3 units from 8 units leave 5 units, which write in the units' place; 6 tens from 9 tens leave 3 tens, which write in the tens' place; 5 hundreds from 7 hundreds leave 2 hundreds, which write in the hundreds' place. Hence the remainder is 235.

The remainder 235 added to the subtrahend 563 equals 798, the minuend. Hence the work is correct. (PRIN. 2.)

UNIVERSITY

Copy, subtract, and prove,

	(2.)	(3.)	(4.)	(5.)
Minuend	426	573	784	837
Subtrahend	214	321	434	315

	(6.)	(7.)	(8.)	(9.)
From	624 feet.	795 tons.	864 men.	$976
Take	211 "	352 "	413 "	$525

91. Before subtracting dollars and cents, the numbers must be written as in Addition, in such order that the points will stand in the same *vertical* line.

In like manner, subtract and prove,

10. $54.26 from $68.37.
11. 2714 from 5945.
12. $30.52 from $81.76.
13. 1763 tons from 3886 tons.
14. 6245 feet from 8569 feet.
15. 7301 days from 9625 days.

16. $93.64 — $52.41 = ?
17. $270.59 — $40.16 = ?
18. $703.42 — $501.30 = ?
19. 437615 — 213502 = ?
20. 732740 — 11520 = ?
21. 242674 — 32142 = ?

Find the difference between

22. 1204 and 5379.
23. 1320 and 1471.
24. 8673 and 3560.
25. $57.46 and $18.00 + $24.25.
26. $50.20 + $4.01 and $76.31.
27. $98.76 and $30.46 + $43.04.

28. From five thousand seven hundred and forty, take 3 thousand and 30.

29. From 46 thousand 5 hundred and 27, take 12 thousand 3 hundred and fourteen.

30. Two men bought a piece of property for $358.50. One paid $146.30; how much did the other pay?

31. A house and lot sold for $7856, which was one thousand one hundred and ten dollars more than the cost. What was the cost?

32. A certain city has a population of 246857, which is 25324 more than it had last year. What was its population last year?

ORAL EXERCISES.

92. 1. A man having $20, paid $7 for a hat, and $8 for a vest. How many dollars had he left?

Analysis.—The difference between $20, and the sum of $7 and $8, which is $5.

2. A boy had 25 cents, and gave 15 cents for a slate and 10 cents for some paper. How many cents had he left?

3. Ella having 16 cents, Jane gave her 9 more, and James gave her enough to make her number 36. How many did James give her?

4. Subtract by 7's from 63 to 0.
5. By 7's from 80 to 3.
6. By 8's from 64 to 0.
7. By 8's from 85 to 5.
8. By 9's from 90 to 0.
9. By 9's from 86 to 5.
10. By 10's from 100 to 0.
11. By 11's from 119 to 9.
12. By 11's from 125 to 4.
13. By 12's from 129 to 9.
14. By 12's from 150 to 6.
15. Count by 7's from 2 to 86, and back from 86 to 2.
16. Count by 8's from 4 to 100, and back to 4.
17. Count by 9's from 7 to 115, and back to 7.
18. Count by 10's from 16 to 136, and back to 16.
19. Count by 11's from 9 to 119, and back to 9.
20. Count by 12's from 20 to 140, and back to 20.
21. How many are 5 tens less 3 tens? 50—30?

22. How many are 6 tens less 4 tens? $60-40$?

23. From 6 tens 5 units subtract 4 tens 3 units.

24. From 8 tens 7 units subtract 5 tens 6 units.

25. From a cask containing 52 gallons, 27 gallons were drawn out. How many gallons remained?

ANALYSIS.—The difference between 52 gallons and 27 gallons 27 is 2 tens and 7 units. 2 tens or 20 from 52 leaves 32, and 7 from 32 leaves 25. Hence 25 gallons remained in the cask.

26. From a piece of cloth containing 46 yards, 24 yards were cut. How many yards were left?

27. A man bought a watch for $40, and a chain for $15, and sold both for $63. How much did he gain?

28. How many are 6 and 40, less 5 and 20?

29. How many are 7 and 30, taken from 5 and 50?

30. Eighteen plus 12 equals 40 minus how many?

31. Twenty-two plus 15, equals how many plus 10?

32. William having 75 cents, gave 25 cents for a book and 20 cents for a slate. How many cents had he left?

33. A farmer sold a horse for $96, which was $23 more than the horse cost. What did he cost?

Find the *omitted* term in the following equations:

34. $12+8-6=?$
35. $46+12-14=?$
36. $57-13+8=?$
37. $60-(24+6)=?$
38. $28+6=40-?$
39. $42-12=18+?$
40. $30+25=?+40$
41. $27-11=19-?$
42. $36+16=60-?$
43. $54-12=?+12$
44. $17+23=56-?$
45. $18+25=23+?$
46. $64-48=30-?$
47. $75-30=?+15$
48. $16+38=60-?$
49. $43+?=27+28$
50. $80-?=100-40$
51. $22+54=64+?$

WRITTEN EXERCISES.

93. When any figure of the subtrahend is greater than the corresponding figure of the minuend.

1. From 953 subtract 674.

OPERATION.

	8 14 13
Minuend	9 5 3
Subtrahend	6 7 4
Remainder	2 7 9

ANALYSIS.—Write the numbers as before (90), and subtract each order of units separately.

Since 4 units cannot be subtracted from 3 units, increase the 3 units by a unit from the next higher order, or 10 units, making 13 units 4 units from 13 units leave 9 units, which write in the units' place.

Since 1 of the tens was united with the units, there are 4 tens left. As 7 tens cannot be subtracted from 4 tens, increase the 4 tens by a unit from the next higher order, or 10 tens, making 14 tens. 7 tens from 14 tens leave 7 tens, which write in the tens' place.

Since 1 of the hundreds was united with the tens, there are 8 hundreds left. 6 hundreds from 8 hundreds leave 2 hundreds, which write in the hundreds' place. Hence the remainder is 279.

In like manner, solve and prove the following :

	(2.)	(3.)	(4.)	(5.)
From	3273	6345	5702	7465
Subtract	1425	2462	4384	3270

	(6.)	(7.)	(8.)
From	42670 miles.	51062 acres.	246700 feet.
Take	14384 "	24300 "	18030 "

When one of the given numbers contains *cents,* and the other does not, fill the vacant places with two ciphers.

	(9.)	(10.)	(11.)	(12.)
From	$325.17	$279.00	$105.08	$7.00
Take	84.36	183.42	67.00	.84

RULE.—I. *Write the subtrahend under the minuend, placing units of the same order in the same column.*

II. *Begin at the right, and subtract the units of each order of the subtrahend from the units of the corresponding order of the minuend, and write the result beneath.*

III. *If the units of any order of the subtrahend are greater than the units of the corresponding order of the minuend, increase the latter by* 10, *and subtract; then diminish by* 1 *the units of the next higher order in the minuend, and proceed as before.*

PROOF.—*Add the remainder to the subtrahend, and if the sum is equal to the minuend, the work is correct.*

Instead of *diminishing* by 1 the units of the next higher order in the *minuend*, we may *increase* by one the units of the next higher order in the *subtrahend*.

Subtract

13. 20762 from 53120.
14. $73.16 from $138.
15. $247 from $382.28.

From

16. $430.09, take $272.46.
17. 15200 rods, take 6472 rods.
18. 120764 tons, take 75028 tons.

How many years from the date of each of the following events to the present year?

19. Figures were used by the Arabs in the year 890.

20. Decimal fractions were invented in 1464.

21. Printing was invented in 1441.

22. The telescope was invented by Galileo in 1610.

23. The electric telegraph was first used in the United States in 1844.

24. The first passage of the Atlantic Ocean by steam was in 1839.

What is the difference between

25. 34726 and 47062?
26. 57600 and 20012?
27. 70361 and 1005?
28. 7620 and 12420?
29. \$4027 and \$703.41?
30. \$1076 and \$2340.50?
31. 2762 + 10341 and 45701 + 1200?
32. 3000 + 42301 and 720 + 1684 + 7342?

33. A merchant bought a quantity of goods for \$1248.65, and sold them for \$1540. How much did he gain?

34. Sold a horse for \$250.75, which was \$28 more than he cost. How much did he cost?

35. A man having \$15740.80, gave \$5085 for a store, and \$7640.75 for goods. How much money had he left?

36. If a piece of property bought for \$7086.86 is sold at a loss of \$1562.09, for how much is it sold?

Find the *second member* of the following equations:

37. 12346 + 840 + 1046 − 3846 = ?
38. \$210 + \$809.76 − (\$15.21 + \$308.76) = ?
39. \$600.09 − \$276.25 + \$5682 − \$654 = ?
40. \$1032.07 + \$68.05 + \$.98 − \$1000 = ?
41. 476281 − 12672 − 8720 + 20000 = ?

REVIEW.

ORAL EXAMPLES.

94. 1. The sum of two numbers is 46, and one of them is 18; what is the other?

2. The difference of two numbers is 16, and the greater is 32; what is the less?

3. The difference of two numbers is 24, and the less is 26; what is the greater?

4. A boy having 28 peaches gave 8 to his brother, 7 to his sister, and lost 4; how many had he left?

5. If a lady buy some thread for 10 cents, some needles for 5 cents, and some ribbon for 20 cents, and give the clerk 50 cents, how much change should he return?

6. In a garden are 47 fruit trees; 15 of them are peach trees, 12 plum trees, and the remainder pear trees. How many pear trees are there?

7. A lady having 3 ten-dollar bills and 1 five-dollar bill, bought a bonnet for $11, a pair of gaiters for $7, and a scarf for $3. How much money had she left?

8. A man died at the age of 64 years, having been married 36 years. What was his age when he married?

9. In a public school there are 75 pupils, and 47 of them are girls; how many of them are boys?

10. A man sold 25 sheep, then bought 12, and then had 20. How many had he at first?

11. A merchant gave $52 for a box of goods, and paid $5 freight; for how much must he sell them to gain $15?

12. A man gave his watch and $10 in money for a harness valued at $75. How much did he get for his watch?

13. A man having received $45 for labor, paid $15 for a coat, $7 for a barrel of flour, and $6 for a ton of coal. How much had he left?

14. A man bought a vest for $7, a pair of pants for $12, and three shirts for $9, and gave in payment 3 ten-dollar bills. How much change should he receive?

Find the *required* term in the following equations:

15. $42-(10+12)=?$
16. $9+16=30-?$
17. $36-14=15+?$
18. $36-8+9+12=?$
19. $7+16-8=22-?$
20. $14+28-16-9=?$

WRITTEN EXAMPLES.

95. 1. The subtrahend is 260346, and the remainder 72304. What is the minuend?

2. The difference is $310.62, and the minuend $1206.28. What is the subtrahend?

3. What is the sum of 4062 and 12356 increased by the difference between 15000 and 975?

4. From the sum of 23462 and 9030, subtract the difference between 34000 and 7640.

5. From the difference between 19876 and 6032, subtract the difference between 12000 and 673.

6. From what sum must $.62 be taken to leave a remainder of $14.60?

7. There were 67374 miles of railway in the United States in 1872, and 71564 miles in 1873. How much was the gain in one year?

8. A man has $10000. How much must he add to this, to be able to pay for a farm worth $13640?

9. California contains 158933 square miles, and Texas 237321 square miles. How much larger is Texas than California?

10. Mt. Blanc is 15572 feet high, and Pike's Peak 12000 feet. What is the difference in their height?

11. A man willed $125000 to his wife and two children. To his son he gave $44675, to his daughter $26380, and the remainder to his wife. What was his wife's share?

12. A merchant of Nashville goes to New Orleans with $21600. He invests $7638.50 in groceries, $3210.65 in crockery, $1245.18 in woodenware, and the remainder in hardware. How much does he invest in hardware?

13. The population of London in 1870 was 3250000; of New York, 944292; and of Brooklyn, 396099. How much greater was the population of London than of New York and Brooklyn?

14. A man owns property valued at $75860, of which $45640 is invested in real estate, $25175.75 in personal property, and the remainder he has in bank. How much has he in bank?

15. Three persons bought a hotel valued at $42075. The first agreed to pay $8375.50, the second agreed to pay twice as much, and the third the remainder. How much was the third to pay?

16. A had $725.40, B had $180.36 more than A, and C had as much as A and B together minus $214. How much had C?

17. 376 + 1684 + 573 − (931 + 1000) = ?

18. $27.62 + $30.50 − $14.00 − $7.62 = ?

19. 17300 + 6840 − (5800 + 1386) = 25300 − ?

20. (48036 − 7690) − (3600 + 1873) = 18321 + ?

96. SYNOPSIS FOR REVIEW.

SUBTRACTION.

- 1. Definitions.
  - 1. Subtraction. 2. Difference, or Remainder. 3. Minuend. 4. Subtrahend. 5. Sign of Subtraction. 6. A Parenthesis, or Vinculum.
- 2. Principles, 1 and 2.
- 3. Subtraction of Dollars and Cents.
  - 1. How the numbers should be written.
  - 2. If one number contains cents, and the other does not
- 4. Rule, I, II, III
- 5. Proof.

MULTIPLICATION

ORAL EXERCISES.

97. 1. If a man earns $3 a day, how many times $3 does he earn in 4 days? $3 + $3 + $3 + $3 are how many?

2. There are 7 days in 1 week. How many days are there in 3 weeks? How many are three 7's, or 3 times 7?

3. There are 4 pecks in 1 bushel. How many pecks in 4 bushels. Four 4's, or 4 times 4 are how many?

4. Is the result the same whether we say 4 times 6, or 6 times 4?

5. What is the difference between six 5's and five 6's?

6. How many are three 8's? Eight 3's?

7. Add by 2's from 0 to 24.

8. Multiply from 0 times 2, to 12 times 2.

Operation.—0 times 2 is 0, once 2 is 2, twice 2 are 4, 3 times 2 are 6, 4 times 2 are 8, 5 times 2 are 10, and so on.

9. Subtract by 2's back from 24 to 0.

10. Multiply back from 12 times 2 to 0 times 2.

Operation.—12 times 2 are 24, 11 times 2 are 22, 10 times 2 are 20, 9 times 2 are 18, 8 times 2 are 16, and so on.

11. Multiply from 0 times 3 to 12 times 3, and back.

12. Multiply from 0 times 4 to 12 times 4, and reverse.

13. Multiply from 0 times 5 to 12 times 5, and reverse.

14. Multiply from 0 times 6 to 12 times 6, and reverse.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

DEFINITIONS.

98. ***Multiplication*** is the process of taking one of two numbers as many times as there are units in the other. Or, it is a short method of *adding equal* numbers.

99. The ***Multiplicand*** is the number to be multiplied.

100. The ***Multiplier*** is the number by which to multiply. It shows *how many times* the multiplicand is to be taken.

101. The ***Product*** is the result obtained by the multiplication.

The multiplicand and multiplier are called the *factors* of the product.

102. The ***Sign of Multiplication*** is ×. It is read *times*, or *multiplied by*.

When placed between two numbers, it shows that they are to be multiplied together. Thus 9×7 is read 9 *multiplied by* 7, or 7 *times* 9.

Since changing the order of the factors does not change the result, 9×7 may be read, 7 *times* 9, or 9 *times* 7.

103. PRINCIPLES.—1. *The multiplier is always regarded as an abstract number.*

2. *The multiplicand and product are like numbers, and may be either concrete or abstract.*

In examples containing concrete numbers, the concrete number is the *true multiplicand*, but when it is the smaller, it is often, for convenience, used *abstractly* as the multiplier.

ORAL EXERCISES.

104. $8\times4=?$

$7\times7=?$

$6\times9=?$

$0\times5=?$

$10\times8=?$

$10\times0=?$

$9\times5=?$

$7\times11=?$

$12\times6=?$

$9\times7=?$

$11\times8=?$

$9\times12=?$

$11\times10=?$

$10\times12=?$

$9\times11=?$

$8\times8+10=?$

$9\times4-10=?$

$12\times6+15=?$

$10\times12-25=?$

$8\times11-12=?$

$9\times9+19=?$

$10\times10-14=?$

$7\times12+16=?$

$8\times0-7=?$

$0\times12\times8=?$

$1\times12+8=?$

$12\times10-30=?$

$15\times2+15=?$

$11\times11-9=?$

$12\times0+25=?$

$10\times12-16=?$

$12\times11-12=?$

$12\times12+6=?$

1. At 7 cents each, what is the cost of 5 pencils?

ANALYSIS.—Since 1 pencil costs 7 cents, 5 pencils will cost 5 times 7 cents, or 35 cents.

2. What is the cost of 4 tons of coal, at $8 a ton?

3. What is the cost of 5 hats, at $5 a piece?

4. At 9 cents each, what will 3 melons cost?

5. What will 5 yards of gimp cost, at 11 cents a yard?

6. 12 inches make a foot. How many inches in 4 feet?

7. At $4 a cord, what will 9 cords of wood cost?

8. Multiply from 0 times 6 to 12 times 6, and reverse.

9. Multiply from 0 times 7 to 12 times 7, and reverse.

10. Multiply from 0 times 8 to 12 times 8, and reverse.

11. Multiply from 0 times 9 to 12 times 9, and reverse.

12. What cost 6 pairs of boots at $8 a pair? At $9?

13. At 8 cents each, what cost 9 books? 10 books?

14. What cost 9 barrels of flour at $9 a barrel? At $10?

15. 7 days make a week. How many days in 7 weeks?

16. If a man earn $12 in 1 week, how much will he earn in 8 weeks? In 9 weeks?

17. Multiply from 0 times 10 to 12 times 10, and reverse.

18. Multiply from 0 times 11 to 12 times 11, and reverse.

19. Multiply from 0 times 12 to 12 times 12, and reverse.

20. At 12 cents a yard, what cost 9 yards of calico?

21. What cost 10 pounds of ginger, at 11 cents a pound?

22. At $11 a hundred, what will 11 hundred posts cost?

23. How many bushels of grain can be put in 8 bins, each containing 12 bushels?

24. How many are 8 times $4, minus $7?

25. How many are 7 times 9 pounds, plus 10 pounds?

26. How many are 6 times 12 rods, less 20 rods?

27. James gave 5 cents each for 6 oranges. How much change should he receive from 50 cents?

28. How much more than $35 will 7 tons of coal cost, at $6 a ton?

WRITTEN EXERCISES.

105. When the multiplier consists of but one order of units.

1. How many are 4 times 73?

1ST OPERATION.

73
73
73
73
Sum 292

ANALYSIS.—To obtain the result by Addition.

First find the sum of four 3's, or 4 *times* 3 units, which is 12 units, equal to 1 ten and 2 units. Write the 2 units in the units' place, and reserve the 1 ten to be added to the sum of the tens.

Next, the sum of four 7's, or 4 *times* 7 tens, is 28 tens, and 28 tens plus 1 ten reserved are 29 tens, or 2 hundreds and 9 tens, which write in the hundreds' and tens' place. Hence the *sum* is 292.

2D OPERATION.

Multiplicand 73
Multiplier 4
Product 292

ANALYSIS.—In this operation, the multiplicand 73 is written but once; and as it is to be taken 4 times, write the multiplier 4 under it, and commence at the right to multiply. 4 times 3 units are 12 units, or 1 ten and 2 units. Write the 2 units in units' place and reserve the 1 ten to add to the product of the tens.

Next, 4 times 7 tens are 28 tens, and 28 tens plus 1 ten reserved are 29 tens, or 2 hundreds and 9 tens, which write in the hundreds' and tens' places. Hence the *product* is 292, equal to the *sum* in the first operation.

Solve by both methods,

2. 3 times 84.
3. 4 times 135.
4. 5 times 234.
5. 6 times 352.
6. 4 times $204.
7. 5 times $425.
8. Multiply 4621 by 4; by 5; by 6; by 7.
9. Multiply 3062 by 6; by 7; by 8; by 9.

What is the product

10. Of $5642 by 6? by 5? by 7? by 9?
11. Of 20372 feet by 7? by 9? by 5? by 6?

12. What cost 527 barrels of flour, at $9 a barrel?

Although $9 is the *true multiplicand*, for convenience, we may use 9 for the multiplier, and 527 as the multiplicand (**103,** Note), but the *product* is *dollars,* since the true multiplicand is dollars. This is obvious, since 527 barrels at $1 a barrel, would cost $527, and at $9 a barrel, 9 times $527, etc.

13. What cost 326 tons of coal, at $6 a ton?

14. What cost 1238 cords of wood, at $5 a cord?

15. What cost 752 pounds of nails, at 7 cents a pound?

OPERATION.—7 cents × 752 = 5264 cents = $52.64.

When *either* factor contains cents, the *product* is *cents,* and may be changed to dollars and cents by putting the point (.) *two* places from the *right,* and prefixing the sign ($).

	(16.)	(17.)	(18.)	(19.)
Multiply	$43.72	$136.04	87 cents.	$2.06
By	8	7	9	6
Product	$349.76	$952.28	$7.83	$12.36

20. At 6 cents a pound, what cost 675 pounds of rice?

21. At $4.37 a yard, what is the cost of 7 yards of cloth?

22. At $124.50 an acre, what will 5 acres of land cost?

23. What is the cost of 8 building lots, at $2015 each?

ORAL EXERCISES.

106. 1. 9 times $12 are $108. Which number is the Multiplicand? The Multiplier? The Product?

2. If 6 men can build a wall in 7 days, in how many days can 1 man build it?

ANALYSIS.—It will take 1 man 6 times as many days as it will 6 men, to build the wall; and 6 times 7 days are 42 days. Hence it will take 1 man 42 days.

3. If 7 men can do a piece of work in 10 days, how many days will it take 1 man to do the same work?

4. How many horses will consume as many bushels of oats in one day as 7 horses will consume in 5 days?

5. If 3 barrels of flour last 9 persons 4 months, how long will the same quantity of flour last 1 person?

6. If a man earns $18 a week, and spends $9 for board and other expenses, how much will he save in 8 weeks?

7. If Henry earn $5 a week, and James $4, how much will both earn in 7 weeks?

8. What is the difference in the cost of 6 yards of ribbon at 9 cents a yard, and of 6 yards at 11 cents a yard?

9. What will be the cost of 6 cows at $26 each?

ANALYSIS.—Six cows will cost 6 times $26. 6 times 6 units are 36 units, or 3 tens and 6 units, and 6 times 2 tens are 12 tens, which plus 3 tens and 6 units, are 15 tens and 6 units, or 156 Hence 6 cows will cost $156.

10. What cost 7 pounds of figs, at 23 cents a pound?

11. What cost 8 pounds of coffee, at 42 cents a pound?

12. At $36 a ton, what will 6 tons of guano cost?

13. At $18 a barrel, what will 9 barrels of pork cost?

14. At $5 a barrel, what are 33 barrels of apples worth.

15. At $7 a week, what is the cost of 21 weeks board?

16. What cost 20 pounds of beef, at 12 cents a pound.

17. Two men start from the same place, and travel in opposite directions, one at the rate of 6 miles an hour, the other, of 8 miles an hour. How far apart will they be at the end of 6 hours? 8 hours? 9 hours?

18. A woman sold a grocer 5 pounds of butter at 30 cents a pound, and received in payment 12 pounds of sugar at 9 cents a pound. How much was still due her?

Find the *second member* of the following equations :*

19. $20+12-3\times 6=?$
20. $16-7+4\times 0=?$
21. $7\times 12-6\times 11=?$
22. $60-(0\times 12)+15=?$
23. $20\times 3+(40-7\times 5)=?$
24. $3\times 0+4\times 7=?$
25. $(55-7)-\overline{20-8}=?$
26. $(7+5)-(6+4)=?$
27. $14\times 0+45-15=?$
28. $100-12\times 7+20-4=?$

WRITTEN EXERCISES.

107. When the multiplier consists of two or more orders of units.

1. Multiply 678 by 46.

OPERATION.

Multiplicand	678	
Multiplier	46	
1st Partial Prod.	4068	$=678\times 6$
2d Partial Prod.	2712	$=678\times 40$
Entire Prod.	31188	$=678\times 46$

ANALYSIS.—Write the numbers as before.

Since 46 is composed of 6 units and 4 tens, 46 times any number is equal to 6 times the number, *plus* 4 tens, or 40 times the number.

6 times 678 is 4068, the first *partial* product. 4 tens times 8 units are 32 *tens*, or 3 hundreds and 2 tens. Write the 2 tens in the tens' place, in the second *partial* product, and reserve the 3 hundreds to add to the product of hundreds.

4 tens times 7 tens are 28 *hundreds*, and 28 hundreds plus 3 hundreds reserved, are 31 hundreds, or 3 thousands and 1 hundred. Write the 1 hundred in the hundreds' place in the second *partial* product, and reserve the 3 thousands to add to the product of thousands.

4 tens times 6 hundreds are 24 *thousands*, and 24 thousands plus 3 thousands reserved, are 27 thousands, or 2 tens of thousands and 7 thousands, which write in the second *partial* product. The sum of the *partial* products is the *entire product* 31188.

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* The operations of multiplication and division, indicated by signs, must be performed before those of addition and subtraction, unless otherwise indicated by a parenthesis or vinculum.

In like manner, multiply

2. 473 by 27.
3. 738 by 35.
4. 609 by 56.
5. $36.45 by 34; by 47.
6. $70.65 by 55; by 64.
7. $29.07 by 76; by 82.

RULE.—I. *Write the multiplier under the multiplicand, so that units of the same order stand in the same column.*

When the multiplier consists of one figure.

II. *Begin at the right and multiply the units of each order of the multiplicand by the multiplier. Write in the product the units of each result, and reserve the tens to add to the next result.*

When the multiplier consists of more than one figure.

III. *Multiply the multiplicand by the units of each order of the multiplier successively, beginning at the right, and write the right-hand figure of each partial product under the order of the multiplier used.*

The sum of the partial products is the required product.

PROOF.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results are the same, the work is probably correct.*

When there are ciphers in the multiplier, multiply by the significant figures only, since the product of any number by 0 is 0.

8. Multiply 6432 by 75; by 67; by 136.
9. Multiply 23072 by 128; by 243; by 307.
10. Multiply $420.06 by 204; by 666; by 408.

What is the value

11. Of 67 hogsheads of sugar, at $37.75 a hogshead?
12. Of 2347 acres of land, at $136 an acre?
13. Of 64 horses, at $219.75 each?

14. What will be the cost of building a line of telegraph 274 miles long, at $967 a mile?

15. If 1049 pounds of seed cotton be raised from an acre of land, how many pounds will 386 acres produce?

16. If a cotton mill manufactures 628 yards of cloth in a day, how many yards can it make in 297 days?

What is the product

17. Of 2572 bushels by 94?
18. Of $403.06 by 127?
19. Of 86072 pounds by 208?
20. Of 316 times $487.46?
21. Of 507 times 30975 days?
22. Of 325 times 6408 cents?
23. Of 370607 by 4071?
24. Of 600326 by 2645?
25. Of 730096 by 5006?
26. Of 2407068 by 3406?
27. Of 408091 by 2407?
28. Of 73069 by 46035?

29. Find the cost of 386 railway coaches, at $7034.75 each.

30. What cost 802 tubs of butter, at $27.08 each?

31. $236 \times 63 \times 28 = ?$
32. $439 \times 0 \times 142 = ?$
33. $1927 \times 613 \times 802 = ?$
34. $4605 \times 2034 \times 576 = ?$

35. How many yards of shirting in 49 bales, each bale containing 26 pieces, and each piece 57 yards?

36. What is the cost of 128 barrels of beef, each containing 216 pounds, worth 13 cents a pound?

37. Three schooners, ship 239 cords of wood each, and a fourth ships 248 cords. What is the value of the whole at $4.25 a cord?

38. If it require 108 tons of iron rail for 1 mile of track, how many tons will be required for 476 miles, and what will be its value at $145 a ton?

39. A crop of cotton was put up in 472 bales, the average weight of which was 588 pounds. What was the weight of the whole crop, and its value at 18 cents a pound?

108. To multiply by the factors of a number.

The ***Factors*** of a number are the numbers which multiplied together will produce it. Thus, 6 and 7 are factors of 42; 2, 4, and 5 are factors of 40.

The pupil should carefully distinguish between the *factors* and the *parts* of a number. The *factors* are *multiplied*, but the *parts* are *added*, to produce a number. A factor is always a part, but a part is not always a factor.

Thus, 2 and 9, 3 and 6, 2, 3, and 3, are factors of 18; but the *parts* of 18 are 9 and 9, 10 and 8, 6 and 12, 7 and 11, etc.

109. PRINCIPLE.—*The product of any number of factors will be the same in whatever order they are multiplied.*

1. Multiply 468 by 36.

OPERATION.

$36 = 6 \times 6$, or 9×4, or 12×3.

468	468	468	468
36	6	9	12
2808	2808	4212	5616
1404	6	4	3
16848	16848	16848	16848

It will be observed that the multiplicand, multiplied by the given multiplier, or by any *set of factors* into which it can be separated, produces the same result.

In like manner, multiply

2. $73.04 by $48 = 8 \times 6$.
3. 50076 by $72 = 6 \times 4 \times 3$.
4. 46502 by $84 = 7 \times 4 \times 3$.
5. $206.14 by $96 = 4 \times 4 \times 6$.
6. $780.91 by 108.
7. 140086 by 120.
8. 380509 by 144.
9. $457.52 by 240.

RULE.—I. *Separate the multiplier into two or more factors.*

II. *Multiply the multiplicand by one of the factors, the resulting product by another factor, and so continue until all the factors have been used.*

The last product will be the product required.

10. What will 56 acres of land cost, at $164.50 an acre?

11. At 28 cents a pound, what will be the cost of 24 sacks of coffee, each containing 64 pounds?

12. What is the value of 107 pieces of cloth, each piece containing 42 yards, at $4.28 a yard?

110. When either the multiplicand or multiplier, or both, have ciphers on the right.

1. Multiply 286 by 100.

OPERATION.

$$\begin{array}{r} 286 \\ 100 \\ \hline 28600 \end{array}$$

ANALYSIS.—Since removing a figure *one place* to the *left*, increases its value *ten times* (**44**), annexing a cipher to a number multiplies it by 10; annexing *two* ciphers multiplies it by 100, etc. Hence 286 × 100=28600, the product required.

2. Multiply 3240 by 600.

OPERATION.

$$\begin{array}{r} 3240 \\ 600 \\ \hline 1944000 \end{array}$$

ANALYSIS.—3240=324 × 10, and 600=6 × 100. First multiply together the two factors 324 and 6, and then multiply their product 1944, by 10 × 100, or by 1000, by annexing *three* ciphers, which gives 1944000, the required product.

What is the product

3. Of 372 by 10? By 100? By 1000? By 10000?

4. Of 860 by 50? By 400? By 1500? By 3000?

RULE.—*To the product of the significant figures, annex as many ciphers as there are ciphers on the right of either or of both of the factors.*

What is the product

5. Of $4.72 by 100 ?

6. Of $30.40 by 60 ?

7. Of $1200 by 700 ?

Find

8. 120 times 5000.

9. 600 times 21000.

10. 1000 times 104000.

11. $42030090 \times 3020 = ?$ 12. $7000600 \times 50040 = ?$

13. There are 640 acres in 1 square mile. How many acres in 150 square miles ? In 200 ? In 420 ?

14. The salary of the president is $50000 a year. How much does he receive in 8 years ?

REVIEW.

ORAL EXAMPLES.

111. 1. The sum of $8+12+16$ equals the product of $9 \times$ what number ?

2. The sum of $40 - 14$ and $12 + 4$ equals $7 \times$ what number ?

3. The difference between $35+15$ and $24-10$ is equal to the product of what two factors ? Three factors ?

4. The product of what two factors is equal to the sum of 9, 20, and 11 ?

5. The product of 8 times 9 is equal to 6 times what number ?

6. The sum of 25, 13, 8, and 10 is equal to the product of what three factors ?

7. What is the sum of 3 times 3×4, and 5 times 4×3 ? What is the difference ?

8. What is the product of $15+24-14$ by $16-12$?

9. Which is greater, $9 \times \overline{13-6}$, or 12 times $8-20$?

10. How much less is $60-5 \times 8$ than $16+14-10$?

11. Charles is twice as old as George, and George is 12 years old. What is the sum of their ages?

12. What is the cost of 4 brooms at 30 cents each, and 6 pounds of sugar at 11 cents a pound?

13. Mary had 18 cents, and Belle had 3 times as many less 9 cents. How many had both?

14. A young man earned $9 a week, and spent $5 a week for board. How much did he save in 12 weeks?

15. A woman sold a grocer 4 dozen of eggs at 24 cents a dozen, and received in payment half a pound of tea worth 50 cents, and 2 pounds of sugar at 11 cents a pound. How much was still due her?

16. A boy bought a book for 36 cents, a slate for 20 cents, and a pencil for 4 cents. How much change should he receive from a 1 dollar bill?

17. A lady bought 9 yards of silk at $3 a yard, 3 pairs of kid gloves at $2 a pair, 4 pairs of hose at half a dollar a pair. She gave in payment 4 ten dollar bills. How much change should she receive?

Find the *required term* in the following equations:

18. $19-7+28-11=?$
19. $8\times 9-16=7\times ?$
20. $21+6\times 7=40+?$
21. $10\times 12-9\times 11=?$
22. $75-5\times 12=35-?$
23. $44+19-(50-23)=?$
24. $7\times 12-25\times 0=?$
25. $3\times 0\times 5+16=2\times ?$
26. $28+12-\overline{6\times ?}=16$
27. $9\times 12+10=120-?$
28. $42-20+14=?\times 9$
29. $8+55-(?\times 8)=7$

112. By a little practice, numbers containing *three* or *four* figures may be multiplied *mentally*, by first multiplying the highest order of units, and adding the product of each lower order as found.

1. Multiply 324 by 2.

Operation.—2 times 3 hundreds are 600; 2 times 2 tens are 4 tens, or 40, and 600 + 40 are 640; 2 times 4 are 8, and 640 + 8 are 648.

Omitting all but results, the required product will be easily and promptly obtained by a strictly mental process. Thus, 600, 640, 648.

In like manner, find the product of

2. 3 times 230.
3. 3 times 342.
4. 4 times 150.
5. 4 times 425.
6. 6 times 241.
7. 5 times 615.
8. 234 × 2.
9. 501 × 3.
10. 255 × 4.

WRITTEN EXAMPLES.

113. 1. If I receive $1500 salary, and pay $370 for board, $281.50 for clothing, $112.75 for books, and $196.65 for other expenses annually, what can I save in 3 years?

2. A merchant bought 7 hogsheads of sugar at $46.45 a hogshead, and sold it for $53.62 a hogshead. How much did he gain?

3. Paid $2709 for 388 barrels of flour, and sold the same at $9.12 a barrel. How much was the gain?

4. If a man have an income of $5670 a year and his daily expenses average $7.25, how much can he save in a year of 365 days?

5. What number must be added to 272×400 to make the amount 126720?

6. What is the difference between $40706 - 308 \times 56$, and $97 \times 340 - 12400$?

7. Multiply $98 + 6 \times (37 + 50)$ by $\overline{64 - 50} \times 5 - 10$.

8. Multiply $675 - (77 + 56)$ by $(3 \times 155) - (214 - 28)$.

9. A man owing $15760, gave in payment 5 lots of land, worth $730 each, 5 horses, valued at $236.50 each, an interest he had in a coal mine worth $2000, and $1728.75 in money. How much remained unpaid?

10. A farm-house is worth \$3246, the farm is worth 3 times as much plus \$1200, and the stock is worth twice as much as the house, less \$1875. What is the value of the whole, and of the farm and stock?

11. What is the difference in the cost of 48 horses at \$184.50 each, and of 130 sheep at \$4.80 a head?

12. Bought 150 barrels of flour for \$1150, and finding 25 barrels of it worthless, sold the remainder at \$9 a barrel. Did I gain or lose, and how much?

Complete the following equations:

13. $(142 + 405) \times (1000 - 850) - 5000 = ?$

14. $(97 \times 1000) - (75 \times \overline{500 - 420}) + 1500 = ?$

15. $\$73.46 - (\$.94 + \$3.02) + \$47 \times 35 = ?$

16. $\$246.08 \times 104 + (\$2000 - \$240.50) \times 10 = ?$

114. SYNOPSIS FOR REVIEW.

MULTIPLICATION.

- 1. DEFINITIONS.
  - 1. Multiplication. 2. Multiplicand. 3. Multiplier. 4. Product. 5. Sign of Multiplication.
- 2. PRINCIPLES, 1 and 2.
- 3. RULE—I, II, III.
- 4. PROOF.
- 5. WHEN EITHER FACTOR CONTAINS CENTS.
- 6. BY FACTORS.
  - 1. Definition of factors.
  - 2. Principle.
  - 3. Rule.
- 7. MULTIPLIER AND MULTIPLICAND.
  - 1. When one or both have ciphers on the right.
  - 2. Rule.

DIVISION

ORAL EXERCISES.

115. 1. How many 4's are 12? Are 16? Are 24?

2. How many lots, of 5 acres each, in 20 acres?

3. How many 5's in 15? In 30? In 35? In 50?

4. How many barrels, each holding 3 bushels, will be required for 18 bushels of apples? 21 bushels?

5. How many times can 6 yards of cloth be taken from a piece containing 30 yards.

6. How many times can 6 cents be taken from 23 cents, so as to have 5 cents remaining?

7. Distribute $28 equally among 7 men. How many dollars will each receive?

Do you find how many times 7 men are contained in $28, or do you find *one* of 7 equal parts of $28?

8. How do you find one of 8 equal parts of a number? Of 9 equal parts? Of 6 equal parts?

9. What is one of 4 equal parts of 40? Of 36? Of 48?

10. What is one of 6 equal parts of 30? Of 42? Of 48?

11. What is one of 7 equal parts of 56 pounds?

12. How many times 8 cents are 48 cents? Is the result a concrete or an abstract number?

13. What is one of 8 equal parts of 48 cents? Is the result a concrete or an abstract number?

DEFINITIONS.

116. ***Division*** is the process of finding *how many times* one number is contained in another of the same kind, or of finding *one* of the *equal parts* of a number.

117. The ***Dividend*** is the number to be divided.

118. The ***Divisor*** is the number by which to divide.

119. The ***Quotient*** is the result of the division, and shows *how many times* the dividend contains the divisor.

The division is said to be *exact* when there is no remainder.

The part of the dividend remaining when the division is not *exact* is called the *Remainder,* and must always be *less* than the divisor.

120. The ***Sign of Division*** is ÷. It is read *divided by.*

It shows that the number before it is to be divided by the one after it; thus 54 ÷ 9 is read 54 *divided by* 9.

121. Division is also indicated by placing the dividend above the divisor with a line between them; thus, $\frac{72}{8}$ is read 72 *divided by* 8.

122. PRINCIPLES.—In finding *how many times* one number is contained in another:

1. *The divisor and dividend are like numbers, and the quotient an abstract number.*

In finding one of the *equal parts* of a number:

2. *The dividend and quotient are like numbers, and the divisor an abstract number.*

3. *The dividend is equal to the product of the divisor by the quotient, plus the remainder.*

ORAL EXERCISES.

123. $36 \div 9 = ?$ $63 \div 9 = ?$ $64 \div 8 = ?$ $84 \div 7 = ?$
$42 \div 7 = ?$ $56 \div 8 = ?$ $66 \div 6 = ?$ $72 \div 12 = ?$
$40 \div 5 = ?$ $45 \div 5 = ?$ $72 \div 9 = ?$ $96 \div 8 = ?$

$\frac{63}{7} = ?$ $\frac{96}{8} = ?$ $\frac{77}{7} = ?$ $\frac{120}{10} = ?$
$\frac{72}{9} = ?$ $\frac{70}{7} = ?$ $\frac{96}{12} = ?$ $\frac{108}{12} = ?$
$\frac{48}{12} = ?$ $\frac{84}{12} = ?$ $\frac{72}{6} = ?$ $\frac{132}{11} = ?$

1. Divide by 2, from 2 in 2 to 2 in 24.

Operation.—2 in 2, once; 2 in 4, twice; 2 in 6, 3 times; 2 in 8, 4 times; 2 in 10, 5 times, and so on to 2 in 24, 12 times.

In the same manner, divide

2. By 3, from 3 in 3, to 3 in 36.
3. By 4, from 4 in 4, to 4 in 48.
4. By 5, from 5 in 5, to 5 in 60.
5. By 6, from 6 in 6, to 6 in 72.
6. By 7, from 7 in 7, to 7 in 84.
7. By 8, from 8 in 8, to 8 in 96.
8. By 9, from 9 in 9, to 9 in 108.
9. By 10, from 10 in 10, to 10 in 120.

The pupil may *reverse* the above; thus, 2 in 24, 12 times; 2 in 22, 11 times; 2 in 20, 10 times, and so on.

Also *combine* the two; thus, 3 in 3, once; 3 in 6, twice, 2 in 6, 3 times; 3 in 12, 4 times, 4 in 12, 3 times; and so on to 3 in 36, 12 times, 12 in 36, 3 times.

124. Division may also be regarded as a short method of performing several *subtractions* of a number.

Thus, $24 - 6 = 18$; $18 - 6 = 12$; $12 - 6 = 6$; $6 - 6 = 0$. We have performed *four* subtractions of 6, hence there are *four* 6's in 24, or 6 is contained in 24, 4 times.

125. Since one number is contained in another as many times as it is a *factor* of the other, division may be regarded as the *reverse* of multiplication.

In Multiplication, *both factors* are given to find the *product;* in Division, *one factor* and the *product* (answering to the *dividend*) are given to find the *other factor,* which answers to the *quotient.*

Thus, $6 \times 4 = 24$, the factor 6 being taken 4 times; hence there are *four* 6's in 24, or 6 is contained in 24, 4 times.

126. The Object of Division is *twofold.*

First. To find *how many times* one number is contained in another of the *same kind.*

Ex. At 5 cents each, how many pencils can be bought for 20 cents.

Since 5 cents taken 4 times equals 20 cents ($5 \times 4 = 20$), it follows that 5 cents is contained in 20 cents 4 times.

ANALYSIS.—As many pencils can be bought for 20 cents, as 5 cents are contained times in 20 cents, which are 4 times. Hence, etc.

127. Second. To *separate* a given number into as many *equal parts* as there are units in another.

Ex. If 4 pencils cost 20 cents, what is the cost of 1 pencil?

Since 5 cents taken 4 times equals 20 cents, it follows that 5 cents is one of the four *equal parts* of 20 cents ($5 + 5 + 5 + 5 = 20$), and we say *one-fourth* of 20 cents is 5 cents.

ANALYSIS.—Since 4 pencils cost 20 cents, 1 pencil costs *one-fourth* of 20 cents, which are 5 cents.

128. The *equal parts* into which a unit or whole thing is divided are called *fractions.*

129. The *names* of these equal parts of a unit vary according to the *number* of these parts; thus, *one-half* is one of *two* equal parts, *one-third* is one of *three* equal parts into which the whole thing or number is divided.

So in like manner we have *fourths, fifths, sixths, sevenths, eighths, tenths, twelfths, twentieths*, etc.

130. These parts are expressed by writing the number denoting the *name* of the parts below a short horizontal line as a *divisor*, and the *number* of parts taken or used, above the line as a *dividend.*

Thus, $\frac{1}{2}$, signifies 1 *divided by* 2, and is read, *one-half.*

$\frac{2}{3}$, signifies 2 *divided by* 3, and is read *two-thirds.*

$\frac{7}{12}$, signifies 7 *divided by* 12, and is read, *seven-twelfths*, etc.

ORAL EXERCISES.

131. 1. If a number is separated into *two* equal parts, what is each part called?

Ans. ***One-half*** of the number, written $\frac{1}{2}$.

2. If $18 are equally divided between two poor families, how much does each receive? What *part* of the whole?

3. What is *one-half* of 12? Of 16? Of 20? Of 24?

4. If a number is separated into *three* equal parts, what is each part called? ***One-third*** of the number, $\frac{1}{3}$.

5. If 15 peaches are equally distributed among 3 boys, what *part* of the whole will each receive?

6. What is *one-third* of $15? Of 21 days? Of 30 rods?

7. Divide an acre of land into *four* equal parts. What is one of the parts called? ***One-fourth*** of an acre, $\frac{1}{4}$.

8. What are 2 of the parts called? ***Two-fourths***, $\frac{2}{4}$. Three of the parts? ***Three-fourths***, $\frac{3}{4}$.

9. If 48 marbles are given to 4 boys, to each an equal number, what *part* of the whole does 1 boy receive? Two boys? Three boys? How many marbles?

10. What is *one-fourth* of 24? Of 48 miles?

11. If a number is divided into *five* equal parts, what is each part called? ***One-fifth*** of the number, $\frac{1}{5}$. Two parts? ***Two-fifths***, $\frac{2}{5}$.

12. If \$20 are paid for 5 barrels of apples, what *part* of \$20 is paid for 1 barrel? For 2 barrels? For 3 barrels?

13. What is $\frac{1}{5}$ of 30? Of \$40? Of 45 rods?

14. If a number is divided into *six* equal parts, what is each part called? ***One-sixth*** of the number, $\frac{1}{6}$.

15. If into *seven* equal parts? ***One-seventh***, $\frac{1}{7}$.

16. If into *eight* equal parts? ***One-eighth***, $\frac{1}{8}$.

17. If into *nine* equal parts? ***One-ninth***, $\frac{1}{9}$.

18. If into *ten* equal parts? ***One-tenth***, $\frac{1}{10}$.

19. If into *twelve* equal parts? ***One-twelfth***, $\frac{1}{12}$.

20. Find one-half of 2, one-half of 4, one-half of 6, one-half of 8, and so on to one-half of 20.

21. Find one-third of 3, one-third of 6, one-third of 9, one-third of 12, and so on to one-third of 30.

22. Find $\frac{1}{4}$ of 4, $\frac{1}{4}$ of 8, $\frac{1}{4}$ of 12, $\frac{1}{4}$ of 16, to $\frac{1}{4}$ of 40.

23. Find $\frac{1}{5}$ of 5, $\frac{1}{5}$ of 10, $\frac{1}{5}$ of 15, $\frac{1}{5}$ of 20, to $\frac{1}{5}$ of 50.

24. Find $\frac{1}{6}$ of 6, $\frac{1}{6}$ of 12, $\frac{1}{6}$ of 18, $\frac{1}{6}$ of 24, to $\frac{1}{6}$ of 60.

25. Find $\frac{1}{7}$ of 7, $\frac{1}{7}$ of 14, $\frac{1}{7}$ of 21, $\frac{1}{7}$ of 28, to $\frac{1}{7}$ of 70.

26. Find $\frac{1}{8}$ of 8, $\frac{1}{8}$ of 16, $\frac{1}{8}$ of 24, $\frac{1}{8}$ of 32, to $\frac{1}{8}$ of 80.

27. Find $\frac{1}{9}$ of 9, $\frac{1}{9}$ of 18, $\frac{1}{9}$ of 27, $\frac{1}{9}$ of 36, to $\frac{1}{9}$ of 90.

28. Find $\frac{1}{10}$ of 10, $\frac{1}{10}$ of 20, $\frac{1}{10}$ of 30, to $\frac{1}{10}$ of 100.

29. How do you find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc., of any number?

30. How many yards of cloth, at \$4 a yard, can be bought for \$36?

ANALYSIS.—As many yards as \$4 are contained times in \$36, which are 9 times. Hence 9 yards can be bought for \$36.

31. At \$6 a ton, how many tons of coal can be bought for \$24? For \$30? For \$54? For \$72?

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32. If 7 cords of wood cost $42, what does 1 cord cost.

ANALYSIS.—Since 7 cords of wood cost $42, 1 cord costs 1 *seventh* of $42, or $6. Hence 1 cord costs $6.

33. A man sold 8 bushels of cranberries for $32. How much did he receive a bushel for them?

34. A farmer gathered 108 bushels of apples from 9 trees. What was the average number of bushels to each tree.

35. A merchant paid $96 for 8 pieces of dress goods. What was the cost of each piece?

36. If a farm of 120 acres is divided into 12 equal lots, how many acres does each lot contain?

WRITTEN EXERCISES.

132. When the divisor consists of but one order of units.

1. Divide 875 by 7.

OPERATION.

```
Divisor. Dividend. Quotient.
     7)875(125
       7
       --
       17
       14
       --
        35
        35
        --
```

ANALYSIS.—Write the divisor at the left of the dividend with a line between them.

7 is contained in 8 hundreds, 1 hundred times, with a remainder. Write the 1 hundred at the right of the dividend, for the first figure of the quotient. Multiply the divisor 7 by the 1 hundred of the quotient, and write the product, 7 hundreds, under the hundreds of the dividend. Subtract, and to the remainder 1 hundred, annex the 7 tens of the dividend, making 17 tens.

7 is contained in 17 tens, 2 tens times, with a remainder. Write the 2 tens in the quotient. Multiply the divisor 7 by the 2 tens, and subtract the product from the partial dividend, 17 tens. To the remainder 3 tens, annex the 5 units of the dividend, making 35 units.

7 is contained in 35 units, 5 times, which write in the quotient. Multiplying and subtracting as before, nothing remains. Hence, etc.

The solution of the preceding example may be abbreviated by what is termed *Short Division*, as follows:

OPERATION.
$7\underline{)875}$
Quotient 125

ANALYSIS.—7 is contained in 8, once, and 1 remainder. 1 prefixed to 7 makes 17. 7 is contained in 17, 2 times and 3 remainder. 3 prefixed to 5 makes 35, and 7 is contained in 35, 5 times. Hence the quotient is 125.

133. In ***Short Division*** only the quotient is written, the operations being performed *mentally*. It is generally used when the divisor does not exceed 12.

In like manner, divide and analyze the following:

(2.)	(3.)	(4.)	(5.)
$6\underline{)7944}$	$7\underline{)9464}$	$8\underline{)8928}$	$5\underline{)6895}$

6. Divide 92352 by 8; by 6; by 4.

7. Divide 83762 by 7; 79880 by 6; 3263 by 8.

OPERATION.
$8\underline{)3263}$
Quotient $407\frac{7}{8}$

ANALYSIS.—Since 8 is not contained in 3 thousands, unite the 3 thousands and 2 hundreds, making 32 hundreds. 8 is contained in 32 hundreds, 4 hundreds times, which write in the hundreds' place in the quotient. Next, 8 is not contained in 6 tens, so write a cipher in tens' place in the quotient, and unite the 6 tens and 3 units. 8 is contained in 63 units 7 times and 7 units remainder, which write over the divisor and add as a part of the quotient. Hence the quotient is $407\frac{7}{8}$.

PROOF.—Multiply the quotient 407 by the divisor 8, and the product is 3256; 3256 plus the remainder 7, equals the dividend 3263. (PRIN. 3.)

8. Divide 8135464 by 6; by 8; by 7; by 5; by 9.

9. Divide $48.56 by 8 cents.

OPERATION.
$\$.08\underline{)\$48.56}$
607 times.

Eight cents may be written $.08 (**73**). When the divisor and dividend are like numbers, the quotient is an abstract number (PRIN. 1). Hence 8 cents are contained in $48.56, 607 *times*.

10. Divide $48.56 by 8.

OPERATION.

8) $48.56

$6.07

When the divisor is an abstract number, the dividend and quotient are like numbers (PRIN. 2). Hence 1 *eighth* of $48.56 is $6.07.

Solve and prove,

(11.)	(12.)	(13.)	(14.)
9) $217.62	7) $6.44	$7) $644	$.07) $6.44
$24.18	$.92	92 times.	92 times.

How many times

15. Are $8 contained in $15096? In $58424? In $23064?

16. Is 7 contained in 330457? In 19278? In 918271?

17. Is 9 contained in 436281? In 605675? In 1039126?

Find

18. 1 *fifth* of $863.25.

19. 1 *sixth* of 34807 tons.

20. 1 *eighth* of 20673 days.

21. 1 *ninth* of $7384.50.

What is

22. $\frac{1}{7}$ of 500322 miles?

23. $\frac{1}{4}$ of 32876 men?

24. $\frac{1}{3}$ of 60349 acres?

25. $\frac{1}{12}$ of 760344 rods?

26. How many barrels of flour at $8 a barrel, can be bought for $12736? For $7068?

27. If 75000 bushels of grain are put into 8 bins of equal size, how many bushels does each bin contain?

28. If 9 acres of land cost $976.50, what is the cost of 1 acre?

29. How many oranges can be bought for $3.72, at 4 cents a piece?

30. At 8 cents a yard, how many yards of ribbon can be bought for $7.28?

31. Paid $1792 for 7 horses. What did each cost?

ORAL EXERCISES.

134. 1. The quotient of two numbers is 15, and the divisor is 8. What is the dividend?

2. The dividend is 96, and the quotient is 6. What is the divisor?

3. The quotient is 12, the remainder is 9, and the divisor is 11. What is the dividend?

4. If 12 yards of cloth cost $35, for how much a yard must it be sold to gain $13?

5. A man received $50 for 5 barrels of pears, and paid all but $14 for 4 chairs. What did each chair cost?

6. If 4 weeks' board cost $28, what will 9 weeks' board cost?

Analysis.—One week's board will cost 1 *fourth* of $28, or $7; and 9 weeks' board will cost 9 times $7, or $63.

7. If 8 yards of silk cost $32, what will 12 yards cost?

8. What will 15 sheep cost, if 5 sheep cost $35?

9. How many cords of wood at 4 dollars a cord, will pay for 6 barrels of flour at $8 a barrel?

Analysis.—Six barrels of flour will cost 6 times $8, or $48; and $4, the price of 1 cord of wood, are contained in $48, 12 times. Hence, etc.

10. How many days' labor at $4 a day will pay for 3 tons of coal at $6 a ton, and 2 tons of hay at $15 a ton?

11. How many pounds of meat at 12 cents a pound, will cost as much as 9 pounds of cheese at 8 cents a pound?

Complete the following equations:

12. $8 \times 0 + 6 \times 4 \div 8 = ?$
13. $10 \times 12 - 0 \times 6 \div 6 = ?$
14. $9 \times 11 - \overline{54 \div 6} + 20 = ?$
15. $\overline{63 \div 7} \times 0 + 12 = ?$
16. $(108 \div 12) \times 11 - 25 = ?$
17. $90 - 18 \div (44 - 7 \times 6) = ?$

WRITTEN EXERCISES.

135. When the divisor consists of more than one order of units.

1. Divide 5437 by 26.

OPERATION.

Divisor. Dividend. Quotient.

$26)5437(209\frac{3}{26}$

52

$\overline{237}$

$\underline{234}$

3 Remainder.

ANALYSIS.—26 is contained in 54 hundreds, 2 hundred times, with a remainder. Write the 2 hundreds in the quotient, and multiply the divisor 26 by this quotient figure, and subtract the product, 52 hundreds, from 54 hundreds, the *first* partial dividend, and there remains 2 hundreds. To this annex the 3 tens of the dividend, making 23 tens for the *second* partial dividend.

26 is not contained in 23, so write a cipher in the quotient and bring down the 7 units of the dividend, making 237 units for the *third* partial dividend.

26 is contained in 237 units 9 times, with a remainder. Write the 9 units in the quotient, and multiplying and subtracting as before, there remain 3 units, which write over the divisor, and annex as a part of the quotient. Hence the quotient is $209\frac{3}{26}$.

136. ***Long Division*** is the process of dividing when the subtractions *are written.*

2. Find how many times 204 is contained in 1041835.

OPERATION.

Divisor. Dividend. Quotient.

$204)1041835(5107\frac{7}{204}$

$\underline{1020}$

218

$\underline{204}$

1435

$\underline{1428}$

7 Remainder.

PROOF.

5107 Quotient.

$\underline{204}$ Divisor.

20428

$\underline{10214}$

1041828

$\underline{7}$ Remainder.

1041835 Dividend.

3. Divide 32762 by 14; by 16; by 23; by 28.
4. Divide 130426 by 58; by 63; by 81; by 74.

RULE.—I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the least number of the left hand orders of the dividend that will contain it, and write the result for the first figure of the quotient.*

III. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder annex the figure of the next lower order of the dividend for a new partial dividend, and divide as before.*

IV. *Proceed in the same manner until all the orders of the dividend have been used.*

V. *If any partial dividend does not contain the divisor, write a cipher in the quotient, and annex the next order of the dividend, and proceed as before.*

VI. *If there be at last a remainder, write it after the quotient with the divisor underneath.*

PROOF.—*Multiply the divisor by the quotient, and to the product add the remainder, if any. If the work is correct, the result will be equal to the dividend.*

1. If the product of the divisor and quotient be *greater* than the *partial* dividend, the quotient is *too large*, and must be *diminished.*
2. If any remainder is *equal to* or *greater* than the divisor, the quotient is *too small* and must be *increased.*

137. When the divisor and dividend are both concrete numbers, they must be of the *same name.* Hence, if one be *dollars*, and the other *cents*, or dollars and cents, before dividing, change so that both may be *cents.*

138. Since 100 cents make 1 dollar, there are 100 times as many cents as dollars. Hence,

To change a number representing dollars to a number representing cents, annex two ciphers (110), omit the sign ($) and write the word *cents* after it.

To change dollars and cents to the same form, omit the sign ($) and the point (.) and write the word *cents* at the right.

5. Divide $46.92 by 23.

OPERATION.

```
23)$46.92($2.04
   46
   ----
     92
     92
     --
```

6. Divide $46.92 by 23 cents.

OPERATION.

```
23)4692(204 times.
   46
   ----
     92
     92
     --
```

7. Divide $46.92 by $23.

OPERATION.

```
2300)4692(2 92/2300 times.
     4600
     ----
       92
```

8. Divide $46 by 23 cents.

OPERATION.

```
23)4600(200 times.
   46
   ----
     00
```

In like manner divide, and prove the following:

9. $325.72 by 34.
10. $938.07 by 63.
11. $3176.46 by 126.
12. $49.56 by 14 cents.
13. $87.36 by 21 cents.
14. $288.96 by $.43.
15. $810.98 by $.46.
16. $594 by 18 cents.
17. $1385 by $105.
18. $32.48 by $7.

How many times

19. Is 47 contained in 30176? In 27865? In 103474?

20. Is 185 contained in 200376? In 4701625?

21. The annual receipts of a company are $570685. What is the average a day, if there are 313 working days?

22. If 867 shares of railroad stock are valued at $84099, what is the value of each share?

23. A plantation of 736 acres was sold for $55936. What was the price of an acre?

24. Paid $17100 for a farm, at the rate of $36 an acre. How many acres did it contain?

25. How many horses, at $125 each, will $4735 buy, and how much money will be left?

Divide

26. 33490 by 85.
27. 740070 by 135.
28. 1554768 by 216.
29. 5497800 by 175.
30. 3931476 by 556.
31. 5120401 by 587.
32. 1018090 by 1669.
33. 73484248 by 2624.

Divide

34. 863256 by 736.
35. 1646301 by 381.
36. 5226412 by 2567.
37. 11214887 by 3076.
38. 75862500 by 10115.
39. 313194105 by 7153.
40. 1246038849 by 269181.
41. 2331883954 by 6739549.

139. To divide by the factors of a number.

1. Divide 644 by 28, using the factors.

OPERATION.

```
4)6 4 4
  -----
7)1 6 1
  -----
    2 3
```

ANALYSIS.—Since 28 is equal to 4 times 7, divide either by 28, or by its factors 4 and 7. Now, 644 ÷ 4 = 161; but this quotient is 7 times too great, and must therefore be divided by 7; hence, 161 ÷ 7 = 23 the true quotient.

Factors.

2. Divide 6228 by 36, or by 4, and 9.
3. Divide 27360 by 96, or by 3, 4, and 8.
4. Divide 526050 by 126, or by 2, 7, and 9.
5. Divide 73416 by 168, or by 4, 6, and 7.

6. Divide 5831 by 84, using the factors, 3, 4, and 7.

OPERATION.

$$\begin{array}{rlr} 3)\underline{5831} & & \\ 4)\underline{1943} & \ldots\ldots & 2 \\ 7)\underline{485} & \ldots 3\times 3 = & 9 \\ 69 & \ldots 2\times 4\times 3 = & \underline{24} \\ & \text{True Remainder.} & 35 \\ 69\frac{35}{84} & \text{Quotient.} & \end{array}$$

ANALYSIS.—Since 84 is equal to 3×4×7, divide by 84, or by its factors 3, 4, and 7.

$5831 \div 3 = 1943$, and a remainder of 2, which being a part of the dividend, is also a part of the true remainder.

$1943 \div 4 = 485$, and a remainder of 3. Since a unit of the first quotient 1943, equals 3 units of the dividend, this second remainder 3 being a part of 1943, equals 3 × 3, or 9 units of the dividend.

$485 \div 7 = 69$, and a remainder of 2. Since a unit of the second quotient 485 equals 4 units of the first quotient 1943, this third remainder 2 being a part of 485, equals 2 × 4 × 3, or 24 units of the dividend. Hence the first *partial* remainder is 2, the second is 9, the third is 24, and the *true* remainder 35; and the quotient $69\frac{35}{84}$.

7. Divide 139074 by 72, using its factors 3, 4, and 6.

8. Divide 7360479 by 96, using its factors 2, 6, and 8.

RULE.—I. *Separate the divisor into two or more factors.*

II. *Divide the dividend by one of these factors, and the quotient thus obtained by another factor, and so on until all the factors have been used as divisors.*

III. *If there be remainders, multiply each remainder by all the divisors preceding the one that produced it.*

IV. *Add the products and the remainder from the first division, if any, and the sum will be the true remainder.*

9. Divide 1315125 by 315, or by 5, 7, and 9.

10. Divide 73522 by 135, or by 3, 5, and 9.

11. Divide 401976 by 245, or by 5, 7, and 7.

140. When the divisor has ciphers on the right.

1. Divide 4067 by 10.

OPERATION.

$$1|0)\underline{406|7}$$
$$406\ ..\ 7 \text{ Rem.}$$
$$406\tfrac{7}{10} \text{ Quotient.}$$

Analysis.—Since removing any order of figures *one place* to the *right*, diminishes its value *ten times* (**45**), by cutting off, or taking away, the right-hand figure of a number, each of the remaining figures, being removed one place to the right, is diminished in value ten times, or *divided by* 10.

For similar reasons, cutting off *two* figures divides by 100, cutting off *three* figures, divides by 1000, and so on. The remaining figures are the quotient, and those cut off, the remainder.

Divide

2. 37684 by 100.
3. 103076 by 1000.
4. 267104 by 10000.
5. 5023001 by 100000.

6. Divide 2416700 by 6000.

OPERATION.

$$6|000)\underline{2416}|700$$
$$402\ \ .\ \ .\ \ 4700 \text{ Rem.}$$
$$402\tfrac{4700}{6000} \text{ Quotient.}$$

Analysis.—Resolve 6000 into the factors 1000, and 6. First divide by 1000, by cutting off the three right-hand figures of the dividend. The quotient is 2416, and a remainder of 700. Next divide 2416 by 6; the quotient is 402 and a second remainder of 4 thousands, which prefixed to the *first* remainder 700 gives a *true* remainder of 4700. Hence the quotient is $402\frac{4700}{6000}$.

In like manner, divide

7. 307200 by 900.
8. 7820305 by 28000.
9. 5761321 by 2040.
10. 8073160 by 14800.

Rule.—I. *Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend.*

II. *Divide the remaining part of the dividend by the remaining part of the divisor.*

III. *Prefix the remainder, if any, to the figures cut off, and the result will be the true remainder.*

11. If it require $34400 to pay a regiment of 800 men, how much does each man receive?

12. At $3400, how many lots can be bought for $68000?

13. How many bales, each weighing 470 pounds, can be made of 39500 pounds of cotton?

GENERAL PRINCIPLES OF DIVISION.

141. The quotient depends upon the *relative* values of the dividend and divisor. Hence, any change in the value of either dividend or divisor, will produce a change in the value of the quotient. But some changes may be made upon both dividend and divisor, which will not affect their *relative* values, and consequently will not affect the quotient. To illustrate, let $54 \div 9 = 6$, be the fundamental equation, with which the following are to be compared:

1. $(54 \times 3) \div 9 = 162 \div 9 = 18.$	Multiplying the dividend by 3 multiplies the quotient by 3.
2. $54 \div (9 \div 3) = 54 \div 3 = 18.$	Dividing the divisor by 3 multiplies the quotient by 3.
3. $(54 \div 3) \div 9 = 18 \div 9 = 2.$	Dividing the dividend by 3 divides the quotient by 3.
4. $54 \div (9 \times 3) = 54 \div 27 = 2.$	Multiplying the divisor by 3 divides the quotient by 3.
5. $(54 \times 3) \div (9 \times 3) = 162 \div 27 = 6.$	Multiplying both dividend and divisor by 3 does not change the quotient.
6. $(54 \div 3) \div (9 \div 3) = 18 \div 3 = 6.$	Dividing both dividend and divisor by 3 does not change the quotient.

These six equations illustrate the following

142. General Principles of Division.

1. *Multiplying the dividend,* or *Dividing the divisor,*	*Multiplies the quotient.*
2. *Dividing the dividend,* or *Multiplying the divisor,*	*Divides the quotient.*
3. *Multiplying or dividing both dividend and divisor by the same number,*	*Does not change the quotient.*

These *three principles* may be embraced in one

GENERAL LAW.

143. *A change in the dividend produces a* LIKE *change in the quotient, but a change in the divisor produces an* OPPOSITE *change in the quotient.*

GENERAL REVIEW.

ORAL EXAMPLES.

144. 1. The sum of three numbers is 40. One of the numbers is 12, and another is 15. What is the third?

2. The difference of two numbers is 16, and the smaller is 12. What is the larger?

3. The difference of two numbers is 18, and the larger is 30. What is the smaller?

4. The product of two numbers is 132, and one of the numbers is 11. What is the other?

5. What five numbers less than 10 will divide 120 without a remainder?

6. The sum of two numbers is 21, and the greater 12. What is the product of the two numbers?

7. The quotient of two numbers is 45, and the divisor 8. What is the dividend?

8. How many times can 8 bushels of grain be taken from a bin containing 52 bushels, and what will remain?

9. A news-boy sold 24 papers at 4 cents each, and thereby gained 48 cents. At what rate did he buy the papers?

10. The dividend is 240 and the quotient 12. What is the divisor?

11. The quotient is 20, the remainder 8, and the divisor 9. What is the dividend?

12. A drover bought 10 sheep at $8 a head, and sold them for $96. How much did he gain a head?

How many

13. In each of 5 equal parts of $(9 \times 12 - 8 \times 6)$?

14. In each of 9 equal parts of $(56 - 0 \times 7 + 16)$?

15. In each of 7 equal parts of $(72 - 40 + \overline{37 - 20})$?

16. If 5 men can build a wall in 9 days, in how many days can 3 men build it?

ANALYSIS.—It will take 1 man 5 times 9 days, or 45 days; and 3 men can build it in 1 *third* of 45 days, or 15 days.

17. How long will it take 7 men to do the same work that 14 men can perform in 3 days?

18. If 9 days' work will pay for 6 tons of coal at $6 a ton, what is the price of a day's labor?

19. How much pork can be bought for 96 cents, if 9 pounds cost 72 cents?

20. If 5 men can build a wall in 8 days, how many men can build it in 4 days?

ANALYSIS.—It will require 8 times 5 men, or 40 men, to build it in 1 day, and 1 *fourth* of 40 men, or 10 men, to build it in 4 days.

21. How many men will be required to do the same work in 5 days that 4 men can do in 40 days?

22. If 6 men can dig a ditch in 5 days, how many men would be required to dig it in 1 day? In 2 days? In 3 days? In 6 days? In 10 days?

23. At the rate of 24 miles in 8 hours, how many miles would a man walk in 12 hours?

24. If a woman pay 60 cents for some lemons, at the rate of 10 cents for 6, and sell them at the rate of 9 for 20 cents, how many cents will she gain?

25. If 5 barrels of flour are worth $60, how many cords of wood at $4 a cord will pay for 3 barrels?

26. If 12 yards of cloth cost $40, for how much must it be sold a yard to gain $20?

27. What cost 9 quarts of milk, if 4 quarts cost 24 cents?

28. How many bags will be required to hold 108 bushels of wheat, if 4 bags hold 9 bushels?

29. To 6 add 8, subtract 4, multiply by 5, add 6, divide by 8, and what is the result?

30. How much greater is 7 times 8 plus 4, than 72 divided by 9, multiplied by 7?

31. How much less is 10 times 10, diminished by 4 times 10, plus 12, than 100 divided by 10, plus 8 times 11?

Find the *required* term in the following equations:

32. $25+9-32+4=?$
33. $4\times 12+3\times 9=?$
34. $\overline{60-12}\div 6\times ?=56$
35. $\overline{72\div 9}\times\overline{22-10}=?$
36. $120\div 20+48\div ?=9$
37. $\overline{96\div 8}\times 9=?\times 12$
38. $(\overline{32+12}\div 11)\times ?=80$
39. $(132\div 11-4)\times 9=60+?$
40. $42+24-15=?+10$
41. $\overline{48+36}\div\overline{48-36}=16-?$
42. $(120-7\times 12)\div 6=?\div 11$
43. $\overline{49+14}\div(28-19)=25-?$

WRITTEN EXAMPLES.

145. 1. Subtract 2520 from the sum of 3472, 450, 1254, and 56; divide the remainder by 113, and multiply the quotient by 205. What is the result?

2. How many times can 236 be subtracted from 2124?

3. How many times 236 will produce 2124?

4. The factors of a number are 36+114, and 5640 −3007. What is the number?

5. The product of two numbers is 30128, and one of the numbers 4200÷75. What is the other?

6. Divide the product of 204 and 378 by their difference.

7. What must be added to the sum of $12.36 and $7.62, to amount to $30.76?

8. What is the difference between 746 × 23 and 18975 ÷25?

9. A man owing a debt of $3000, paid $756.50 at one time, $1289.75 at another, and then made a third payment large enough to reduce the debt to $925.60. What was the third payment?

10. How many pounds of butter at 40 cents a pound are worth as much as 1600 bushels of oats at 75 cents a bushel?

11. If a man gain $638.75 by selling 365 barrels of flour at $9.25 a barrel, at what price did he buy it?

12. The multiplier is 36, and the product 170352; if the multiplier is 1 fourth as great, what is the product?

13. The multiplier is 204, and the multiplicand is 17605; if the multiplicand were one-fifth as great, what would be the product?

14. If a mechanic receives $1500 a year for his labor, and his expenses are $968, in what time can he save enough to buy 28 acres of land at $133 an acre?

15. With the multiplier 48, the product is 166656; with a multiplicand 1 third as great, what would be the product?

16. The divisor is 16, the quotient 12624; with a divisor 1 fourth as great, what would be the quotient?

17. The divisor is 24, and the quotient is 43950; if the divisor be made 6 times as large, what will be the quotient?

18. The quotient is 91864; with a divisor 1 ninth as great, what would be the quotient?

19. A grocer bought two kinds of syrup; one for 54 cents a gallon, and the other for 62 cents. What was the *average* cost a gallon?

OPERATION.—(54 cents + 62 cents) ÷ 2 = 58 cents.

The *average* of *two* numbers is *one-half* their sum, the average of *three* numbers is *one-third* their sum, etc.

20. A merchant bought equal quantities of 3 kinds of tea, some at 60 cents, some at 78 cents, and some at 90 cents a pound. What was the average cost a pound?

21. A keeper of a toll bridge received $104 toll on Monday, $97 on Tuesday, $128 on Wednesday, and $99 on Thursday. What were the average daily receipts?

22. Sold 3 city lots for $1500, $2976, and $1895, respectively. What was the average price?

23. If a young man receive a salary of $25 a week, and he pays $8.75 for his board, and $4.65 for other expenses, in how many weeks can he pay a debt of $487.20?

24. A man having $4578 paid out all but $1642 in 8 weeks. What was the average amount paid out each week?

25. Bought 140 acres of land for $7560, and sold 86 acres of it at $75 an acre, and the remainder at cost. How much was gained?

26. A father gave his property to his 4 children. To the first he gave $6780, to the second $8200, to the third $1526 more than to the first, and to the fourth $1345 less than to the third. What was the value of his property?

27. The sum of two numbers is 184, and their difference is 42. What are the numbers?

Analysis.—Since 184 is the sum of the numbers, if the difference 42 be subtracted from the sum 184, the remainder 142 will be *twice* the *less* number. 142 ÷ 2 = 71 the less number; and 71 + 42 = 113 the *greater* number.

Or, if the *difference* 42 be added to the *sum* 184, the amount 226, will be *twice* the *greater* number. 226 ÷ 2 = 113 the greater number; and 113 − 42 = 71 the less number.

Proof.—113 + 71 = 184 the sum.

28. The sum of two numbers is 5672, and their difference is 1974. What are the numbers?

29. A man paid $1250 for a horse and carriage, the horse being valued at $190 more than the carriage. What was the value of each?

30. At a town election the whole number of votes cast for two candidates was 3789, and the majority for the successful candidate was 227. How many votes did each receive?

31. Two men are worth $28475, and one is worth $4625 more than the other. How much is each man worth?

32. A grocer wishes to put 240 pounds of tea into three kinds of boxes, containing respectively 5, 10, and 15 pounds, using the same number of boxes of each kind. How many boxes will be required?

33. Sold a quantity of wood for $2492, that cost $1424, thus gaining $3 a cord. How many cords were there, and what was the cost per cord?

34. What number divided by 36, the quotient increased by 48, the sum diminished by 37, the remainder multiplied by 14, and the product increased by 216÷72, is 269?

Find the missing *term* in the following equations:

35. $(15341 \div 29) \times (8430 \div 1405) = 1587 \times ?$

36. $[4500 + (12000 - 1375) \div \overline{121 \times 25}] \times 48 = ? \quad \times 24$

37. $732 \times 6 \div (\overline{15 \times 24} \div \overline{9 \times 10}) + (42 \times \overline{234 \div 26}) = ?$

38. $450 + (\overline{24 - 12) \times 5} \div (90 \div 6) + (\overline{3 \times 11} - 18 = ?$

146. The pupil should illustrate the following problems by original examples:

PROBLEM 1. Given several numbers, to find their sum.

2. Given the sum of several numbers and all of them but one, to find that one.

3. Given the parts, to find the whole.

4. Given the whole and all the parts but one, to find that one.

5. Given two numbers, to find their difference.

6. Given the greater of two numbers and their difference, to find the less.

7. Given the less of two numbers and their difference, to find the greater.

8. Given the minuend and subtrahend, to find the remainder.

9. Given the minuend and remainder, to find the subtrahend.

10. Given the subtrahend and remainder, to find the minuend.

11. Given two or more numbers, to find their product.

12. Given the product and one of two factors, to find the other factor.

13. Given the multiplicand and multiplier, to find the product.

14. Given the product and multiplicand, to find the multiplier.

15. Given the product and multiplier, to find the multiplicand.

16. Given two numbers, to find their quotient.

17. Given the divisor and dividend, to find the quotient.

18. Given the divisor and quotient, to find the dividend.

19. Given the dividend and quotient, to find the divisor.

20. Given the divisor, quotient, and remainder, to find the dividend.

21. Given the dividend, quotient, and remainder, to find the divisor.

22. Given the final quotient of a continued division and the several divisors, to find the dividend.

23. Given the quotient of a continued division, the first dividend, and all the divisors but one, to find that divisor.

24. Given the dividend and several divisors of a continued division, to find the quotient.

25. Given two or more sets of numbers, to find the difference of their sums.

26. Given two or more sets of factors, to find the sum of their products.

27. Given two or more sets of factors, to find the difference of their products.

28. Given the sum and the difference of two numbers, to find the numbers.

147. SYNOPSIS FOR REVIEW.

DIVISION.

- 1. DEFINITIONS.
  - 1. Division. 2. Dividend. 3. Divisor. 4. Quotient. 5. Remainder 6. Sign of Division.
- 2. PRINCIPLES, 1, 2, and 3.
- 3. RELATION OF DIVISION TO SUBTRACTION.
- 4. RELATION OF DIVISION TO MULTIPLICATION.
- 5. OBJECTS OF DIVISION.
  - 1. Illustrate.
  - 2. "
- 6. EQUAL PARTS. 1, 2.
- 7. SHORT DIVISION.
  - 1. Definition.
  - 2. Method.
- 8. LONG DIVISION.
  - 1. Definition.
  - 2. Method.
  - 3. Rule, I—VI.
  - 4. Proof.
- 9. DIVISION OF DOLLARS AND CENTS.
  - 1. When divisor and dividen are concrete, but unlike.
  - 2. How to change dollars to cents.
  - 3. How to change dollars and cents to cents.
- 10. DIVISION BY FACTORS.
  - 1. Method.
  - 2. Rule, I, II, III, IV
- 11. WHEN THE DIVISOR HAS CIPHERS ON THE RIGHT.
  - 1. Method.
  - 2. Rule, I, II, III
- 12. GENERAL PRINCIPLES OF DIVISION, 1, 2, 3.
- 13. GENERAL LAW.

PROPERTIES OF NUMBERS

148. 1. What two numbers, besides the number itself and 1, will give a product of 8? 16? 25? 42? 64?

2. What numbers, other than the given number and 1, will exactly divide 9? 15? 36? 48? 55?

3. Of what sets of two numbers is 24 the product?

4. Of what sets of three numbers is 36 the product?

5. What are the smallest numbers, other than 1, that will exactly divide 18? 21? 49? 55?

6. What is the largest number, other than the given number itself, that will exactly divide 22? 24? 30? 40?

7. Name the numbers between 12 and 30, that are the product of *two* factors greater than 1. Between 30 and 50.

8. Name the numbers between 5 and 20, that have no other factors than the numbers themselves and 1.

9. Of what number are 7 and 8 the factors? 2, 5, and 7? 4, 5, and 3? 2, 3, 5, and 10?

DEFINITIONS.

149. The ***Properties of Numbers*** are those qualities or elements which necessarily belong to numbers.

Numbers are either *Integral, Fractional,* or *Mixed.*

150. An ***Integral Number*** or ***Integer*** is a number representing whole things. (4.)

Thus, 8, 23, 30 men, 45 pounds are integral numbers.

Integral numbers are either *Even* or *Odd, Prime* or *Composite.*

151. An ***Even Number*** is a number that is exactly divisible by 2.

All numbers whose unit figure is 0, 2, 4, 6, or 8, are *even*.

152. An ***Odd Number*** is a number that is not exactly divisible by 2.

All numbers whose unit figure is 1, 3, 5, 7, or 9, are *odd*.

153. A ***Prime Number*** is a number that has no integral factors except unity and itself.

Thus, 2, 3, 5, 11, 23, etc., are prime numbers.
2 is the only even prime number.

154. A ***Composite Number*** is a number that has other *integral* factors besides unity and itself.

Thus, 21 is a composite number, since $21 = 7 \times 3$.

155. The ***Factors*** of a number, are the numbers which multiplied together will produce it. **(108.)**

Thus, 7 and 8 are factors of 56; 3, 4, and 7, of 84.

156. A ***Prime Factor*** is a prime number used as a *factor*. **(153.)**

The *prime factors* of a number are also the *prime divisors* of it.

157. An ***Exact Divisor*** of a number is one that will divide that number without a remainder.

Thus, 6 is an exact divisor of 48, and 9 an exact divisor of 72.

1. The *Exact Divisors* of a number are also the *factors* of that number.

2. An exact divisor of a number is sometimes called the *measure* of that number.

3. When a number is a factor, or divisor, of each of *two* or *more* numbers, it is called a *common* factor, or divisor, of those numbers.

158. Numbers are ***prime to each other*** when they have no *common integral factors, or divisors.*

Thus, 9 and 14, 16 and 25 are prime to each other.

DIVISIBILITY OF NUMBERS.

159. A number is said to be divisible by another, when there is no remainder after dividing. Any number is *divisible*

1. By 2, if it is an *even* number.

Thus, 20, 24, 36, and 44 are divisible by 2.

2. By 3, if the sum of its digits is divisible by 3.

Thus, 135, 471, and 1134 are divisible by 3.

3. By 4, if its two right-hand figures are ciphers, or express a number divisible by 4.

Thus, 300, 432, and 1548 are divisible by 4.

4. By 5, if it ends with a cipher or 5.

Thus, 30, 45, and 235 are divisible by 5.

5. By 6, if it is an *even* number and divisible by 3.

Thus, 168, 402, and 1314 are divisible by 6.

6. By 8, if its three right-hand figures are ciphers, or express a number divisible by 8.

Thus, 3000, 2728, and 10576 are divisible by 8.

7. By 9, if the sum of its digits is divisible by 9.

Thus, 217683 and 401301 are divisible by 9.

8. By 10, if it ends with one or more ciphers.

Thus, 40, 500, 3000 are respectively divisible by 10, 100, and 1000.

9. By 7, 11, and 13, if it consists of but *four* places, the first and fourth being occupied by the same significant figures, and the second and third by ciphers.

Thus, 2002, 3003, and 5005, are divisible by 7, 11, and 13.

10. An *odd* number is not divisible by an *even* number.

11. If an *even* number is divisible by an *odd* number, the quotient will be an *even* number.

Thus, the quotient of 36 divided by 9, is 4; of 42 by 7, is 6.

12. If an even number is divisible by an odd number, it is also divisible by *twice* that number.

Thus, 28 is divisible by 7, and also by *twice* 7.

13. Every odd number except 1, increased or else diminished by 1, is divisible by 4.

Thus, 11 *increased* by 1, or 17 *diminished* by 1, is divisible by 4.

14. Every prime number except 2 and 3, increased or else diminished by 1, is divisible by 6.

Thus, 23 increased by 1, or 31 diminished by 1, is divisible by 6.

EXERCISES.

160. Find by inspection some of the exact divisors of the following numbers :

1. 1536.
2. 1683.
3. 3348.
4. 6105.
5. 12936.
6. 43560.
7. 32472.
8. 71460.
9. 197200.

FACTORING.

ORAL EXERCISES.

161. 1. What are the even numbers from 12 to 36?

2. What are the odd numbers from 12 to 36?

3. What are the prime numbers from 12 to 36?

4. What are the composite numbers from 12 to 36?

5. Name all the prime factors of 36.

6. Name all the composite factors of 36.

7. What are the prime factors of 35? 49? 60?

8. What are the composite factors of 32? 48? 72?

9. What prime factors are common to 21 and 42?

10. What composite factors are common to 36 and 72?

11. What factors are common to 18 and 30? To their *sum* and *difference*?

12. What factors are common to the sum and difference of 20 and 40?

13. What prime factors are common to 14 and 4 times 14?

14. What two composite factors are common to 24 and 3 times 24?

15. What is the largest, and what the smallest prime factor of 15, 30, and 45?

DEFINITIONS AND PRINCIPLES.

162. ***Factoring*** is the resolving of a composite number into its factors, and is performed by division.

163. An ***Exponent*** is a small figure written at the right of a number, and a little above, to show how many times the number is used as a factor.

Thus, $2^3 = 2 \times 2 \times 2$, and denotes that 2 is used as a factor 3 *times*. 5^4, denotes that 5 is used as a factor 4 *times*.

164. PRINCIPLES.—1. *The prime factors of a number, or the product of any two or more of them, are the only exact divisors of that number.*

2. *A factor of a number is a factor also of any number of times that number.*

3. *A factor common to two or more numbers is a factor of their sum, and also of the difference of any two of them.*

4. *Every composite number is equal to the product of its prime factors.*

WRITTEN EXERCISES.

165. To find all the prime factors of a composite number.

1. What are the prime factors of 2772?

OPERATION.

2)2772
2)1386
3)693
3)231
7)77
11

ANALYSIS.—Since the given number is even, divide it by 2, the least prime factor, and the result also by 2, which gives an odd number for a quotient.

Next divide by the prime factors 3, 3, and 7, successively, obtaining for the last quotient 11, which not being divisible, is a prime factor of the given number. Hence the divisors 2, 2, 3, 3, 7, and the last quotient 11, are all the prime factors, or divisors, of 2772, and may be written 2^2, 3^2, 7, 11.

In like manner find the prime factors or divisors

2. Of 1050.
3. Of 1140.
4. Of 2445.
5. Of 2366.
6. Of 2205.
7. Of 2310.

RULE.—*Divide the given number by any prime factor of it, and the resulting quotient by another, and so continue the division until the quotient is a prime number. The several divisors and the last quotient are the prime factors.*

PROOF.—*The product of all the prime factors is equal to the given number.* (PRIN. 4.)

Resolve the following numbers into their prime factors

8. 1155.
9. 2934.
10. 6300.
11. 2205.
12. 13981.
13. 32320.
14. 21504.
15. 29925.
16. 12673.
17. 10010.
18. 28665.
19. 31570.

COMMON DIVISORS.

ORAL EXERCISES.

166. 1. Name two exact divisors of 12. Of 15. Of 20.

2. Name three exact divisors of 24. Of 48. Of 72.

3. What number is an exact divisor of 27 and of 56?

4. What are the prime divisors of 15? 55? 49? 77?

5. What are the composite divisors of 72? 84? 120?

6. What prime divisor is common to 28, 35, and 42?

7. Name a common measure of 22, 44, and 66.

8. Name the greatest common measure of 16, 32, and 64.

9. Of what three numbers is 12 a common divisor?

10. What two numbers will exactly divide 15 and 30? Their *sum* and *difference?*

11. What is the smallest exact divisor of the sum and difference of 10 and 15? Of 21 and 56?

12. What is the greatest exact divisor of the sum and difference of 16 and 24? Of 18 and 45?

13. Find the greatest common measure of 14, 42, and 56.

14. Find the greatest common divisor of 27, 36, and 45.

DEFINITIONS AND PRINCIPLES.

167. A ***Common Divisor*** of two or more numbers is a *common factor* of each of them.

168. The ***Greatest Common Divisor*** of two or more numbers is the *greatest common factor,* and is the product of *all* the common *prime* factors.

169. PRINCIPLES.—1. *The only exact divisors of a number are its prime factors, or the product of two or more of them.*

2. *An exact divisor divides any number of times its dividend.*

3. *A common divisor of two or more numbers will divide their sum, and also the difference of any two of them.*

4. *The greatest common divisor of two or more numbers is the product of all their common prime factors.*

WRITTEN EXERCISES.

170. When the numbers can be readily factored.

1. What is the greatest common divisor of 42, 63, and 126?

1ST OPERATION.

$$\begin{aligned} 42 &= 7 \times 3 \times 2 \\ 63 &= 7 \times 3 \times 3 \\ 126 &= 7 \times 3 \times 6 \end{aligned}$$

ANALYSIS.—By factoring the given numbers, the prime factors common to *all* of them are 7 and 3. Hence $7 \times 3 = 21$ is the greatest common divisor of 42, 63, and 126. (PRIN. 4.)

2D OPERATION.

$$\begin{array}{r|rrr} 3 & 42 & 63 & 126 \\ \hline 7 & 14 & 21 & 42 \\ \hline & 2 & 3 & 6 \end{array}$$

ANALYSIS.—Since the given numbers are exactly divisible by 3, and the resulting quotients by 7, they are also divisible by 7×3, or 21. (PRIN. 1.)

If there were other factors of the greatest common divisor, then the quotients 2, 3, and 6 would be exactly divisible by them.

Find the greatest common divisor

2. Of 42 and 112.

3. Of 96 and 544.

4. Of 40, 75, and 100.

5. Of 72, 126, and 216.

RULE.—*Separate the numbers into their prime factors and find the product of all that are common.* Or,

I. *Write the numbers in a line, and divide by any prime factor common to all the numbers.*

II. *Divide the quotients in like manner, and so continue the division till all the quotients are prime to each other.*

III. *The product of all the divisors will be the greatest common divisor.* (PRIN. 4.)

What is the greatest common divisor

6. Of 144 and 720?
7. Of 308 and 506?
8. Of 126, 210, and 252?
9. Of 72, 96, 120, and 384?

171. When the numbers cannot be readily factored.

1. Find the greatest common divisor of 527 and 1207.

OPERATION.

		1207
527	2	1054
459	3	153
68	2	136
68	4	17

ANALYSIS.—Draw two vertical lines, and place the greater number on the right, and the less on the left, one line lower down. Divide 1207 by 527, and write the quotient 2 between the vertical lines, the product, 1054, under the greater number, and the remainder 153, below.

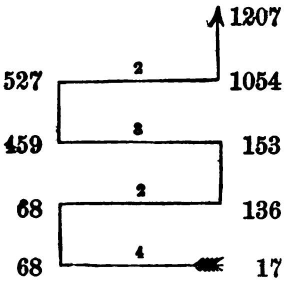
Next, divide 527 by this remainder 153, writing the quotient 3 between the verticals, the product 459, on the left, and the remainder 68, below.

Again, divide the last divisor 153, by 68, and write the product, and remainder in the same order as before.

Finally, dividing the last divisor 68, by the last remainder 17, there is no remainder. Hence 17, the last divisor, is the greatest common divisor of 537 and 1207.

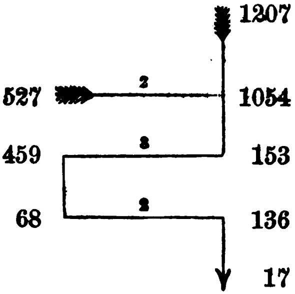
PROOF.—Now, observing that the *dividend* is always the *sum* of the product and remainder, and that the *remainder* is always the *difference* of the dividend and product, trace the work in the reverse order, as indicated by the arrow line in the diagram below.

ILLUSTRATION.



17 divides 68, as proved by the last division; it will also divide 2 times 68, or 136 (PRIN. 2). Since 17 divides both itself and 136, it will divide 153, their sum (PRIN. 3). It will also divide 3 times 153, or 459 (PRIN. 2); and, since it is a common divisor of 459 and 68, it must divide their *sum*, 527, which is one of the given numbers. It will also divide 2 times 527, or 1054 (PRIN. 2); and, since it divides 1054 and 153, it must divide their *sum*, 1207, the greater number (PRIN. 3). Hence, 17 is a *common divisor* of the given numbers.

Again, tracing the work in the direct order, as indicated in the following diagram, the *greatest* common divisor, *whatever it is*, must divide 2 times 527, or 1054 (PRIN. 2). And since it will divide both 1054 and 1207, it must divide their difference, 153 (PRIN. 3). It will also divide 3 times 153, or 459 (PRIN. 2); and as it will divide both 459 and 527, it must divide their *difference*, 68 (PRIN. 3). It will also divide 2 times 68, or 136 (PRIN. 2); and as it will divide both 136 and 153, it must divide their *difference*, 17 (PRIN. 3); *hence, it cannot be greater than* 17.



Thus, it has been shown,

1st. That 17 is a *common divisor* of the given numbers.

2d. That their greatest common divisor, whatever it be, cannot be *greater than* 17. Hence it must be 17.

In like manner, find the greatest common divisor

2. Of 316 and 664.
3. Of 679 and 1869.
4. Of 1080 and 189.
5. Of 2192 and 458.
6. Of 825 and 1372.
7. Of 2041 and 8476.
8. Of 7241 and 10907.
9. Of 2373 and 6667.

RULE.—I. *Draw two vertical lines, and write the two numbers, one on each side, the greater number one line above the less.*

II. *Divide the greater number by the less, writing the quotient between the verticals, the product under the dividend, and the remainder below.*

III. *Divide the less number by the remainder, the last divisor by the last remainder, and so on, till nothing remains. The last divisor is the greatest common divisor.*

IV. *If more than two numbers are given, first find the greatest common divisor of two of them, and then of this divisor and one of the remaining numbers, and so on to the last; the last common divisor found is the greatest common divisor of all the given numbers.*

10. What is the greatest number that will divide 3281 and 10778? 10353 and 14877?

11. What is the greatest number that will divide 620, 1116, and 1488? 396, 5184, and 6914?

12. A man having a piece of land, the sides of which are 240 feet, 648 feet, and 420 feet, wishes to inclose it with a fence having panels of the greatest possible uniform length; what will be the length of each panel?

13. A farmer wishes to put 231 bushels of corn, 393 bushels of wheat, and 609 bushels of oats into the largest bags of equal size, that will exactly hold each kind. How many bushels must each bag hold?

14. A forwarding merchant has 15292 bushels of wheat, 1520 bushels of corn, and 504 bushels of beans, which he wishes to ship, in the fewest bags of equal size that will exactly hold either kind of grain; how many bags will it take?

15. Three persons have respectively $630, $1134, and $1386, with which they agree to purchase horses, at the highest price per head, that will allow each man to invest all his money. How many horses can each man buy?

16. How many rails will inclose a field 5850 feet long by 1729 feet wide, the fence being straight, and 7 rails high, and the rails of equal length, and the longest that can be used?

MULTIPLES.

ORAL EXERCISES.

172. 1. What numbers between 5 and 30 are exactly divisible by 4? By 6? 7? 8? 9?

2. What numbers less than 40 are exactly divisible by 7?

3. What prime factors are common to 6, and 5 times 6?

4. Name some numbers exactly divisible by 4 and 6; by 3 and 7; by 5 and 7; by 8 and 10.

5. By what three prime numbers can 42 be divided?

6. Name some numbers of which 3 and 4 are factors.

7. Find the least number exactly divisible by 3, 4, and 5.

DEFINITIONS AND PRINCIPLES.

173. A ***Multiple*** of a number is a number exactly divisible by the given number; or, it is any product or dividend of which a given number is a *factor*.

1. A number may have an *unlimited number* of multiples.
2. A number is a *divisor* of all its multiples, and a *multiple* of all its divisors.

174. A ***Common Multiple*** of two or more given numbers is a number exactly divisible by each of them.

175. The ***Least Common Multiple*** of two or more given numbers is the least number exactly divisible by each of them.

Two or more numbers can have but *one* least common multiple.

176. PRINCIPLES.—1. *A multiple of a number contains each of the prime factors of that number.*

2. *A common multiple of two or more numbers contains each of the prime factors of those numbers.* Hence,

3. *The least common multiple of two or more numbers is the least number that contains each of the prime factors of those numbers.*

4. *A common multiple of two or more numbers may be found by multiplying the given numbers together.*

WRITTEN EXERCISES.

177. To find the least common multiple.

FIRST METHOD.

1. Find the least common multiple of 30, 42, and 66.

OPERATION.

$30 = 2 \times 3 \times 5$
$42 = 2 \times 3 \times 7$
$66 = 2 \times 3 \times 11$

$2 \times 3 \times 11 \times 7 \times 5 = 2310$

ANALYSIS.—The least common multiple cannot be less than the largest number 66, since it must contain 66; hence it must contain all the prime factors of 66, which are 2, 3, and 11. (PRIN. 1.) But the least common multiple of 66 must also contain all the prime factors of each of the other numbers, and since the prime factors 2 and 3 of 66 are common also to 42 and 30 omit them, and annex the factors 7 and 5 to those of 66, and the series 2, 3, 11, 7, and 5 are all the prime factors of the given numbers, and their product $2 \times 3 \times 11 \times 7 \times 5 = 2310$, is the least common multiple of the given numbers. (PRIN. 3.)

2. Find the least common multiple of 24, 42, and 17.
3. Find the least common multiple of 8, 12, 20, and 30.
4. Find the least common multiple of 10, 45, 75, and 90.

RULE.—I. *Resolve each of the given numbers into its prime factors.*

II. *Multiply together all the prime factors of the largest number, and such prime factors of the other numbers as are not found in the largest number, and their product will be the least common multiple.*

Find the least common multiple

5. Of 30, 66, 78, and 42.
6. Of 21, 30, 44, and 126.
7. Of 16, 60, 140, and 210.
8. Of 16, 48, 80, 32, and 66.

SECOND METHOD.

178. 1. Find the least common multiple of 18, 24, and 54.

OPERATION.

$$\begin{array}{r|rrr} 2 & 18 & 24 & 54 \\ \hline 3 & 9 & 12 & 27 \\ \hline 3 & 3 & 4 & 9 \\ \hline & & 4 & 3 \end{array}$$

ANALYSIS.—Write the numbers in a horizontal line, with a vertical line at the left

Since 2 is a prime factor of one or more of the given numbers, it must also be a factor of the least common multiple of those numbers. (PRIN. 3.) Hence, divide by 2 and write the quotients underneath. For a like reason divide again successively by 3 and 3, writing the quotients and undivided numbers in a line below, omitting to write any quotient when it is 1.

Since there is no factor common to 4 and 3, they are prime to each other, and hence the divisors 2, 3, and 3, with the numbers 4 and 3 in the last line, are all the prime factors of the given numbers, and their product 216 is the least common multiple. (PRIN. 3.)

If in any example, any of the smaller numbers are exactly contained in the larger, they may be omitted in finding the least common multiple, inasmuch as a number that will contain a given number, will contain any factor of that number.

Thus, if required to find the least common multiple of 8, 12, 24, 72, and 120, omit all the numbers except 72 and 120, since the others are factors of these, and the least common multiple of 72 and 120, will be the least common multiple of *all* the numbers.

2. Find the least common multiple of 32, 34, and 36.

3. Find the least common multiple of 84, 100, and 224.

RULE.—I. *Write the numbers in a horizontal line, omitting such of the smaller numbers as are factors of the larger, and draw a vertical line at the left.*

II. *Divide by any prime factor that will exactly divide two or more of the given numbers, and write the quotients and undivided numbers in a line underneath.*

III. *In like manner divide the quotients and undivided numbers until they are prime to each other.*

IV. *The product of the divisors and the final quotients and undivided numbers, is the least common multiple.*

What is the least common multiple

4. Of 4662, and 5698?

5. Of 312, 260, and 390?

6. Of 24, 10, 32, 45 and 25?

7. Of 153,204,102,and 1020?

8. Find the least common multiple of the first eight even numbers.

9. Find the least common multiple of the first five odd numbers.

10. What is the least number of oranges that can be equally distributed among 16, 20, 24, or 30 boys?

11. What is the shortest piece of rope that can be cut exactly into pieces either 15, 18, or 20 feet long?

12. What is the smallest sum of money which can be exactly expended for books at $5, or $3, or $4, or $6 each?

13. What is the product of the least common multiple of 12, 16, 24, and 32, multiplied by their greatest common divisor?

14. Divide the least common multiple of 7, 42, 6, 9, 10, and 630, by the greatest common divisor of 110, 140, and 680.

15. What is the smallest sum of money which can be exactly expended for sheep at $8, or cows at $28, or oxen at $54, or horses at $162 each?

16. What is the smallest quantity of grain that will fill an exact number of bins, whether they hold 36, 48, 80, or 144 bushels?

CANCELLATION.

ORAL EXERCISES.

179. 1. Divide 72 by 24. One-half of 72 by one-half of 24. One-third of 72 by one-third of 24.

2. Divide one-fourth of 72 by one-eighth of 24.

3. Divide 36 by 9. One-third of 36 by one-third of 9.

4. What factors are common to 72 and 24?

5. What is the quotient of 12 × 6 divided by 12 × 2?

6. Divide 3 × 3 × 3 by 3 × 3. 4 × 5 × 2 by 2 × 2 × 5.

7. Divide 7 × 6 × 2 by 2 × 6 × 7. 5 × 6 × 4 by 3 × 4 × 6.

DEFINITIONS AND PRINCIPLES.

180. ***Cancellation*** is the process of abridging operations in division by rejecting equal factors from both dividend and divisor.

181. PRINCIPLES.—1. *Rejecting a factor from any number divides the number by that factor.*

2. *Rejecting equal factors from both dividend and divisor does not change the quotient.*

WRITTEN EXERCISES.

182. 1. Divide 56 × 24, by 48 × 7.

1ST OPERATION.

$$\frac{56\times24}{48\times7}=\frac{\cancel{8}\times\cancel{7}\times\cancel{6}\times4}{\cancel{8}\times\cancel{6}\times\cancel{7}}=4$$

ANALYSIS.—Indicate the operation to be performed in the example, by writing the numbers that constitute the dividend, above a line, and those that constitute the divisor below it.

Resolve these numbers into their factors, and the dividend will consist of 8 × 7 × 6 × 4, and the divisor of 8 × 6 × 7. Rejecting equal factors from both dividend and divisor, there remains the factor 4 in the dividend. Hence the quotient is 4.

2D OPERATION.

$$\frac{\overset{\cancel{7}}{\cancel{56}} \times \overset{4}{\cancel{24}}}{\underset{\cancel{6}}{\cancel{48}} \times \cancel{7}} = 4$$

ANALYSIS.—Since it is evident that 8 will divide both 56 and 48, reject 8 as a factor of 56, retaining the factor 7, and also of 48, retaining the factor 6.

Again, since 6 will divide both 24 in the dividend and 6 in the divisor, reject 6 as a factor from both, retaining the factor 4 in the dividend. Finally, rejecting the factor 7, common both to the dividend and to the divisor, there remains only the factor 4 in the dividend, which is the required quotient.

2. Divide the product of 44, 30, 7, and 6, by the product of 33, 18, and 14; or, divide 55440 by 8316.

OPERATION.

$$\frac{55440}{8316} = \frac{\overset{2}{\cancel{44}} \times \overset{10}{\cancel{30}} \times \cancel{7} \times \cancel{6}}{\underset{3}{\cancel{33}} \times \underset{\cancel{3}}{\cancel{18}} \times \underset{\cancel{2}}{\cancel{14}}} = \frac{10 \times 2}{3} = 6\tfrac{2}{3}$$

Or,

$$\begin{array}{r|l} 3\ \cancel{3}\cancel{3} & \cancel{4}\cancel{4}\cancel{4}^{2} \\ \cancel{3}\ \cancel{1}\cancel{8} & \cancel{3}\cancel{0}^{10} \\ \cancel{2}\ \cancel{1}\cancel{4} & \cancel{7} \\ & \cancel{6} \\ \hline 3 & 20 \\ \hline & 6\tfrac{2}{3} \end{array}$$

By many it is thought more convenient to write the factors of the *dividend* on the *right* of a *vertical* line, and the factors of the *divisor* on the *left*.

3. Divide 13 × 7 × 5 × 3 by 3 × 5 × 7.

4. Divide 42 × 18 × 6 × 4 by 36 × 21 × 6.

RULE.—I. *Cancel all the factors common to both dividend and divisor.*

II. *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

When a factor equal to the number itself is canceled, the unit 1 remains, since a number divided by itself gives a quotient of 1. If the 1 occur in the dividend, it must be *retained;* if in the divisor, it need not be regarded.

5. What is the quotient of 35 × 33 × 28, divided by 15 × 14 × 11?

6. What is the quotient of 140 × 39 × 13 × 7, divided by 7 × 26 × 21?

7. Multiply 11 times 21 by 26, and divide the product by 14 times 13.

8. How many times is the continued product of 14, 9, 3, 20, 5, and 6 contained in the continued product of 183, 18, 70, 12, and 5?

9. If 213 × 190 × 84 × 264 is the dividend, and 56 times 36 multiplied by 30 is the divisor, what is the quotient?

10. $(240 \times 56 \times 18) \div (60 \times 28 \times 9) = ?$

11. $(72 \times 48 \times 28 \times 5) \div (84 \times 15 \times 7 \times 6) = ?$

12. $(66 \times 18 \times 27 \times 25) \div (84 \times 45 \times 7 \times 30) = ?$

13. $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20) = ?$

14. Multiply 64 by 7 times 31, divide the product by 8 times 56, multiply this quotient by 15 times 88, divide the product by 55, multiply this quotient by 13, and divide the product by 4 times 6. What is the quotient?

15. Find the quotient of $\frac{12 \times 60 \times 27 \times 35}{7 \times 15 \times 42 \times 108}$.

16. Find the quotient of $\frac{77 \times 100 \times 18 \times 64}{25 \times 11 \times 49 \times 16}$.

17. How many tons of hay at $18, must be given for 45 cords of wood at $4 a cord?

18. How many flour barrels at $.80 each, will pay for 112 bushels of corn at $.70 a bushel?

19. How many tubs of butter, each containing 56 pounds, at 30 cents a pound, must be given for 7 barrels of sugar, each containing 195 pounds, at 10 cents a pound?

20. A laborer gave 12 days' work for 48 bushels of potatoes, worth 50 cents a bushel. What were his daily earnings?

21. A grocer sold 24 boxes of soap, each containing 55 pounds, at 10 cents a pound, and received as pay 88 barrels of apples, each containing 3 bushels. How much were the apples worth a bushel?

22. Sold 20 pounds of butter at 27 cents a pound, which exactly paid for 15 pounds of coffee. What was the price of the coffee a pound?

23. A farmer exchanged 240 bushels of corn, worth $.75 a bushel, for an equal number of bushels of barley, worth $1 a bushel, and oats, worth $.50 a bushel. How many bushels of each did he receive?

24. A farmer bought two kinds of cloth, one kind at $.75 a yard, and the other at $.90, buying twice as many yards of the first kind as of the second. He paid for the cloth, 132 pounds of butter at 40 cents a pound. How many yards of each kind of cloth did he buy?

25. A merchant bought 6 loads of oats, each load containing 22 bags, and each bag 2 bushels, worth 56 cents a bushel. He gave in payment 8 boxes of tea, each containing 24 pounds. What was the tea worth a pound?

26. How many bushels of oats at $.60 a bushel, will pay for 12 tons of coal at $7.20 a ton?

27. How many chests of tea, each containing 63 pounds worth 87½ cents a pound, must be given for 21 bags of coffee, each weighing 28 pounds, worth 37½ cents a pound?

28. How many days' work, at $1.25 a day, will pay for 75 bushels of corn, at $.80 a bushel?

183. SYNOPSIS FOR REVIEW.

PROPERTIES OF NUMBERS.

- 1. DEFINITIONS.
  - 1. Properties of Numbers. 2. Integral Number, or Integer. 3. Even Number. 4. Odd Number. 5. Prime Number. 6. Composite Number. 7. Factors. 8. Prime Factor. 9. Exact Divisor.
- 2. DIVISIBILITY OF NUMBERS.
  - 1. How to find whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10; also by 7, 11, or 13.
  - 2. Other properties of even and of prime numbers.
- 3. FACTORING.
  - 1. Definitions.
    - 1. *Factoring.*
    - 2. *Exponent.*
  - 2. Principles, 1, 2, 3, 4.
  - 3. Rule.
  - 4. Proof.
- 4. COMMON DIVISORS.
  - 1. Definitions.
    - 1. *Divisor.*
    - 2. *Common Divisor.*
    - 3. *G. C. Divisor.*
  - 2. Principles, 1, 2, 3, 4.
  - 3. Rule (1st), I, II, III.
  - 4. Rule (2d), I, II, III, IV.
- 5. MULTIPLES.
  - 1. Definitions.
    - 1. *Multiple.*
    - 2. *C. Multiple.*
    - 3. *L. C. Multiple.*
  - 2. Principles, 1, 2, 3.
  - 3. Rule (1st), I, II.
  - 4. Rule (2d), I, II, III, IV.
- 6. CANCELLATION.
  - 1. Definition.
  - 2. Principles, 1, 2.
  - 3. Rule, I, II.

FRACTIONS

ORAL EXERCISES.

184. 1. If any unit, as an apple, or a yard, be divided into 2 equal parts, what is each part named? *One-half.*

2. If the unit be divided into 3 equal parts, what name is given to 1 of the parts? To 2 of the parts?

3. If the unit be divided into 5 equal parts, what is each part named? What name is given to 3 of the parts?

4. How many *halves* are there in a unit? How many *thirds? Fourths? Fifths? Sixths? Sevenths?*

5. If a mile be divided into 4 equal parts, what part of the whole mile is 1 of the parts? 3 of the parts?

6. What is 1 of 5 equal parts of a unit called? What are 2 of 6 equal parts called? 4 of 10 equal parts?

7. What is meant by 1 sixth of a unit? By 3 fourths?

8. What are 3 of the 7 equal parts of a week called?

9. Which is the *smaller,* one-third or one-fourth? One-fifth or one-third?

10. Which is the *greater,* one-fourth or one-sixth?

185. PRINCIPLES.—1. *The LESS the NUMBER of equal parts into which a unit is divided, the GREATER is the VALUE of each part.*

2. *The GREATER the NUMBER of equal parts into which a unit is divided, the LESS is the VALUE of each part.*

DEFINITIONS.

186. A ***Fraction*** is one or more of the equal parts of a unit. Thus, 1 *half* and 2 *thirds* are fractions.

187. A ***Fractional Unit*** is *one* of the equal parts into which any unit is divided. Thus, 1 fourth and 1 fifth are fractional units of *fourths* and *fifths.*

Fractional units take their *name* and their *value* from the *number* of parts into which the *integral* unit is divided.

188. A *fraction* is usually expressed by two numbers, called the *Numerator* and the *Denominator,* one written over the other with a line between them. A fraction written in this form is sometimes called a *Common Fraction.* Thus,

One-third	is written	$\frac{1}{3}$	Nine-tenths	is written	$\frac{9}{10}$
Three-fourths	"	$\frac{3}{4}$	Seven-twentieths	"	$\frac{7}{20}$
Five-sixths	"	$\frac{5}{6}$	Twelve-thirty-fifths	"	$\frac{12}{35}$
Seven-eighths	"	$\frac{7}{8}$	Thirty-six forty-ninths	"	$\frac{36}{49}$

189. The ***Denominator*** of a fraction shows the number of *equal parts* into which the unit is divided, and also indicates the *name* of these parts. It is written *below* the line.

Thus, in the fraction $\frac{7}{8}$, 8 is the *denominator* and shows that the unit is divided into *eight* equal parts, named *eighths.*

190. The ***Numerator*** of a fraction shows the *number* of equal parts taken to form the fraction. It is written *above* the line.

Thus, in $\frac{7}{8}$, 7 is the *numerator,* and shows that 7 of the 8 equal parts are taken, or expressed by the fraction

191. The ***Terms*** of a fraction are its numerator and denominator. Thus, 6 and 7 are the *terms* of the fraction $\frac{6}{7}$

Express by figures,

1. Five-ninths.
2. Seven twenty-fifths.
3. Nine-eighteenths.
4. Twelve twentieths.
5. Eight thirty-sixths.
6. Twenty-six forty-eighths.
7. Twenty-seven two-hundredths.
8. Forty-three ninety-ninths.
9. Sixteen one-hundred-eighths.
10. Fifty-five eighty-ninths.

Copy and read,

7. $\frac{7}{11}$; $\frac{9}{13}$; $\frac{11}{27}$; $\frac{16}{73}$; $\frac{27}{112}$; $\frac{64}{148}$; $\frac{95}{208}$.

8. $\frac{14}{116}$; $\frac{9}{84}$; $\frac{23}{180}$; $\frac{120}{450}$; $\frac{225}{876}$; $\frac{32}{960}$; $\frac{204}{600}$.

192. *Fractions* are *Proper* or *Improper*.

193. A ***Proper Fraction*** is a fraction whose numerator is less than its denominator. Its *value* is less than a unit. Thus, $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{11}{14}$ are proper fractions.

194. An ***Improper Fraction*** is a fraction whose numerator equals or exceeds its denominator. Its value is equal to, or greater than a unit. Thus, $\frac{8}{8}$, $\frac{10}{4}$, and $\frac{21}{9}$ are improper fractions.

195. A ***Mixed Number*** is an integer and a fraction united. Thus, $12\frac{5}{8}$ is equivalent to $12+\frac{5}{8}$.

196. The ***Reciprocal*** of a number is 1 divided by that number. Thus, the reciprocal of 9 is $1\div9=\frac{1}{9}$; of 16, it is $1\div16=\frac{1}{16}$, etc.

197. The ***Reciprocal of a Fraction*** is 1 divided by that fraction, or it is the fraction *inverted*. Thus, the reciprocal of $\frac{3}{4}$ is $1\div\frac{3}{4}=\frac{4}{3}$; of $\frac{7}{12}$, it is $\frac{12}{7}$.

198. The ***Value*** of a fraction is the quotient of its numerator divided by its denominator. Thus, $\frac{12}{3}=4$.

1. Analyze the fraction $\frac{7}{8}$.

Analysis.—$\frac{7}{8}$ is a fraction; 8 is the *denominator*, and shows that the unit is divided into 8 equal parts; $\frac{1}{8}$ is the *fractional unit*, since it is *one* of the *eight* equal parts into which the unit is divided; 7 is the *numerator*, and shows that *seven* of these equal parts are taken; 7 and 8 are the *terms* of the fraction. It is a *proper* fraction, since the numerator is less than the denominator; its *value* is *less* than 1; and it is read *seven-eighths.*

In like manner, analyze

2. $\frac{8}{9}$. | 3. $\frac{11}{12}$. | 4. $\frac{2}{4}$. | 5. $\frac{21}{26}$. | 6. $\frac{11}{9}$.

199. Since fractions indicate division, all changes in the *terms* of a fraction will affect the *value* of the fraction according to the laws of division; hence if we substitute the General Principles of Division (**142**), we shall have the following

200. GENERAL PRINCIPLES OF FRACTIONS.

1. *Multiplying the numerator,* or *Dividing the denominator,*	*Multiplies the fraction.*
2. *Dividing the numerator,* or *Multiplying the denominator,*	*Divides the fraction.*
3. *Multiplying or dividing both numerator and denominator by the same number,*	*Does not change the value of the fraction.*

201. These *three principles* may be embraced in one

GENERAL LAW.

A change in the NUMERATOR *produces a* LIKE *change in the value of the fraction; but a change in the* DENOMINATOR *produces an* OPPOSITE *change in the value of the fraction.*

REDUCTION.

202. To reduce fractions to higher or lower terms.

ORAL EXERCISES.

1. One-half is equal to how many fourths?

ANALYSIS.—Since 1 is equal to 4 *fourths*, $\frac{1}{2}$ is equal to 1 half of 4 fourths or 2 fourths.

2. One-third of a mile is how many *sixths* of a mile?

3. One-half of a dollar is how many *fourths* of a dollar?

4. Name some equivalent fractions for halves. Thirds.

5. Express $\frac{2}{3}$ in terms 3 times as great. 4 times as great.

6. The denominators four, six, eight, and ten, are multiples of what number?

7. Multiply both terms of $\frac{3}{4}$ by 3, and show that the *value* of the fraction is not changed.

ANALYSIS.—If both terms of $\frac{3}{4}$ are multiplied by 3, the resulting fraction is $\frac{9}{12}$, which is equivalent to $\frac{3}{4}$, since the *fractional unit* is $\frac{1}{3}$ as great, while the number taken is 3 times as great.

8. Name three equivalent fractions for $\frac{4}{5}$; for $\frac{1}{6}$; for $\frac{3}{8}$.

9. Change $\frac{5}{6}$ to twelfths. To eighteenths.

10. 8 *twelfths* are how many *thirds*?

ANALYSIS.—Since 1 third is equal to 4 twelfths, 8 twelfths are equal to as many thirds as 4 twelfths are contained times in 8 twelfths, which is 2 times. Hence there are $\frac{2}{3}$ in $\frac{8}{12}$.

11. How many *fourths* of a rod are 9 *twelfths* of a rod?

12. Divide both terms of $\frac{15}{20}$ by 5, and show that the *value* of the fraction is not changed.

ANALYSIS.—If both terms of $\frac{15}{20}$ are divided by 5, the resulting fraction is $\frac{3}{4}$, which is equivalent to $\frac{15}{20}$, since the *fractional unit* is 5 times as great, while the number taken is $\frac{1}{5}$ as great.

13. Change $\frac{18}{36}$ to an equivalent fraction having a denominator 1 *half* as great. 1 *third* as great.

14. Change $\frac{9}{27}$ to a fraction having lower terms. $\frac{14}{21}$. $\frac{25}{30}$.

15. In what lower terms can $\frac{12}{24}$ be expressed ?

16. Change $\frac{9}{36}$ to its lowest terms. $\frac{16}{20}$. $\frac{32}{40}$. $\frac{14}{35}$. $\frac{20}{25}$.

17. Name two common divisors of $\frac{18}{36}$. $\frac{24}{36}$. $\frac{30}{45}$. $\frac{33}{66}$.

18. Express $\frac{8}{16}$ in terms 4 times as great.

19. Express $\frac{12}{18}$ in terms 6 times as great.

DEFINITIONS.

203. ***Reduction of Fractions*** is the process of changing their *form* without altering their *value.*

204. A fraction is reduced to ***Higher Terms*** when the numerator and denominator are expressed in larger numbers. Thus, $\frac{3}{4}=\frac{6}{8}$, or $\frac{9}{12}$.

205. A fraction is reduced to ***Lower Terms*** when the numerator and denominator are expressed in smaller numbers. Thus, $\frac{8}{12}=\frac{4}{6}$, or $\frac{2}{3}$.

206. A fraction is reduced to its ***Lowest Terms*** when its numerator and denominator are *prime to each other*. Thus, $\frac{4}{10}=\frac{2}{5}$; $\frac{12}{16}=\frac{3}{4}$.

207. Fractions are changed to *higher terms* by Multiplication, and to *lower terms* by Division.

All higher terms of a fraction are multiples of its lowest terms.

208. PRINCIPLE.—*Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.* **(200, 3.)**

WRITTEN EXERCISES.

209. 1. Change $\frac{5}{6}$ to a fraction whose denominator is 30.

OPERATION.

$30 \div 6 = 5$

$\frac{5 \times 5}{6 \times 5} = \frac{25}{30}$

ANALYSIS.—First, divide 30, the required denominator, by 6, the denominator of the given fraction. The quotient 5 is the factor employed to produce the required denominator. Hence, multiply both terms of $\frac{5}{6}$ by 5 (**200,** 3), and $\frac{25}{30}$ is the required fraction.

2. Change $\frac{7}{12}$ to a fraction whose denominator is 96.
3. Change $\frac{11}{15}$ to a fraction whose denominator is 105.
4. Reduce $\frac{72}{120}$ to its lowest terms.

OPERATION.

$\frac{72 \div 8}{120 \div 8} = \frac{9}{15}$; $\frac{9 \div 3}{15 \div 3} = \frac{3}{5}$

Or, $\frac{72 \div 24}{120 \div 24} = \frac{3}{5}$

ANALYSIS.—Dividing both terms of the given fraction $\frac{72}{120}$, by 8, (**200,** 8) the result is $\frac{9}{15}$. Again, dividing both terms of $\frac{9}{15}$ by 3, the result is $\frac{3}{5}$. Since the terms of $\frac{3}{5}$ are *prime to each other*, the lowest terms of $\frac{72}{120}$ are $\frac{3}{5}$.

The same result is obtained more directly, by dividing both terms by their greatest common divisor, 24.

5. Reduce $\frac{84}{96}$ to its lowest terms.
6. Reduce $\frac{75}{135}$ to its lowest terms.
7. Reduce $\frac{156}{208}$ to its lowest terms.

RULES.—1. To reduce a fraction to *higher terms.*

Divide the required denominator by the denominator of the given fraction, and multiply the terms of the given fraction by the quotient.

2.—To reduce a fraction to its *lowest terms.*

Reject all factors common to the terms of the given fraction. Or,

Divide the terms of the given fraction by their greatest common divisor.

8. Change $\frac{7}{15}$ to a fraction whose denominator is 180.

9. Reduce $\frac{7}{9}$ and $\frac{5}{21}$ each to sixty-thirds.

10. Reduce $\frac{5}{8}$, $\frac{1}{4}$, and $\frac{7}{10}$, each to 120ths.

11. Reduce $\frac{10}{11}$, $\frac{1}{6}$, $\frac{19}{22}$, and $\frac{21}{33}$, each to 132ds.

12. Change 168 ÷ 252 to the form of a fraction in its lowest terms. 81 ÷ 63. 160 ÷ 400. 324 ÷ 612.

Reduce to their lowest terms,

13.	$\frac{288}{360}$.	17.	$\frac{639}{1737}$.	21.	$\frac{5643}{5940}$.
14.	$\frac{258}{282}$.	18.	$\frac{172}{1118}$.	22.	$\frac{4680}{10600}$.
15.	$\frac{288}{504}$.	19.	$\frac{568}{1544}$.	23.	$\frac{17826}{26640}$.
16.	$\frac{441}{462}$.	20.	$\frac{1344}{1536}$.	24.	$\frac{4692}{161976}$.

210. To reduce an integer or a mixed number to an improper fraction.

ORAL EXERCISES.

1. In 3 units, how many fourths?

ANALYSIS.—Since in 1 unit there are 4 *fourths*, in 3 units there are 3 times 4 fourths, or 12 fourths. Hence $3 = \frac{12}{4}$.

2. In 4 bushels, how many *eighths* of a bushel?

3. How many *sevenths* of a week in 6 weeks?

4. How many 9ths in 5? 6? 8? 10? 12?

5. How many tenths of a dollar in $7? In $9?

6. How many *half* dollars will pay for a ton of coal that cost $7? For a barrel of flour that cost $10?

7. How may an integer be changed to *thirds?* To *sixths?* To *eighths?* To *tenths?*

8. In $5\frac{3}{8}$ how many *eighths?*

ANALYSIS.—Since 1 is equal to 8 *eighths*, 5 equals 5 times 8 eighths, or 40 eighths, and $\frac{3}{8}$ added make 43 eighths. Hence $5\frac{3}{8} = \frac{43}{8}$.

9. In $6\frac{3}{4}$ cords of wood, how many *fourths* of a cord?

10. How many 6ths in \$$8\frac{5}{6}$? In $12\frac{1}{6}$ rods?

11. Among how many boys can you distribute $5\frac{3}{4}$ quarts of chestnuts, if you give $\frac{1}{4}$ of a quart to each?

12. Among how many poor families can $4\frac{5}{6}$ tons of coal be distributed, if each family receive $\frac{1}{6}$ of a ton?

WRITTEN EXERCISES.

211. 1. Change 75 to the form of a fraction having 27 for its denominator.

OPERATION.

$$75 \times 27 = 2025$$

$$75 = \frac{2025}{27}$$

ANALYSIS.—Since 1 is equal to 27 *twenty-sevenths*, 75 is equal to 75 times 27 twenty-sevenths, or 2025 twenty-sevenths. Hence $75 = \frac{2025}{27}$.

2. Change $49\frac{7}{12}$ to *twelfths*.

OPERATION.

$$\begin{array}{r} 49\frac{7}{12} \\ 12 \\ \hline 588 \text{ twelfths} \end{array}$$

$$\frac{588}{12} + \frac{7}{12} = \frac{595}{12}$$

ANALYSIS.—Since 1 is equal to 12 *twelfths*, 49 is equal to 49 times 12 twelfths, or 588 twelfths; to which add $\frac{7}{12}$, and the result is 595 twelfths. Hence $49\frac{7}{12} = \frac{595}{12}$.

An *integer* is reduced to a fractional *form* by writing 1 under it for a denominator. Thus, $9 = \frac{9}{1}$; $23 = \frac{23}{1}$.

3. Change 81 to a fraction having 24 for its denominator.

4. In 78 pounds, how many sixteenths of a pound?

5. In $42\frac{6}{7}$ weeks, how many sevenths of a week?

6. How many 20ths of a ton in $16\frac{17}{20}$ tons? In $21\frac{13}{20}$ tons?

RULE.—*Multiply the integer by the required denominator, and to the product, add the numerator of the fraction, and under the result write the required denominator.*

Reduce

7. 207 to fifteenths.
8. $136\frac{11}{18}$ to eighteenths.
9. $472\frac{7}{26}$ to twenty-sixths.
10. $543\frac{9}{40}$ to fortieths.
11. $184\frac{32}{95}$ to ninety-fifths.
12. $2014\frac{61}{84}$ to eighty-fourths.

13. Reduce $204\frac{11}{24}$ days to twenty-fourths of a day.

14. Change 312 to a fraction whose denominator is 126.

15. Reduce $2146\frac{3}{64}$ to an improper fraction.

16. Change $1006\frac{17}{90}$ to an improper fraction.

212. To reduce an improper fraction to an integer, or a mixed number.

ORAL EXERCISES.

1. How many units are $\frac{18}{4}$?

Analysis.—Since 4 *fourths* equal 1, 18 fourths are as many times 1 as 4 fourths are contained times in 18 fourths, which is $4\frac{2}{4}$ times.

2. How many times 1 are $\frac{28}{7}$? $\frac{46}{8}$? $\frac{75}{12}$? $\frac{60}{9}$? $\frac{90}{15}$?

3. How many yards are $\frac{27}{3}$ of a yard? $\frac{45}{5}$? $\frac{62}{5}$?

4. How many dollars are $\$\frac{21}{4}$? $\$\frac{36}{8}$? $\$\frac{75}{12}$? $\$\frac{84}{20}$?

5. In $\frac{90}{12}$ of a foot, how many feet? In $\frac{108}{9}$ of an acre, how many acres? In $\frac{180}{20}$ of a ton, how many tons?

WRITTEN EXERCISES.

213. 1. Reduce $\frac{218}{9}$ to a mixed number.

OPERATION.

$\frac{218}{9} = 218 \div 9 = 24\frac{2}{9}$

Analysis.—Since 9 ninths equal 1, 218 ninths are $24\frac{2}{9}$ times 9 ninths. Hence $\frac{218}{9} = 24\frac{2}{9}$.

2. Change $\frac{297}{12}$ to a mixed number.

3. In $\frac{407}{25}$ of a dollar, how many dollars?

4. How many rods in $\frac{525}{40}$ of a rod?

Rule.—*Divide the numerator by the denominator.*

Reduce to integers or mixed numbers,

5. $\frac{978}{56}$.
6. $\frac{2431}{85}$.
7. $\frac{738}{24}$.
8. $\frac{1512}{81}$.
9. $\frac{17391}{17}$.
10. $\frac{23280}{464}$.
11. $\frac{22625}{375}$.
12. $\frac{103407}{1046}$.
13. $\frac{2540304}{2468}$.

214. To reduce fractions to equivalent fractions having a common denominator.

ORAL EXERCISES.

1. How many *fourths* in 1? In $\frac{1}{2}$?

2. How many *ninths* in 1? In $\frac{1}{3}$? In $\frac{2}{3}$?

3. Express $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$, each as *twelfths.*

4. Change $\frac{3}{4}$ and $\frac{5}{6}$ to fractions of the same denominator.

5. What is a multiple of 4? Of 6? Of 8? Of 9?

6. What is a common multiple of 3 and 4? Of 4 and 5?

7. What is the least common multiple of 3, 4, and 6?

8. What is the least common multiple of the denominators of $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{8}$? Of $\frac{2}{3}$, $\frac{3}{8}$, and $\frac{5}{6}$?

9. Reduce $\frac{2}{3}$ and $\frac{1}{9}$ to eighteenths. To twenty-sevenths.

10. Name some fractions that can be changed to 16ths.

11. Name four fractions that can be changed to 24ths.

DEFINITIONS AND PRINCIPLES.

215. A ***Common Denominator*** is a denominator common to two or more fractions.

216. The ***Least Common Denominator*** of two or more fractions is the least denominator to which they can all be reduced.

Since all higher terms of a fraction are multiples of its corresponding lowest terms (**207,** Note), hence the following

217. PRINCIPLES.—1. *A common denominator of two or more fractions is a common multiple of their denominators.*

2. *The least common denominator of two or more fractions is the least common multiple of their denominators.*

WRITTEN EXERCISES.

218. 1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{5}$ to equivalent fractions having a common denominator.

OPERATION.

$$2\times3\times5=30$$
$$\frac{1}{2}\times\frac{3}{3}\times\frac{5}{5}=\frac{15}{30}$$
$$\frac{2}{3}\times\frac{2}{2}\times\frac{5}{5}=\frac{20}{30}$$
$$\frac{3}{5}\times\frac{2}{2}\times\frac{3}{3}=\frac{18}{30}$$

ANALYSIS.—Multiply each denominator by the other two, and the product, 30, is a common denominator of the three. (PRIN. 1.)

But since the value of the fractions is not to be changed, each numerator must be multiplied by the same multiplier as its denominator. Hence, multiplying the terms of $\frac{1}{2}$ by 3 and 5, the result is $\frac{15}{30}$; of $\frac{2}{3}$, by 2 and 5, the result is $\frac{20}{30}$; and of $\frac{3}{5}$ by 2 and 3, the result is $\frac{18}{30}$. Or,

To find the numerators, take such part of the common denominator 30, as the given fraction is part of 1. Thus, $\frac{1}{2}$ of 30 is 15, etc.

Reduce to fractions having a common denominator

2. $\frac{3}{7}$ and $\frac{5}{9}$.
3. $\frac{7}{12}$ and $\frac{4}{5}$.
4. $\frac{3}{7}$, $\frac{5}{8}$, and $\frac{2}{3}$.
5. $\frac{7}{16}$, $\frac{2}{3}$, and $\frac{1}{4}$.
6. $\frac{9}{10}$, $\frac{5}{7}$, and $\frac{1}{2}$.
7. $\frac{1}{6}$, $\frac{2}{9}$, and $\frac{2}{15}$.

8. Change $\frac{2}{3}$, $\frac{6}{7}$, and $\frac{5}{14}$ to equivalent fractions having the least common denominator.

OPERATION.

$$\begin{array}{r|rrr} 7 & 3 & 7 & 14 \\ \hline & 3 & 1 & 2 \end{array}\qquad \frac{2}{3}=\frac{28}{42}$$
$$\frac{6}{7}=\frac{36}{42}$$
$$2\times3\times7=42\qquad \frac{5}{14}=\frac{15}{42}$$

ANALYSIS.—First find the least common multiple of the given denominators, which is 42. This must be the least common denominator of the given fractions. (PRIN. 2.)

9. Change $\frac{3}{5}$, $\frac{7}{12}$, and $\frac{11}{15}$ to equivalent fractions having the least common denominator.

RULE.—1. To reduce two or more fractions to equivalent fractions having a common denominator.

Multiply the terms of each fraction by the denominators of all the other fractions.

2. To reduce them to their *least* common denominator.

I. *Find the least common multiple of the denominators of the given fractions for their least common denominator.*

II. *Divide this common denominator by the denominator of each of the given fractions, and multiply its numerator by the quotient. The products are the new numerators.*

Mixed numbers must first be reduced to improper fractions.

Reduce to **fractions having** the least common denominator.

10. $\frac{7}{8}$, $\frac{11}{16}$, and $\frac{17}{24}$.
11. $\frac{4}{13}$, $\frac{15}{26}$, and $\frac{7}{39}$.
12. $\frac{20}{21}$, $\frac{9}{56}$, and $\frac{51}{84}$.
13. $1\frac{11}{12}$, $\frac{7}{8}$, $\frac{11}{16}$, and $\frac{17}{24}$.
14. $\frac{3}{4}$, $2\frac{5}{7}$, $\frac{3}{5}$, and $1\frac{7}{16}$.
15. $6\frac{1}{4}$, $\frac{7}{20}$, 7, and $1\frac{1}{2}$.
16. $\frac{15}{40}$, $\frac{25}{120}$, $\frac{14}{64}$, and $2\frac{4}{5}$.
17. $\frac{57}{84}$, $\frac{67}{168}$, $\frac{59}{42}$, and $\frac{3}{21}$.

ADDITION.

ORAL EXERCISES.

219. 1. What is the sum of $\frac{3}{8}$ and $\frac{5}{8}$? Of $\frac{4}{9}$ and $\frac{7}{9}$?

2. How many times 1 is the sum of $\frac{6}{7}$, $\frac{3}{7}$, and $\frac{5}{7}$?

3. Sold $\frac{4}{15}$ of an acre of land to one man, $\frac{7}{15}$ to another, and $\frac{8}{15}$ to a third. How much was sold to all?

How are fractions added that have a common denominator?

4. Mary paid \$$\frac{3}{4}$ for some ribbon, and \$$\frac{5}{6}$ for a pair of gloves. How much did she pay for both?

ANALYSIS.—She paid the sum of \$$\frac{3}{4}$ and \$$\frac{5}{6}$. $\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{5}{6}$ is equal to $\frac{10}{12}$; $\frac{9}{12}$ and $\frac{10}{12}$ are $\frac{19}{12}$, or $1\frac{7}{12}$. Hence she paid \$$1\frac{7}{12}$.

5. A man having $\frac{2}{5}$ of a ton of coal, bought $\frac{2}{3}$ of a ton more. How much had he then?

How are fractions added that have different denominators?

6. Henry gave $\$\frac{3}{8}$ for a book, $\$\frac{1}{4}$ for a slate, and $\$\frac{1}{8}$ for a bottle of ink. What did he pay for all?

7. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{6}$? Of $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$?

8. Find the sum of $\frac{5}{8}$, $\frac{1}{3}$, and $\frac{7}{12}$. Of $\frac{4}{5}$, $\frac{3}{4}$, and $\frac{7}{10}$.

9. Find the sum of $\frac{7}{16}$, $\frac{3}{8}$, and $\frac{1}{4}$. Of $\frac{5}{6}$, $\frac{1}{4}$, and $\frac{7}{12}$.

10. A farmer sold $3\frac{1}{4}$ tons of hay to one man, and $5\frac{3}{8}$ to another. How much did he sell to both?

ANALYSIS.—The sum of $3\frac{1}{4}$ tons and $5\frac{3}{8}$ tons. 5 and 3 are 8; and $\frac{1}{4}$ and $\frac{3}{8}$ are $\frac{5}{8}$, which added to 8 makes $8\frac{5}{8}$ tons.

11. A man bought $5\frac{1}{8}$ cords of wood at one time, and $7\frac{9}{16}$ at another. How much did he buy in all?

How are mixed numbers added?

12. A man paid $\$25\frac{7}{8}$ for a watch, and sold it for $\$6\frac{1}{4}$ more than he gave for it. What did he sell it for?

Find the sum

13. Of $\frac{3}{4}$ and $3\frac{1}{2}$.
14. Of $5\frac{1}{3}$ and $\frac{7}{8}$.
15. Of $1\frac{5}{6}$ and $\frac{2}{3}$.
16. Of $2\frac{1}{2}$ and $6\frac{3}{8}$.
17. Of $8\frac{1}{7}$ and $\frac{9}{14}$.
18. Of $15\frac{1}{3}$ and $\frac{5}{6}$.
19. Of $2\frac{1}{4}$ and $1\frac{2}{3}$.
20. Of $5\frac{1}{3}$ and $\frac{5}{6}$.
21. Of $1\frac{9}{16}$ and $12\frac{5}{8}$.

220. PRINCIPLE.—*Fractions can be added only when they have a common denominator, and when they express parts of like units.*

WRITTEN EXERCISES.

221. 1. Find the sum of $\frac{2}{5}$, $\frac{7}{12}$, and $\frac{4}{15}$.

OPERATION.

$$\frac{2}{5}+\frac{7}{12}+\frac{4}{15}=\frac{24+35+16}{60}$$

$$\frac{24+35+16}{60}=\frac{75}{60}=1\frac{1}{4}$$

ANALYSIS.—Reduce the given fractions to equivalent fractions having the least common denominator, which is 60 (**217**, 2). Then add their numerators, and write the sum, 75, over the common denominator 60, and $\frac{75}{60}=1\frac{1}{4}$ is the required result.

2. What is the sum of $14\frac{3}{4}$, $25\frac{7}{8}$, and $7\frac{5}{8}$?

OPERATION.

$$\begin{aligned}14\tfrac{3}{4} &= 14\tfrac{18}{24}\\ 25\tfrac{7}{8} &= 25\tfrac{21}{24}\\ 7\tfrac{5}{8} &= \underline{7\tfrac{15}{24}}\\ & \ 46+\tfrac{54}{24}=48\tfrac{1}{4}\end{aligned}$$

Analysis.—The sum of the fractions is $\frac{54}{24}=2\frac{1}{4}$, which added to the sum of the integers 46, gives $48\frac{1}{4}$ the required sum.

Find the sum

3. Of $\frac{5}{12}$, $\frac{13}{15}$, and $\frac{7}{20}$.

4. Of $\frac{3}{5}$, $\frac{7}{8}$, and $\frac{2}{3}$.

5. Of 42, $31\frac{5}{12}$, and $9\frac{7}{18}$.

6. Of $204\frac{7}{15}$, $50\frac{10}{21}$, and $7\frac{8}{35}$.

Rule.—I. *Reduce the given fractions to equivalent fractions having the least common denominator, and write the sum of the numerators over the common denominator.*

II. *When there are mixed numbers or integers, add the fractions and integers separately, then add the results.*

7. $\frac{1}{2}+\frac{2}{5}+\frac{4}{15}+\frac{9}{17}=?$

8. $\frac{3}{4}+7\frac{5}{6}+\frac{31}{42}+\frac{9}{14}=?$

9. $18\frac{5}{14}+2\frac{4}{7}+1\frac{11}{35}=?$

10. $\frac{5}{8}+6\frac{7}{12}+21\frac{5}{6}+77=?$

11. $\frac{15}{48}+\frac{11}{16}+7\frac{3}{8}+60+\frac{75}{96}=?$

12. $124\frac{4}{7}+325\frac{5}{12}+40\frac{10}{21}=?$

13. Bought 3 pieces of cloth containing $105\frac{7}{8}$, $86\frac{3}{4}$, and $58\frac{2}{3}$ yards respectively; how many yards in all?

14. If it takes $5\frac{1}{8}$ yards of cloth for a coat, $3\frac{1}{6}$ yards for a pair of pantaloons, and $\frac{7}{8}$ of a yard for a vest, how many yards does it take for all?

15. Four cheeses weighed respectively $46\frac{5}{8}$, $48\frac{2}{3}$, $49\frac{7}{16}$, and $57\frac{1}{4}$ pounds. What was their entire weight?

16. What number is that from which if $24\frac{4}{7}$ is taken, the remainder is $63\frac{22}{35}$?

17. A farm is divided into 4 fields: the first contains $29\frac{7}{12}$ acres, the second $50\frac{10}{21}$ acres, the third $41\frac{4}{7}$ acres, and the fourth $69\frac{3}{4}$ acres. How many acres in the farm?

SUBTRACTION.

ORAL EXERCISES.

222. 1. What is the difference between $\frac{6}{7}$ and $\frac{3}{7}$?

2. What is the difference between $\frac{3}{12}$ and $\frac{7}{12}$?

How is one fraction subtracted from another, each having the same denominator?

3. A gentleman who owned a sail-boat sold $\frac{5}{16}$ of it. What part did he still own?

4. A boy having $\$\frac{3}{4}$, gave $\$\frac{1}{3}$ for a neck-tie. What had he left?

ANALYSIS.—He had left the difference between $\$\frac{3}{4}$ and $\$\frac{1}{3}$. $\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{1}{3}$ equals $\frac{4}{12}$; $\frac{9}{12}$ less $\frac{4}{12}$ are $\frac{5}{12}$.

5. A man owning $\frac{2}{3}$ of an acre of ground, sold $\frac{1}{6}$ of an acre. What part remained?

How is one fraction subtracted from another having a different denominator?

6. Subtract $\frac{1}{2}$ from $\frac{5}{8}$; $\frac{2}{5}$ from $\frac{1}{2}$; $\frac{3}{7}$ from $\frac{9}{14}$.

7. Find the difference between $\frac{7}{10}$ and $\frac{3}{5}$; $\frac{2}{3}$ and $\frac{7}{8}$.

8. From a piece of cloth containing $12\frac{1}{2}$ yards, $5\frac{1}{8}$ yards were cut. How many yards remained?

ANALYSIS.—The difference between $12\frac{1}{2}$ yards and $5\frac{1}{8}$ yards. $\frac{1}{8}$ from $\frac{1}{2}$ leaves $\frac{3}{8}$, and 5 from 12 leaves 7. Hence $7\frac{3}{8}$ yards remained.

9. If a ton of coal costs $\$7\frac{4}{5}$, and a cord of wood $\$4\frac{1}{3}$, what is the difference in their cost?

How is one mixed number subtracted from another?

10. What is the value of $3\frac{1}{3}—2\frac{1}{5}$? $8\frac{1}{4}—2\frac{1}{6}$? $6\frac{11}{12}—\frac{2}{3}$?

223. PRINCIPLE.—*Fractions can be subtracted only when they have a common denominator, and when they express parts of like units.*

WRITTEN EXERCISES

224. 1. From $\frac{7}{9}$ subtract $\frac{4}{15}$.

OPERATION.

$\frac{7}{9} - \frac{4}{15} = \frac{35}{45} - \frac{12}{45} = \frac{35-12}{45} = \frac{23}{45}$

ANALYSIS.—Reduce the given fractions to equivalent fractions having the least common denominator. Hence $\frac{35}{45} - \frac{12}{45} = \frac{23}{45}$.

2. From $134\frac{1}{3}$ take $76\frac{5}{8}$.

OPERATION.

$$\begin{array}{rl} 134\frac{1}{3} = & 134\frac{8}{24} \\ 76\frac{5}{8} = & \underline{76\frac{15}{24}} \\ & 57\frac{17}{24} \end{array}$$

ANALYSIS.—Reduce $\frac{1}{3}$ and $\frac{5}{8}$ to equivalent fractions having the least common denominator. As $\frac{15}{24}$ cannot be taken from $\frac{8}{24}$, take 1 or $\frac{24}{24}$ from 134, leaving 133, and add it to $\frac{8}{24}$, making $\frac{32}{24}$. Then $\frac{15}{24}$ from $\frac{32}{24}$ leaves $\frac{17}{24}$, and 76 from 133, leaves 57. Hence $57\frac{17}{24}$ is the result.

3. From $\frac{9}{14}$ take $\frac{16}{63}$.
4. From $\frac{7}{12}$ take $\frac{5}{42}$.
5. From $\frac{24}{35}$ take $\frac{7}{24}$.
6. From $36\frac{5}{7}$ take $10\frac{1}{2}$.
7. From $112\frac{7}{102}$ take 56.
8. From $204\frac{3}{16}$ take $39\frac{7}{12}$.

RULE.—I. *Reduce the given fractions to equivalent fractions having the least common denominator, and write the difference of the numerators over the common denominator.*

II. *When there are mixed numbers, subtract the fractional and integral parts separately, and add the results.*

If the mixed numbers are small, they may be reduced to improper fractions and subtracted according to the usual method.

Find the difference between

9. $\frac{5}{8}$ and $\frac{3}{11}$.
10. $1\frac{3}{4}$ and $\frac{5}{6}$.
11. $\frac{14}{15}$ and $2\frac{4}{5}$.
12. 16 and $3\frac{2}{11}$.
13. $63\frac{7}{9}$ and $71\frac{1}{8}$.
14. 106 and $95\frac{15}{36}$.
15. From $\frac{7}{64}$ take $\frac{13}{120}$.
16. From $16\frac{7}{15}$ take $\frac{43}{125}$.
17. From $410\frac{5}{27}$ take $226\frac{5}{9}$.
18. From $428\frac{11}{12}$ take $180\frac{9}{11}$.

19. A farmer having 208 acres of land, sold $92\frac{7}{25}$ acres. How many acres had he left?

ADDITION AND SUBTRACTION.

ORAL EXERCISES.

225. 1. How much less than 2, is $\frac{1}{4}+\frac{2}{3}$?

2. How much greater than 2, is $\frac{2}{3}+\frac{1}{2}+1\frac{1}{6}$?

3. What is the difference between 4 and $2\frac{3}{8}$? $5\frac{1}{7}$ and $7\frac{5}{7}$?

4. Mr. Smith sold $\frac{1}{3}$ of his farm to one man, $\frac{1}{4}$ to another, and $\frac{1}{6}$ to a third. What part had he left?

5. Paid \$$6\frac{1}{2}$ for a ton of coal, and \$$3\frac{1}{4}$ for a load of wood. What change must be returned for a ten-dollar bill?

6. What is the difference between $1\frac{4}{5}+\frac{2}{3}$ and $5\frac{8}{15}$?

7. What is the difference between $\frac{9}{10}+\frac{1}{2}$ and $\frac{1}{3}+\frac{5}{6}$?

Find the second member of the following equations:

8. $\frac{5}{6}+\frac{1}{3}-\frac{1}{2}=?$
9. $\frac{9}{10}-\frac{1}{5}+2=?$
10. $3-(\frac{4}{5}-\frac{1}{2})=?$
11. $6\frac{1}{2}+\frac{4}{5}-1\frac{3}{4}=?$
12. $12-(8-2\frac{5}{8})+1\frac{1}{3}=?$
13. $(26-14)-(\frac{3}{4}+3\frac{3}{8}=?$
14. $(1\frac{3}{8}+2\frac{3}{4})-(\frac{1}{2}+\frac{2}{3})=?$
15. $(5-3\frac{5}{7})+(9\frac{3}{14}-6)=?$
16. $8\frac{9}{16}-\overline{2\frac{1}{4}+\frac{7}{8}}+10=?$
17. $12\frac{7}{20}+\overline{9\frac{1}{3}-1\frac{4}{5}}-2\frac{1}{4}=?$

WRITTEN EXAMPLES.

226. 1. The sum of two numbers is $124\frac{1}{4}$, and the less is $36\frac{2}{15}$. What is the greater?

2. What number added to $147\frac{4}{7}$ will make $216\frac{3}{8}$?

3. What number added to $307\frac{1}{7}+210\frac{3}{4}$ will make $700\frac{5}{8}$?

4. What number must be added to the difference of $186\frac{5}{6}$ and $214\frac{3}{7}$ to make $1042\frac{18}{21}$?

5. What fraction added to the sum of $\frac{1}{8}$, $\frac{5}{12}$, and $\frac{5}{18}$, will make $\frac{133}{144}$?

6. What must be added to $\frac{4}{5}$, that the sum may be $\frac{17}{21}$?

7. Bought a quantity of barrel staves for $\$160\frac{3}{8}$, and of lumber for $\$1136\frac{3}{8}$. Sold the staves for $\$205\frac{1}{2}$ and the lumber for $\$1240\frac{9}{16}$. What was the whole gain?

8. A man bought a ton of hay for $\$15\frac{3}{8}$, a barrel of flour for $\$9\frac{5}{16}$, and a barrel of apples for $\$3\frac{7}{16}$. What change should be given to him for 3 ten-dollar bills?

Complete the following equations:

9. $\overline{\frac{4}{5}+\frac{6}{7}}-\overline{\frac{3}{8}+\frac{5}{7}}=?$
10. $8\frac{5}{6}+2\frac{3}{5}-5\frac{4}{11}=?$
11. $48-(16\frac{4}{7}-3\frac{1}{2})=?$
12. $41\frac{1}{3}+\overline{56-24\frac{7}{30}}-4\frac{11}{16}=?$
13. $\overline{120-51\frac{3}{5}}+\overline{90\frac{10}{27}-\frac{2}{3}}=?$
14. $342-\overline{(21\frac{3}{15}+\frac{4}{5}}-9)=?$
15. $176\frac{9}{20}+132\frac{7}{8}-26\frac{31}{40}-17\frac{1}{2}=?$
16. $\$1000-\$500+\$107\frac{55}{100}+\$91\frac{9}{10}=?$

MULTIPLICATION.

227. When one factor is a fractional number.

ORAL EXERCISES.

1. What part of a mile is 3 times $\frac{1}{4}$ of a mile?
2. What part of a dollar is 4 times $\frac{1}{5}$ of a dollar?
3. How many times 1 is 3 times $\frac{2}{3}$? 4 times $\frac{3}{4}$?
4. At $\$\frac{5}{8}$ a pound, what will 4 pounds of tea cost?

Analysis.—Four pounds will cost 4 times $\$\frac{5}{8}$, or $\$\frac{20}{8}$, equal to $\$2\frac{1}{2}$.

5. At $\$\frac{5}{6}$ a bushel, what is the cost of 6 bushels of oats? Of 7 bushels? Of 8 bushels? Of 9 bushels?

6. If a horse eat $\frac{4}{7}$ of a bushel of grain in a day, how much will 4 horses eat? 6 horses? 8 horses? 10 horses?

7. What cost 12 baskets of pears, at $\$\frac{4}{5}$ a basket?

With each class of *oral* questions in Art. 227, the pupil may solve the corresponding *written* examples on pages 118 and 119.

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8. What cost 9 pounds of butter at $\$\frac{3}{8}$ a pound?

9. Show that multiplying the numerator of $\frac{3}{20}$ by 4 multiplies the fraction by 4.

10. Show that dividing the denominator of $\frac{3}{20}$ by 4 multiplies the fraction by 4.

How many ways to multiply a fraction by an integer?

11. Multiply $\frac{3}{11}$ by 5; $\frac{4}{18}$ by 6; $\frac{9}{20}$ by 5; $\frac{7}{40}$ by 8.

12. At $\$4\frac{3}{4}$ a box, what will 5 boxes of raisins cost?

ANALYSIS.—They will cost 5 times $\$4\frac{3}{4}$. 5 times $\$\frac{3}{4}$ are $\$3\frac{3}{4}$, and 5 times \$4 are \$20. $\$20 + \$3\frac{3}{4} = \$23\frac{3}{4}$. Hence, etc.

13. At $7\frac{1}{3}$ cents a pound, what will 9 pounds of rice cost?

14. At $\$9\frac{3}{8}$ a barrel, what is the cost of 6 barrels of flour? Of 8 barrels? Of 9 barrels? Of 10 barrels?

15. What will 8 yards of cloth cost, at $\$5\frac{7}{10}$ a yard?

16. Multiply $7\frac{1}{6}$ by 9; $9\frac{4}{7}$ by 6; $10\frac{5}{8}$ by 7; $12\frac{7}{10}$ by 8.

17. What is $\frac{1}{3}$ of 12 yards? $\frac{1}{3}$ of 24 men? $\frac{1}{4}$ of \$30?

18. Multiplying by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, etc., is the same as dividing by what integers?

When a *fractional part* of an integer, or of a fraction, is to be taken, the word *of*, and not *times*, should be used.

19. At \$7 a ton, what will $\frac{3}{4}$ of a ton of coal cost?

ANALYSIS.—It will cost $\frac{3}{4}$ of \$7, or 3 times $\frac{1}{4}$ of \$7. $\frac{1}{4}$ of \$7 is $\$1\frac{3}{4}$, and 3 times $\$1\frac{3}{4}$ are $\$5\frac{1}{4}$. Hence, etc.

20. At \$12 a gross, what will $\frac{7}{8}$ of a gross of butts cost?

21. What will $\frac{3}{10}$ of a ton of hay cost, at \$15 a ton?

22. What is $\frac{5}{8}$ of 6? $\frac{4}{5}$ of 2? $\frac{3}{5}$ of 8? $\frac{5}{12}$ of 9?

23. At \$5 a yard, what will $\frac{4}{5}$ of a yard of cloth cost?

24. If a man can build a wall in 28 days, in what time can he build $\frac{3}{5}$ of it? $\frac{3}{4}$ of it? $\frac{5}{8}$ of it?

25. If an acre of land produce 45 bushels of corn, how much will $\frac{2}{3}$ of an acre produce? $\frac{5}{8}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{3}{10}$?

26. What is $\frac{2}{3}$ of \$5? Of \$7? Of \$16? Of \$25?

27. Multiply 50 by $\frac{3}{4}$; 49 by $\frac{6}{7}$; 63 by $\frac{5}{21}$; 81 by $\frac{7}{10}$.

28. In \$1 are 100 cents; how many cents in $\frac{1}{2}$ of a dollar? In $\frac{1}{4}$? $\frac{1}{8}$? $\frac{1}{5}$? $\frac{1}{10}$? $\frac{1}{12}$? $\frac{1}{20}$?

29. How many cents in $\frac{3}{10}$ of a dollar? In $\frac{3}{20}$? $\frac{3}{4}$? $\frac{4}{5}$?

30. Which is greater, $\frac{4}{5}$ of 15, or 15 × $\frac{4}{5}$?

31. Show that a fraction of an integer equals the product of the integer by the fraction.

How is an integer multiplied by a fraction?

32. At \$12 a ton, what will 5$\frac{3}{8}$ tons of cheese cost?

ANALYSIS.—It will cost 5$\frac{3}{8}$ times \$12. 5 times \$12 are \$60, and $\frac{3}{8}$ of \$12 are \$4$\frac{1}{2}$, which added to \$60, make \$64$\frac{1}{2}$.

33. At 15 cents each, what will 4$\frac{3}{4}$ melons cost?

34. What will 7$\frac{3}{8}$ weeks' board cost, at \$9 a week?

35. How much is 6$\frac{2}{3}$ times 12? 5$\frac{4}{5}$ times 20?

36. Multiply 4 by 8$\frac{1}{6}$; 6 by 7$\frac{5}{6}$; 8 by 9$\frac{3}{10}$.

What

37. Is $\frac{7}{12}$ of 10 gallons?
38. Is $\frac{3}{8}$ of 47 pounds?
39. Is $\frac{5}{9}$ of 90 rods?
40. Is $\frac{7}{8}$ of 56 days?

How many

41. Are 10 times 6$\frac{4}{5}$ tons?
42. Are 9 times 11$\frac{2}{13}$ miles?
43. Are 7$\frac{3}{4}$ times 12 men?
44. Are 12$\frac{1}{2}$ times 9 minutes?

Find the value

45. Of $\frac{3}{5}$ × 15.
46. Of 28 × $\frac{6}{7}$.
47. Of 56 × $\frac{7}{8}$.
48. Of $\frac{4}{21}$ × 7.
49. Of $\frac{5}{6}$ × 7 + $\frac{1}{6}$.
50. Of $\frac{7}{8}$ × 9 — 2$\frac{1}{8}$.
51. Of 27 × $\frac{5}{9}$ + 3$\frac{1}{4}$.
52. Of 6$\frac{3}{7}$ × 7 — 1$\frac{7}{8}$.
53. Of $\frac{7}{10}$ + $\frac{2}{5}$ × 5.
54. Of $\frac{14}{15}$ — 2 × $\frac{1}{5}$.
55. Of 4$\frac{5}{18}$ + 2 × 5$\frac{5}{9}$
56. Of 9 × 9$\frac{1}{7}$ + 20$\frac{1}{2}$

228. PRINCIPLES.—1. *Multiplying the numerator or dividing the denominator multiplies the fraction.* (**200,** 1.)

2. *The product of an integer by a fraction is equal to such part of the integer as the fraction is of a unit.*

WRITTEN EXERCISES

229. 1. Multiply $\frac{7}{27}$ by 9.

OPERATION.

$\frac{7}{27}\times 9=\frac{7\times 9}{27}=\frac{63}{27}=2\frac{1}{3}$

Or,

$\frac{7}{27}\times 9=\frac{7}{27\div 9}=\frac{7}{3}=2\frac{1}{3}$

Or,

$$\begin{array}{r|l} \overset{3}{\cancel{27}} & 7 \\ & \cancel{9} \\ \hline 3 & 7 \\ \hline & 2\frac{1}{3} \end{array}$$

Analysis.—Multiply the numerator 7 by 9, or divide the denominator 27 by 9; either operation will give $2\frac{1}{3}$, the required result. (Prin. 1.)

By using the vertical line and cancellation, both operations are combined and shortened.

In the first operation, the *number* of parts or of fractional units is increased, while their *size* or value remains the same; in the second operation, the *size* of the parts is increased, while their *number* remains unchanged.

In like manner multiply

2. $\frac{3}{14}$ by 12.
3. $\frac{17}{63}$ by 9.
4. $\frac{15}{17}$ by 13.
5. $\frac{7}{75}$ by 15.
6. $\frac{45}{180}$ by 36.
7. $\frac{51}{168}$ by 21.
8. $\frac{4}{102}$ by 17.
9. $\frac{101}{132}$ by 22.
10. $\frac{54}{88}$ by 44.

11. Multiply 72 by $\frac{4}{9}$.

OPERATION.

$72\times\frac{4}{9}=72\div 9\times 4=32$

Or, $72\times\frac{4}{9}=\frac{\overset{8}{\cancel{72}}\times 4}{\cancel{9}}=32$

Or,

$$\begin{array}{r|l} & \overset{8}{\cancel{72}} \\ \cancel{9} & 4 \\ \hline & 32 \end{array}$$

Analysis.—To multiply 72 by $\frac{4}{9}$, is to find $\frac{4}{9}$ of 72. $\frac{4}{9}$ of 72 is 4 times $\frac{1}{9}$ of 72, which is 32. (Prin. 2.)

Find the product

12. Of 75 by $\frac{3}{15}$.
13. Of 7 by $\frac{8}{21}$.
14. Of 56 by $\frac{5}{14}$.
15. Of 168 by $\frac{23}{42}$.
16. Of 200 by $\frac{3}{14}$.
17. Of 315 by $\frac{17}{21}$.
18. Of 19 by $\frac{13}{57}$.
19. Of 448 by $\frac{3}{56}$.
20. Of 572 by $\frac{5}{44}$.

A fraction is *multiplied* by a number equal to its *denominator* by *cancelling* the *denominator*. Thus, $\frac{7}{8}\times 8=7$.

Cancelling a *factor* of the denominator *multiplies* the fraction by that factor. Thus, $\frac{5}{12}\times 4=\frac{5}{3}$.

21. Multiply $17\frac{2}{9}$ by 6.

OPERATION.

$$\begin{array}{r} 17\frac{2}{9} \\ 6 \\ \hline 103\frac{1}{3} \end{array}$$

Or, $17\frac{2}{9} = \frac{155}{9}$

$$\begin{array}{r|l} {}^{3}\cancel{9} & 155 \\ & \cancel{6}^{2} \\ \hline 3 & 310 \\ \cline{2-2} & 103\frac{1}{3} \end{array}$$

Analysis.—To multiply $17\frac{2}{9}$ by 6, multiply the fraction $\frac{2}{9}$, and the integer 17 separately and add their products, which gives $103\frac{1}{3}$, the required product. Or,

Reduce the mixed number to an improper fraction, and multiply as in Ex. 1, which gives the same result.

Multiply

22. $127\frac{8}{9}$ by 12.
23. $85\frac{4}{5}$ by 15.
24. $128\frac{5}{21}$ by 42.
25. $246\frac{5}{8}$ by 16.
26. $314\frac{9}{16}$ by 48.
27. $750\frac{9}{25}$ by 17.

28. Multiply 140 by $9\frac{2}{3}$.

OPERATION.

$$\begin{array}{r} 140 \\ 9\frac{2}{3} \\ \hline 93\frac{1}{3} \\ 1260 \\ \hline 1353\frac{1}{3} \end{array}$$

Or, $9\frac{2}{3} = \frac{29}{3}$

$$\begin{array}{r|l} & 140 \\ 3 & 29 \\ \hline 3 & 4060 \\ \cline{2-2} & 1353\frac{1}{3} \end{array}$$

Analysis.—To multiply 140 by $9\frac{2}{3}$, multiply by the fraction $\frac{2}{3}$, and by the integer 9, separately, and add their products, which gives $1353\frac{1}{3}$, the required product. Or,

Reduce the mixed number to an improper fraction, and multiply as in Ex. 1, which gives the same result.

Multiply

29. 96 by $12\frac{5}{8}$.
30. 216 by $16\frac{7}{9}$.
31. 304 by $24\frac{7}{16}$.
32. 198 by $18\frac{5}{6}$.
33. 560 by $23\frac{7}{10}$.
34. 715 by $14\frac{4}{15}$.

35. Multiply $327\frac{7}{12}$ by 72; 2466 by $84\frac{3}{4}$; 759 by $\frac{14}{33}$.

36. What will 120 dozen of hose cost at \$$4\frac{5}{6}$ a dozen?

37. At \$20 a ton, what will $\frac{12}{25}$ of a ton of hay cost?

38. If a city lot is worth \$3145, what is $\frac{9}{16}$ of it worth?

39. What will 142 yards of curbing cost at \$$6\frac{5}{8}$ a yard?

40. At \$$\frac{15}{16}$ a yard, what is the cost of 8 yards of cloth? Of 24 yards? Of 64 yards? Of 120 yards?

230. When both factors are fractional numbers.

ORAL EXERCISES.

1. A boy having $\frac{1}{2}$ of a melon, gave $\frac{1}{3}$ of it to his sister. What part of the melon did she receive?

2. What part of 1 is $\frac{1}{3}$ of $\frac{1}{2}$? Is $\frac{1}{3}$ of $\frac{1}{3}$? Is $\frac{1}{4}$ of $\frac{1}{2}$?

3. What part of 1 is $\frac{1}{2}$ of $\frac{4}{5}$? $\frac{1}{3}$ of $\frac{5}{7}$? $\frac{1}{4}$ of $\frac{3}{5}$? $\frac{1}{5}$ of $\frac{5}{7}$?

4. Which is greater, $\frac{1}{3}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{4}{5}$, or $\frac{1}{5}$ of $\frac{3}{5}$? $\frac{1}{4}$ of $\frac{1}{3}$, or $\frac{1}{6}$ of $\frac{1}{2}$?

5. If I own $\frac{6}{7}$ of an acre of land, and sell $\frac{1}{3}$ of it, what part of an acre do I sell? What part do I retain?

6. If a yard of silk is worth $\$\frac{3}{8}$, what is $\frac{1}{3}$ of a yard worth?

7. A boy having $\$\frac{3}{4}$ gave $\frac{5}{6}$ of it for a knife. What part of a dollar did he pay for the knife?

ANALYSIS.—He paid $\frac{5}{6}$ of $\$\frac{3}{4}$, or 5 times $\frac{1}{6}$ of $\$\frac{3}{4}$. $\frac{1}{6}$ of $\$\frac{3}{4}$ is $\$\frac{3}{24}$, and 5 times $\$\frac{3}{24}$ are $\frac{15}{24}$, or $\$\frac{5}{8}$.

8. At $\$\frac{4}{5}$ a gallon, what will $\frac{3}{4}$ of a gallon of syrup cost?

Fractions with the word *of* between them are sometimes called *Compound Fractions.* The word *of* is equivalent to the sign (×) of multiplication. Thus, $\frac{5}{6}$ *of* $\frac{3}{8} = \frac{5}{6} \times \frac{3}{8}$; $\frac{4}{5}$ *of* $9 = \frac{4}{5} \times 9$, etc.

9. What is $\frac{3}{5}$ of $\frac{6}{7}$? $\frac{4}{5}$ of $\frac{3}{4}$? $\frac{5}{6}$ of $\frac{3}{8}$? $\frac{3}{7}$ of $\frac{5}{9}$?

10. What is $\frac{4}{7} \times \frac{3}{5}$? $\frac{9}{10} \times \frac{2}{3}$? $\frac{3}{8} \times \frac{5}{12}$? $\frac{7}{9} \times \frac{3}{14}$?

11. A man owning $\frac{4}{5}$ of a mill, sold $\frac{5}{8}$ of his share to his brother. What part of the mill did each then own?

12. At $\$8\frac{4}{5}$ a barrel, what will $\frac{3}{4}$ of a barrel of flour cost?

ANALYSIS.—It will cost $\frac{3}{4}$ of $\$8\frac{4}{5}$, or 3 times $\frac{1}{4}$ of $\$8\frac{4}{5}$. $\frac{1}{4}$ of $\$8\frac{4}{5}$ is $\$2\frac{1}{5}$, and 3 times $\$2\frac{1}{5}$ are $\$6\frac{3}{5}$.

Or, $\$8\frac{4}{5}$ equal $\$\frac{44}{5}$, and $\frac{3}{4}$ of $\$\frac{44}{5} = \$\frac{33}{5} = \$6\frac{3}{5}$.

13. At $\$9\frac{1}{4}$ a case, what will $\frac{3}{5}$ of a case of slates cost?

14. At $\$5\frac{1}{3}$ a yard, what will $\frac{4}{5}$ of a yard of cloth cost?

15. How much is $\frac{5}{6}$ of $7\frac{1}{5}$ miles? $\frac{3}{8}$ of $24\frac{1}{4}$ pounds?

16. If a man travel $26\frac{1}{3}$ miles in a day, how far does he travel in $\frac{1}{4}$ of a day? In $\frac{3}{4}$? In $\frac{5}{6}$? In $\frac{3}{8}$?

17. At $\$\frac{2}{3}$ a yard, what will $6\frac{1}{4}$ yards of flannel cost?

Analysis.—It will cost $6\frac{1}{4}$ times $\$\frac{2}{3}$. 6 times $\$\frac{2}{3}$ are \$4, and $\frac{1}{4}$ of $\$\frac{2}{3}$ is $\$\frac{1}{6}$, which added to \$4, makes $\$4\frac{1}{6}$.
Or, $6\frac{1}{4} = \frac{25}{4}$, and $\frac{25}{4}$ of $\$\frac{2}{3} = \$\frac{25}{6} = \$4\frac{1}{6}$.

18. At $\$\frac{7}{10}$ a pound, what will $8\frac{3}{5}$ pounds of tea cost?

19. What will $9\frac{3}{8}$ pounds of beef cost, at $\$\frac{1}{8}$ a pound?

20. If a man hoe $\frac{7}{8}$ of an acre of corn in a day, how many acres can he hoe in $5\frac{1}{3}$ days? In $6\frac{3}{4}$ days? In $8\frac{4}{5}$ days?

21. What will $2\frac{1}{4}$ yards of silk cost, at $\$3\frac{1}{3}$ a yard?

Analysis.—$2\frac{1}{4}=\frac{9}{4}$, and $3\frac{1}{3}=\frac{10}{3}$; and $\frac{10}{3}\times\frac{9}{4}=\frac{15}{2}=\$7\frac{1}{2}$.

Reduce mixed numbers to improper fractions, and then proceed as in multiplying one fraction by another.

22. What cost $5\frac{3}{8}$ yards of alpaca, at $\$1\frac{2}{3}$ a yard?

23. Multiply $2\frac{1}{3}$ by $3\frac{1}{5}$; $7\frac{1}{3}$ by $1\frac{3}{4}$; $5\frac{1}{6}$ by $2\frac{3}{8}$.

Find the value

24. Of $\frac{3}{4}$ of $\frac{5}{8}$ of a mile.
25. Of $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{3}{4}$ of \$1.
26. Of $\frac{2}{3}$ of $4\frac{7}{10}$ leagues.
27. Of $\frac{3}{4}$ of $6\frac{2}{3}$ pecks.
28. Of $7\frac{1}{4}$ times $\$\frac{4}{5}$.
29. Of $9\frac{3}{5}$ times $\frac{4}{7}$ of a rod.
30. Of $2\frac{1}{4}$ times $5\frac{1}{2}$ acres.
31. Of $4\frac{1}{2}$ times $\frac{3}{4}$ of $2\frac{1}{2}$ feet.

Find the result of

32. $\frac{4}{7}\times\frac{3}{4}$.
33. $1\frac{3}{4}\times\frac{5}{8}$.
34. $\frac{7}{10}\times 6\frac{1}{7}$.
35. $4\frac{1}{9}\times 1\frac{2}{3}$.
36. $10-\frac{4}{5}$ of $3\frac{1}{4}$.
37. $6\frac{7}{8}+\overline{2\frac{1}{2}\times\frac{5}{6}}$.
38. $\frac{15}{16}-\frac{1}{8}$ of $2\frac{3}{4}$.
39. $\frac{11}{12}\times\frac{4}{5}+8\frac{7}{8}$.
40. $2\frac{1}{4}\times\frac{5}{6}-\frac{3}{8}$ of $1\frac{3}{5}$.
41. $16-\frac{3}{10}\times 12\frac{3}{5}$.
42. $4\frac{1}{3}+\frac{5}{4}$ of $1\frac{1}{2}-\frac{7}{15}$.
43. $\frac{18}{33}$ of $\frac{11}{9}+2\frac{1}{3}\times 0$.

231. Prin.—*The product of a fraction by a fraction is such a part of either factor as the other is of a unit.*

WRITTEN EXERCISES.

232. 1. Multiply $\frac{5}{14}$ by $\frac{7}{8}$.

OPERATION.

$\frac{5}{14} \times \frac{7}{8} = \frac{35}{112} = \frac{5}{16}$

Or, $\frac{5}{\underset{2}{\cancel{14}}} \times \frac{\cancel{7}}{8} = \frac{5}{16}$

Analysis.—To multiply $\frac{5}{14}$ by $\frac{7}{8}$ is to find $\frac{7}{8}$ of $\frac{5}{14}$, which is 7 times $\frac{1}{8}$ of $\frac{5}{14}$, and $\frac{1}{8}$ is found by multiplying the denominator (**200**, 1.) Hence $\frac{1}{8}$ of $\frac{5}{14}$ is $\frac{5}{112}$, and $\frac{7}{8}$ of $\frac{5}{14}$ is 7 times $\frac{5}{112}$, or $\frac{35}{112} = \frac{5}{16}$, the required product.

In like manner multiply

2. $\frac{11}{24}$ by $\frac{36}{77}$.
3. $\frac{20}{33}$ by $\frac{11}{15}$.
4. $\frac{7}{12}$ by $\frac{17}{42}$.
5. $\frac{16}{17}$ by $\frac{51}{75}$.
6. $\frac{45}{126}$ by $\frac{7}{360}$.
7. $\frac{9}{56}$ by $\frac{128}{153}$.

8. Find the product of $\frac{7}{16}$, $\frac{4}{21}$, $5\frac{1}{3}$, and 2.

OPERATION.

$$\frac{\cancel{7}}{\cancel{16}} \times \frac{4}{\underset{3}{\cancel{21}}} \times \frac{\cancel{16}}{3} \times \frac{2}{1} = \frac{8}{9}$$

Or,

$$\begin{array}{r|l} \cancel{16} & \cancel{7} \\ {}^{3}\cancel{21} & 4 \\ 3 & \cancel{16} \\ & 2 \\ \hline 9 & 8 = \frac{8}{9} \end{array}$$

Analysis.—Change the mixed number $5\frac{1}{3}$ to an improper fraction, and the integer 2 to the form of a fraction, then multiply as in Ex. 1.

Find the value of the following expressions:

9. $\frac{8}{15}$ of $12\frac{1}{4} \times \frac{1}{5}$ of $7\frac{1}{3}$.
10. $\frac{2}{7}$ of $96 \times \frac{7}{10}$ of $26\frac{2}{3}$.
11. $\frac{3}{4}$ of $\frac{4}{5}$ of $21\frac{1}{3} \times 2\frac{1}{2}$.
12. $42\frac{1}{2} \times 5\frac{1}{3}$ times $6\frac{3}{4}$.
13. $\frac{4}{9}$ of $36\frac{1}{2} \times \frac{3}{7}$ of 9.
14. $\frac{4}{5}$ of $91 \times 72\frac{1}{2}$.
15. $\frac{7}{11}$ of $63\frac{1}{2}$ by $\frac{4}{5}$ of $\frac{55}{56}$.
16. $\frac{6}{65}$ of $21 \times \frac{6}{19}$ of $\frac{11}{8}$ of $\frac{3}{5}$.

From the preceding principles and operations is derived the following *general*

Rule.—I. *Reduce all integers and mixed numbers to improper fractions.*

II. *Multiply together the numerators, for the numerator; and the denominators, for the denominator of the product.* Or,

I. *Multiply by the numerator of the fractional multiplier and divide by the denominator.*

II. *When the multiplier is a mixed number, multiply by the fractional and integral parts separately and add their products.*

Cancel all factors common to numerators and denominators before multiplying, thus *shortening* the operation, and obtaining the answer in its *lowest terms*.

Find the cost

17. Of 15 cords of bark, at $\$4\frac{5}{8}$ a cord.

18. Of $24\frac{3}{4}$ pounds of tea, at $\$\frac{5}{8}$ a pound.

19. Of 80 yards of cloth, at $\$4\frac{5}{16}$ a yard.

20. Of $21\frac{3}{8}$ bushels of corn, at $\$\frac{9}{16}$ a bushel.

21. Of $\frac{3}{4}$ of $5\frac{1}{3}$ tons of hay, at $\$15\frac{3}{16}$ a ton.

22. Of $18\frac{3}{8}$ barrels of crude oil, at $\$7\frac{4}{5}$ a barrel.

23. Of $\frac{1}{3}$ of $18\frac{1}{2}$ yards of silk, at $\frac{3}{4}$ of $5 a yard.

24. Of 126 pounds of beef, at $9\frac{1}{4}$ cents a pound.

25. Of $36\frac{7}{16}$ tons of railroad iron, at $\$62\frac{1}{2}$ a ton.

26. Of 35 horses, at $\$205\frac{3}{8}$ each.

27. Of $\frac{4}{5}$ of $156\frac{2}{3}$ acres of land, at $\frac{5}{6}$ of $\$54\frac{9}{10}$ an acre.

28. Of $28\frac{5}{8}$ bushels of sweet potatoes, at $\$1\frac{2}{3}$ a bushel.

29. Of $\frac{5}{8}$ of a yard of satin, at $\$\frac{13}{20}$ a yard.

30. Of $\frac{9}{40}$ of an acre of land, at $125 an acre.

31. Of $7\frac{9}{20}$ tons of middlings, at $\$26\frac{4}{5}$ a ton?

32. Paid $\$365\frac{5}{8}$ for a horse, and sold him for $\frac{4}{5}$ of what he cost. What was the loss?

33. When peaches are worth $\$\frac{7}{8}$ a basket, what are $126\frac{3}{4}$ baskets worth?

34. Find the value of $(129 - 76\frac{5}{6}) \times \frac{7}{17}$ of $\overline{12\frac{1}{4} - 2\frac{3}{8}} + 21\frac{1}{2} \times 6\frac{1}{3}$.

DIVISION.

233. When the divisor is an integral number.

ORAL EXERCISES.

1. If $\frac{3}{4}$ of an acre of land is divided into 3 equal lots, what part of an acre does each lot contain?

2. $\frac{1}{3}$ of $\frac{3}{4}$ is what part of 1? $\frac{1}{5}$ of $\frac{5}{6}$? $\frac{1}{4}$ of $\frac{7}{8}$? $\frac{1}{5}$ of $\frac{5}{12}$?

3. Dividing by 3, 4, and 5 is the same as multiplying by what fractions?

4. How much is $\frac{9}{12}\times\frac{1}{3}$? $\frac{9}{12}\div 3$? $\frac{8}{15}\times\frac{1}{4}$? $\frac{8}{15}\div 4$?

5. If 4 slates cost $\$\frac{8}{9}$, what will 1 slate cost?

6. If 5 boxes of figs cost $\$\frac{5}{4}$, what will 1 box cost?

7. Divide $\frac{3}{8}$ of a barrel of flour equally among 3 families. What part of a barrel will each family receive?

8. What is $\frac{1}{8}$ of $\frac{9}{10}$? $\frac{9}{10}$ divided by 8?

9. What is the quotient of $\frac{18}{25}$ divided by 2? by 3?

10. Show that dividing the numerator of $\frac{9}{10}$ by 3 divides the fraction by 3.

11. Show that multiplying the denominator of $\frac{5}{7}$ by 3 divides the fraction by 3.

How many ways to divide a fraction by an integer?

12. Divide $\frac{8}{15}$ by 4; $\frac{10}{21}$ by 5; $\frac{3}{11}$ by 7; $\frac{5}{6}$ by 9.

13. If 6 pounds of sugar cost $\$\frac{2}{3}$, what will 1 pound cost?

14. Divide $\frac{5}{8}$ by $\frac{3}{5}$ of 20; $\frac{12}{17}$ by $\frac{2}{5}$ of 15; $\frac{7}{8}$ by $\frac{1}{8}$ of 40.

15. At \$4 a yard, how many yards of silk can be bought for $\$21\frac{5}{7}$?

ANALYSIS.—As many yards as \$4 is contained times in $\$21\frac{5}{7}$, or $\frac{1}{4}$ of $21\frac{5}{7}=\frac{1}{4}$ of $\frac{152}{7}$, or $\frac{38}{7}=5\frac{3}{7}$ times.

Or, 4 is contained in $21\frac{5}{7}$, 5 times and $1\frac{5}{7}$ or $\frac{12}{7}$ remainder; $\frac{1}{4}$ of $\frac{12}{7}$ is $\frac{3}{7}$, which added to 5 makes $5\frac{3}{7}$. Hence, etc.

16. If a man walks $18\frac{4}{5}$ miles in 4 hours, how far does he walk in 1 hour? In 3 hours? In 5 hours?

17. How many times will $16\frac{4}{5}$ gallons of cider fill a vessel that holds 3 gallons?

18. What cost 1 pound of sugar, if 6 pounds cost $\$1\frac{5}{7}$?

19. If a boy earn $\$12\frac{3}{4}$ in 10 days, how much does he earn in 1 day? In 4 days? In 5 days? In 7 days?

20. Divide $8\frac{1}{3}$ by 5; $10\frac{2}{7}$ by 7; $18\frac{1}{2}$ by 12; $4\frac{3}{4}$ by 8.

21. Divide $\frac{3}{4}$ of 21, by $\frac{2}{5}$ of 10; $\frac{7}{8}$ of 29, by $\frac{1}{9}$ of 63.

Find the value of

22. $\frac{15}{16}\div 5$.
23. $\frac{7}{10}\div 8$.
24. $1\frac{11}{12}\div 6$.
25. $4\frac{3}{8}\div 7$.
26. $9\frac{4}{9}\div 5$.
27. $13\frac{5}{7}\div 4-2\frac{2}{3}$.
28. $27\frac{3}{4}\div 6+1\frac{3}{8}$.
29. $\frac{2}{3}\frac{1}{5}\div 7\times 10$.
30. $1\frac{7}{20}\div 9\times\frac{4}{5}$.
31. $5\frac{3}{8}\times 4+13\frac{1}{2}\div 3$.
32. $\frac{14}{15}\div 7\times\frac{2}{3}$ of $1\frac{1}{2}$.
33. $\frac{7}{20}-\frac{1}{5}\times\overline{\frac{5}{6}\div 4}$.

234. PRINCIPLE.—*Dividing the numerator or multiplying the denominator divides the fraction.* (**200**, 2.)

Always divide the numerator when it is a multiple of the divisor; otherwise multiply the denominator.

WRITTEN EXERCISES.

235. 1. Divide $\frac{18}{25}$ by 6.

OPERATION.

$$\frac{18}{25}\div 6=\frac{18\div 6}{25}=\frac{3}{25}$$

Or, $\frac{18}{25}\div 6=\frac{18}{25\times 6}=\frac{3}{25}$

ANALYSIS.—First, to divide $\frac{18}{25}$ by 6, divide the numerator of the fraction by 6, which gives $\frac{3}{25}$ for the quotient. Or,

Multiply the denominator of the fraction by 6, which gives the same result.

In the first operation, the *number* of parts or of fractional units is diminished, while their *size* or value remains the same; in the second operation, the *number* of the parts remains unchanged, while their *size* is diminished.

Divide

2. $\frac{32}{47}$ by 8. | 4. $\frac{18}{17}$ by 18. | 6. $\frac{128}{1025}$ by 64.
3. $\frac{125}{216}$ by 25. | 5. $\frac{105}{311}$ by 21. | 7. $\frac{35}{114}$ by 35.

8. Divide $50\frac{2}{5}$ by 8.

OPERATION.

$$50\tfrac{2}{5} \div 8 = \frac{252}{5 \times 8} = \frac{252}{40} = 6\tfrac{3}{10}$$

$$\text{Or,}\quad 8\overline{)\,50\tfrac{2}{5}}\qquad 6\tfrac{3}{10}$$

ANALYSIS.—Reduce the mixed number to an improper fraction and divide as in Ex. 1. Or,

Divide as in simple numbers; 8 is contained in $50\frac{2}{5}$, 6 times and a remainder of $2\frac{2}{5}$; $2\frac{2}{5}$ equal $\frac{12}{5}$, which divided by 8 gives $\frac{12}{40}$ or $\frac{3}{10}$, which added to the partial quotient 6 gives $6\frac{3}{10}$, the required quotient.

What is the quotient

9. Of $42\frac{7}{8}$ divided by 7? | 12. Of $248\frac{1}{3}$ divided by 48?
10. Of $128\frac{11}{20}$ divided by 8? | 13. Of $306\frac{5}{12}$ divided by 25?
11. Of $85\frac{7}{36}$ divided by 21? | 14. Of $510\frac{5}{6}$ divided by 30?

15. If 20 pounds of rice cost $\$1\frac{2}{3}$, what is the cost of 1 pound?

16. The product of two numbers is $72\frac{5}{6}$, and one of them is 14; what is the other?

17. If $\frac{35}{36}$ of an acre produce 25 bushels of wheat, what part of an acre will produce 1 bushel?

18. If 12 ploughs cost $\$124\frac{3}{4}$, what is the cost of each?

19. What number multiplied by 48 produces $694\frac{3}{8}$?

20. If 54 horses cost $\$4622\frac{2}{5}$, what is the cost of each?

21. The product of two numbers is $1248\frac{7}{8}$, and one of the numbers is 32; what is the other?

22. If 11 men consume $\frac{3}{4}$ of $100\frac{5}{8}$ pounds of meat in 1 week, how much does 1 man consume in the same time?

23. What is the weight of 4 tubs of lard, if 12 tubs weigh $528\frac{1}{4}$ pounds?

236. When the divisor is a fractional number.

ORAL EXERCISES.

1. How many halves in 1 pound? In 4 pounds?

2. 1 is how many times $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$?

3. What is the quotient of 1 divided by $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{8}$?

4. At $\$\frac{4}{5}$ a yard, how many yards of cloth can be bought for \$6?

Analysis.—As many yards as $\$\frac{4}{5}$ is contained times in \$6. 6 is equal to $\frac{30}{5}$, and 4 fifths is contained in 30 fifths $7\frac{1}{2}$ times.

Or, $\$\frac{4}{5}$ is contained in \$1, $\frac{5}{4}$ times, and in \$6, 6 times $\frac{5}{4}$ or $\frac{30}{4}$, equal to $7\frac{1}{2}$ times. Hence, etc.

5. If a boy earn $\$\frac{3}{4}$ a day, in what time will he earn \$5?

6. At $\$\frac{4}{5}$ a pound, how much tea can be bought for \$9?

7. 1 is how many times $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{4}{7}$? $\frac{3}{8}$? $\frac{7}{10}$? $\frac{5}{12}$?

8. 3 are how many times $\frac{2}{3}$? $\frac{3}{7}$? $\frac{5}{8}$? $\frac{7}{9}$? $\frac{4}{15}$? $\frac{7}{20}$?

9. If a horse eat $\frac{3}{7}$ of a bushel of oats in a day, in how many days will he eat 6 bushels? 8 bushels? 9 bushels?

10. Divide 12 by $\frac{3}{8}$; 16 by $\frac{2}{3}$; 25 by $\frac{1}{5}$; 14 by $\frac{4}{7}$.

11. If $\frac{5}{6}$ of a bale of hay cost \$12, what will 1 bale cost?

Analysis.—Since $\frac{5}{6}$ of a bale cost \$12, $\frac{1}{6}$ of a bale will cost $\frac{1}{5}$ of \$12, or $\$\frac{12}{5}$, and 1 bale, or $\frac{6}{6}$, will cost 6 times $\$\frac{12}{5}$, or $\$\frac{72}{5}$, equal to $\$14\frac{2}{5}$.

12. What will 1 ton of coal cost, if $\frac{3}{8}$ of a ton cost \$7?

13. How many times is $\frac{4}{7}$ contained in 6? In 8? In 11?

14. How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$ of 16?

15. How many turkeys can be bought for \$9 at $\$1\frac{2}{3}$ each?

16. How many garments can be made from 15 yards of cloth, if each garment contains $2\frac{1}{2}$ yards?

How is an integer divided by a fraction?

17. How many times is $1\frac{4}{5}$ contained in 5? $2\frac{3}{4}$, in 9?

18. How many times is 2 eighths of a yard contained in 6 eighths of a yard? $\frac{3}{7}$ of a mile, in $\frac{6}{7}$ of a mile?

19. At $\$\frac{3}{16}$ each, how many pine-apples can be bought for $\$\frac{6}{16}$? For $\$\frac{9}{16}$? For $\$\frac{11}{16}$? For $\$\frac{15}{16}$?

20. How many times $\frac{4}{5}$ in $\frac{8}{5}$? $\frac{5}{12}$ in $\frac{10}{12}$? $\frac{7}{16}$ in $\frac{9}{16}$?

How is a fraction divided by a fraction when they have a common denominator?

21. At $\$\frac{2}{5}$ a pound, how much tea can be bought for $\$\frac{3}{4}$?

ANALYSIS.—As many pounds as $\$\frac{2}{5}$ is contained times in $\$\frac{3}{4}$. $\$\frac{2}{5}$ equals $\$\frac{8}{20}$, and $\$\frac{3}{4}$ equals $\$\frac{15}{20}$; 8 twentieths is contained in 15 twentieths $1\frac{7}{8}$ times.

Or, $\$\frac{2}{5}$ is contained in \$1, $\frac{5}{2}$ times, and in $\frac{3}{4}$ of \$1, $\frac{3}{4}$ of $\frac{5}{2}$ or $\frac{15}{8}$, equal to $1\frac{7}{8}$ times. Hence, etc.

22. How many pounds of honey at $\$\frac{1}{4}$ a pound, can be bought for $\$\frac{5}{8}$? For $\$\frac{4}{5}$? For $\$\frac{5}{6}$? For $\$\frac{9}{10}$?

23. Divide $\frac{4}{5}$ by $\frac{2}{3}$; $\frac{3}{4}$ by $\frac{1}{6}$; $\frac{7}{8}$ by $\frac{3}{4}$; $\frac{6}{7}$ by $\frac{1}{3}$; $\frac{7}{12}$ by $\frac{1}{4}$.

How is a fraction divided by a fraction when they have not a common denominator?

24. In $2\frac{1}{4}$ acres of land, how many building lots of $\frac{3}{8}$ of an acre each?

Reduce mixed numbers to improper fractions, then divide as you divide one fraction by another.

25. At $\$\frac{3}{16}$ a yard, how many yards of cambric can be bought for $\$\frac{3}{4}$? For $\$\frac{7}{8}$? For $\$\frac{1}{2}$? For $\$3\frac{1}{4}$?

26. At $\$\frac{2}{3}$ a bushel, how many bushels of onions can be bought for $\$4\frac{1}{2}$? For $\$5\frac{1}{4}$? For $\$3\frac{4}{5}$?

27. If a man chop $1\frac{1}{6}$ cords of wood in a day, in how many days can he chop $5\frac{1}{4}$ cords? $10\frac{1}{2}$ cords? 12 cords?

28. How many times will $4\frac{3}{4}$ gallons of kerosene fill a can that holds $\frac{1}{3}$ of $\frac{2}{3}$ of 1 gallon?

29. If $\frac{2}{5}$ of a box of figs cost $\$\frac{7}{8}$, what will 1 box cost?

ANALYSIS.—Since 2 fifths of a box cost $\$\frac{7}{8}$, 1 fifth of a box will cost $\frac{1}{2}$ of $\$\frac{7}{8}$ or $\$\frac{7}{16}$, and 1 box or $\frac{5}{5}$ will cost 5 times $\$\frac{7}{16}$ or $\$\frac{35}{16}$ equal to $\$2\frac{3}{16}$.

30. If $\frac{3}{7}$ of a yard of cloth cost $\$\frac{4}{5}$, what will 1 yard cost?

31. What cost 1 quart of wine, if $\frac{5}{12}$ of a quart cost $\$\frac{15}{16}$?

What is the quotient
32. Of $\frac{2}{7}$ divided by 6?
33. Of $2\frac{4}{5}$ divided by 7?
34. Of $16\frac{2}{3}$ divided by 10?

How many times
35. Is $\frac{3}{8}$ contained in $\frac{6}{7}$?
36. Is $\frac{7}{8}$ contained in $2\frac{5}{8}$?
37. Is $\frac{4}{5}$ contained in 9?

237. PRINCIPLE.—*To divide by a fraction, multiply by the denominator, and divide the product by the numerator.*

WRITTEN EXERCISES.

238. 1. Divide 180 by $\frac{8}{15}$.

OPERATION.

$$180 \div \frac{8}{15} = \overline{180 \times 15} \div 8 = 337\frac{1}{2}$$

ANALYSIS.—Multiply 180 by the denominator 15, and divide the product by the numerator 8 (PRIN.), which is equivalent to multiplying 180 by the *reciprocal* of $\frac{8}{15}$, or $\frac{15}{8}$. (**197.**)

In like manner divide

2. 63 by $\frac{7}{13}$.
3. 96 by $\frac{16}{21}$.
4. 120 by $\frac{7}{12}$.
5. 276 by $\frac{23}{37}$.
6. 316 by $\frac{9}{25}$.
7. 604 by $\frac{13}{32}$.
8. If $\frac{3}{7}$ of an acre of land cost \$63, what cost 1 acre?
9. Divide 84 by $6\frac{3}{8}$.
10. Divide 195 by $9\frac{7}{15}$.
11. Divide 1260 by $42\frac{6}{7}$.
12. Divide 2400 by $35\frac{5}{8}$.
13. Paid $\frac{1}{8}$ of \$64 for $\frac{1}{7}$ of $17\frac{1}{2}$ cords of wood. What was the cost a cord?

14. A man gave 503 acres of land to his sons, giving them $83\frac{5}{6}$ acres apiece. How many sons had he?

15. Divide $\frac{9}{10}$ by $\frac{3}{8}$.

OPERATION.

$$\tfrac{9}{10} \div \tfrac{3}{8} = \tfrac{36}{40} \div \tfrac{15}{40} = 2\tfrac{2}{5}$$

Or, $\frac{9}{10} \div \frac{3}{8} = \frac{9}{10} \times \frac{8}{3} = \frac{72}{30} = 2\frac{2}{5}$

ANALYSIS.—$\frac{9}{10}$ divided by $\frac{3}{8}$, is equal to 36 fortieths divided by 15 fortieths, which gives $2\frac{2}{5}$ the required quotient. Or,

Since 1 divided by $\frac{3}{8}$ is $\frac{8}{3}$, $\frac{9}{10}$ of 1 divided by $\frac{3}{8}$ is $\frac{9}{10}$ of $\frac{8}{3}$, or $\frac{72}{30}$ equal to $2\frac{2}{5}$, the same result.

It is obvious that the result is obtained by multiplying the numerator of the dividend by the denominator of the divisor, and the denominator of the dividend by the numerator of the divisor. Hence by inverting the terms of the divisor and using its *reciprocal* (**197**), the operation becomes the same as multiplying one fraction by another.

16. Divide $\frac{3}{5}$ of $\frac{5}{9}$ by $\frac{7}{9}$ of $\frac{5}{14}$.

OPERATION.

$\frac{3}{5}$ of $\frac{5}{9} = \frac{1}{3}$; $\frac{7}{9}$ of $\frac{5}{14} = \frac{5}{18}$

$\frac{1}{3} \div \frac{5}{18} = \frac{1}{3} \times \frac{18}{5} = \frac{6}{5} = 1\frac{1}{5}$

Or,

$$\begin{array}{r|l} 5 & 3 \\ \cancel{9} & \cancel{5} \\ \cancel{7} & \cancel{9} \\ \cancel{5} & \cancel{14}^{2} \\ \hline 5 & 6 \\ \hline & 1\frac{1}{5} \end{array}$$

ANALYSIS.—The dividend reduced to its simplest form is $\frac{1}{3}$; the divisor reduced in like manner is $\frac{5}{18}$, and $\frac{1}{3}$ divided by $\frac{5}{18}$ is $1\frac{1}{5}$, the quotient required. Or,

Invert both factors of the divisor, and obtain the result by a single operation.

If the vertical line is used, the numerators of the dividend and the denominators of the divisor are written on the right.

In like manner

17. Divide $\frac{4}{7}$ by $\frac{2}{3}$.
18. Divide $\frac{9}{20}$ by $\frac{3}{5}$.
19. Divide $\frac{2}{3}$ by $\frac{27}{28}$.
20. Divide $\frac{3}{4}$ of $1\frac{1}{3}$ by $\frac{2}{3}$ of $\frac{1}{2}$.
21. Divide $3\frac{1}{4}$ by $\frac{4}{5}$ of $2\frac{1}{2}$.
22. Divide $\frac{4}{7}$ of $2\frac{1}{3}$ by $\frac{7}{9}$ of 3

Hence the following *general*

RULE.—*Reduce the dividend and divisor to fractions having a common denominator, then divide the numerator of the dividend by the numerator of the divisor.* Or,

I. *Reduce the integers and mixed numbers, if any, t impreper fractions.*

II. *Multiply the dividend by the reciprocal of the diviso*

Apply cancellation when practicable.

Divide

23. 56 by $1\frac{5}{9}$. | 26. $\frac{3}{10}$ by $\frac{11}{24}$. | 29. $16\frac{1}{3}$ by $13\frac{1}{6}$.
24. $\frac{128}{135}$ by 80. | 27. $45\frac{3}{4}$ by 8. | 30. $\frac{9}{10}$ of 4 by $\frac{5}{6}$ of $3\frac{1}{4}$.
25. $1\frac{14}{51}$ by $\frac{35}{68}$. | 28. 92 by $5\frac{1}{6}$. | 31. 44 by $\frac{3}{5}$ of $5\frac{1}{2} \times 7$.

32. What number multiplied by $\frac{3}{5}$ will produce $912\frac{15}{16}$?

33. Of what number is $52\frac{1}{2}$ the $\frac{4}{5}$ part?

34. What number multiplied by $33\frac{3}{8}$ will produce $297\frac{1}{2}$?

35. If $\frac{3}{5}$ of a farm cost $6270, what did the whole cost?

36. If 14 acres of meadow yield $32\frac{3}{5}$ tons of hay, how much do $5\frac{1}{4}$ acres produce?

37. If $7\frac{3}{8}$ yards of velvet are worth $$17\frac{1}{5}$, what is $\frac{1}{8}$ of a yard worth?

38. What will be the cost of $24\frac{1}{4}$ pounds of sugar, if $3\frac{3}{8}$ pounds cost $.33.

39. What is the value of $\dfrac{10\frac{1}{8}}{6\frac{3}{4}}$?

OPERATION.

$$\frac{10\frac{1}{8}}{6\frac{3}{4}} = \frac{\frac{81}{8}}{\frac{27}{4}} = \frac{81}{8} \div \frac{27}{4}, \text{ and}$$

$$\frac{81}{8} \div \frac{27}{4} = \frac{\overset{3}{\cancel{81}}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{\cancel{27}} = \frac{3}{2} = 1\frac{1}{2}$$

ANALYSIS.—This example is only another *form* for expressing division of fractions. Hence, after reducing the mixed numbers to improper fractions, treat the numerator $\frac{81}{8}$ as a *dividend*, and the denominator $\frac{27}{4}$ as a *divisor*, then proceed according to the rule for the division of fractions.

Expressions similar to the above are sometimes called *Complex Fractions*, and the process of performing the division is called *reducing a complex fraction to a simple one.*

If either the numerator or the denominator consists of one or more parts connected by + or −, the operations indicated by these signs must first be performed, then the division.

40. What is the value of $\frac{\frac{7}{8}}{\frac{3}{4}}$, or of $\frac{7}{8} \div \frac{3}{4}$?

41. What is the value of $\frac{12\frac{3}{8}}{10\frac{5}{7}}$, or of $12\frac{3}{8} \div 10\frac{5}{7}$?

42. What is the value of $\frac{117\frac{5}{8}}{18}$, or of $117\frac{5}{8} \div 18$?

43. What is the value of $\frac{\frac{3}{4} \text{ of } 2\frac{1}{3}}{\frac{2}{10}}$, or of $\frac{3}{4} \times 2\frac{1}{3} \div \frac{2}{10}$?

44. Find the value of $\frac{\frac{2}{3} \text{ of } \frac{11}{12}}{\frac{1}{18} \text{ of } 5\frac{1}{2}}$, or of $\frac{2}{3} \times \frac{11}{12} \div \frac{1}{18} \times 5\frac{1}{2}$.

45. Find the value of $\frac{\frac{3}{4} - \frac{3}{5}}{\frac{1}{3} + \frac{3}{8}}$, or of $\overline{\frac{3}{4} - \frac{3}{5}} \div \overline{\frac{1}{3} + \frac{3}{8}}$.

46. If a man spend $\$4\frac{3}{5}$ a month for tobacco, in what time will he spend $\$27\frac{1}{3}$?

Find the value

47. Of $\frac{21\frac{1}{3} + 3\frac{1}{4}}{17\frac{1}{2}}$.

48. Of $\frac{\frac{3}{8} + 2\frac{5}{7}}{1\frac{5}{7} - \frac{3}{4}}$.

49. Of $\frac{5}{8} \times \frac{3}{7} \div \overline{6\frac{1}{3} - 5\frac{4}{15}}$.

50. Of $(16\frac{3}{8} \div 18\frac{1}{4}) \times 17$.

51. Of $9\frac{7}{9} \times 8 \times \frac{1}{15} \div 1\frac{6}{7}$.

52. Of $(7\frac{4}{11} - 5\frac{1}{10}) \div (4\frac{1}{3} + 6\frac{2}{3})$.

RELATION OF NUMBERS.

239. Numbers to be compared with each other, must be so far of the same nature, that one may properly be said to be a *part* of the other.

Thus, we may compare a *day* with a *week*, since the *one* is the seventh *part* of the other; but we cannot say, that a *day* is any part of a *mile*, therefore a day cannot be compared with a mile.

240. Principle.—*Only like numbers are so related as to be compared with each other.*

241. To find what part one number is of another.

ORAL EXERCISES.

1. What part of 5 is 3?

Analysis.—Since 1 is $\frac{1}{5}$ of 5, 3 is 3 times $\frac{1}{5}$ or $\frac{3}{5}$ of 5; or it is 3 divided by 5. Hence 3 is $\frac{3}{5}$ of 5.

2. What part of 9 is 5? Of 12 is 7? Of 24 is 18?

3. 10 yards are what part of 25 yards? 8 pounds, of 20 pounds? 9 eggs, of a dozen? 10 ounces, of a pound?

4. $15 are what part of $50? $60, of $72? 9 days, of 90 days? 6 days, of a week? 7 months, of a year?

5. If an acre of land can be bought for $48, what part of an acre can be bought for $8? For $12? $16? $24?

6. What part of 3 is $\frac{5}{8}$?

Analysis.—1 is $\frac{1}{3}$ of 3, and $\frac{5}{8}$ of 1 is $\frac{5}{8}$ of $\frac{1}{3}$ of 3, or $\frac{1}{3} \times \frac{5}{8} = \frac{5}{24}$.

Or, $3 = \frac{24}{8}$; the relation of $\frac{24}{8}$ to $\frac{5}{8}$ is the same as that of their numerators 24 and 5, or $\frac{5}{24}$. Hence $\frac{5}{8}$ is $\frac{5}{24}$ of 3.

7. What part of 9 is $\frac{1}{4}$? Of 8 is $\frac{3}{16}$? Of 20 is $\frac{2}{3}$?

8. What part of 15 is $1\frac{1}{2}$? Of 18 is $2\frac{1}{3}$? Of 25 is $6\frac{1}{4}$?

9. $\frac{4}{5}$ of a month is what part of 8 months?

10. What part of $\frac{4}{5}$ is $\frac{2}{3}$?

Analysis.—1 fifth is $\frac{1}{4}$ of 4 fifths, and 1, or 5 fifths, is 5 times $\frac{1}{4}$ or $\frac{5}{4}$; hence $\frac{2}{3}$ is $\frac{2}{3}$ of $\frac{5}{4}$, or $\frac{10}{12}$, equal to $\frac{5}{6}$.

Or, $\frac{4}{5} = \frac{12}{15}$, and $\frac{2}{3} = \frac{10}{15}$, and the relation of $\frac{2}{3}$ to $\frac{4}{5}$ is the same as that of 10 to 12, or $\frac{10}{12} = \frac{5}{6}$. Hence $\frac{2}{3}$ is $\frac{5}{6}$ of $\frac{4}{5}$.

11. What part of $\frac{3}{4}$ is $\frac{1}{2}$? Of $\frac{9}{16}$ is $\frac{3}{8}$? Of $\frac{5}{6}$ is $\frac{1}{6}$?

12. What part of $\frac{9}{10}$ is $\frac{3}{5}$? Of $\frac{16}{21}$ is $\frac{8}{14}$? Of $\frac{9}{11}$ is $\frac{1}{3}$?

13. What part of $3\frac{1}{2}$ is $\frac{3}{4}$? Of $4\frac{1}{5}$ is 3? Of $2\frac{4}{5}$ is $1\frac{3}{5}$?

14. What part of $7\frac{1}{2}$ is $1\frac{1}{4}$? Of $1\frac{5}{6}$ is $\frac{11}{12}$? Of $3\frac{1}{3}$ is $2\frac{1}{4}$?

15. What part of 9 miles are $\frac{3}{4}$ of 8 miles? $\frac{1}{4}$ of 10 miles?

WRITTEN EXERCISES.

242. What part of

1. 96 is 72?
2. 56 is $\frac{7}{8}$?
3. 120 is 90?
4. $\frac{13}{16}$ is $\frac{26}{42}$?
5. $\frac{15}{17}$ is $\frac{9}{34}$?
6. 6 is $\frac{45}{64}$?
7. 80 is $5\frac{1}{3}$?
8. $13\frac{3}{4}$ is $2\frac{1}{5}$?
9. 150 is $12\frac{1}{2}$?
10. $24\frac{3}{8}$ is $\frac{11}{16}$?
11. 160 is $26\frac{2}{3}$?
12. $212\frac{1}{7}$ is $42\frac{3}{7}$?

13. A man having $150, gave $25 for a robe, and $\frac{3}{5}$ of the remainder for a harness. What part of $150 had he left?

14. Bought a horse for $275, and sold him for $160. For what part of the cost was he sold?

15. If from $18\frac{3}{4}$ yards of cloth $2\frac{1}{4}$ yards are cut, what part of the whole is taken?

16. If 15 tons of coal cost $$112\frac{1}{2}$, what part of $$112\frac{1}{2}$ will $\frac{3}{4}$ of a ton cost?

243. To find a number when a fractional part of it is given.

ORAL EXERCISES.

1. 7 is $\frac{1}{5}$ of what number?

ANALYSIS.—7 is $\frac{1}{5}$ of 5 times 7, which is 35. Hence 7 is $\frac{1}{5}$ of 35.

2. 12 is $\frac{1}{6}$ of what number? $\frac{1}{8}$ of what number?
3. $9\frac{1}{3}$ is $\frac{1}{7}$ of what number? $\frac{1}{4}$ of what number?
4. $7\frac{3}{4}$ is $\frac{1}{10}$ of what number? $\frac{1}{12}$ of what number?
5. 36 is $\frac{3}{4}$ of what number?

ANALYSIS.—Since 36 is $\frac{3}{4}$ of a certain number, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 36, or 12; and the number is 4 times 12, or 48.

6. 42 is $\frac{6}{8}$ of what number? $\frac{6}{11}$ of what number?
7. 75 is $\frac{5}{7}$ of what number? $\frac{3}{4}$ of what number?
8. 84 is $\frac{12}{13}$ of what number? $\frac{7}{10}$ of what number?
9. $15\frac{3}{4}$ is $\frac{7}{8}$ of what number? $\frac{7}{12}$ of what number?

10. $\frac{7}{8}$ is $\frac{2}{3}$ of what number? $\frac{3}{4}$ of what number?

11. $1\frac{3}{8}$ is $\frac{1}{5}$ of what number? $\frac{5}{8}$ of what number?

12. $3\frac{2}{3}$ is $\frac{5}{11}$ of what number? $\frac{5}{6}$ of what number?

13. 36 is $\frac{9}{8}$ of how many times 4?

ANALYSIS.—36 is $\frac{9}{8}$ of 8 times $\frac{1}{9}$ of 36 which is 32, and 4 is contained in 32, 8 times. Hence 36 is $\frac{9}{8}$ of 8 times 4.

14. 28 is $\frac{7}{16}$ of how many times 8? 12? 9? 16?

15. 35 is $\frac{7}{4}$ of how many times $\frac{1}{7}$ of 28? $\frac{1}{6}$ of 30?

16. $16\frac{4}{5}$ is $\frac{7}{3}$ of how many times $\frac{1}{8}$ of 56? $\frac{1}{12}$ of 48?

17. $\frac{3}{5}$ is $\frac{5}{7}$ of how many times $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{3}{5}$?

18. $\frac{3}{8}$ of 56 is $\frac{7}{10}$ of what number?

ANALYSIS.—$\frac{3}{8}$ of 56 is 3 times $\frac{1}{8}$ of 56, which is 21; and 21 is $\frac{7}{10}$ of 10 times $\frac{1}{7}$ of 21, which is 30. Hence $\frac{3}{8}$ of 56 is $\frac{7}{10}$ of 30.

19. $\frac{8}{9}$ of 27 is $\frac{6}{7}$ of what number? $\frac{5}{8}$ of what number?

20. $\frac{3}{8}$ of $\frac{2}{3}$ of 64 is $\frac{2}{3}$ of what number?

21. $\frac{5}{9}$ of $\frac{1}{2}$ of 72 is $\frac{1}{4}$ of $\frac{5}{8}$ of what number?

22. $\frac{7}{9}$ of 54 is $\frac{6}{5}$ of how many times 5? 7? 8? 9?

23. $\frac{3}{7}$ of $\frac{1}{3}$ of 63 is $\frac{1}{4}$ of $\frac{3}{5}$ of how many times 10? 9?

24. $\frac{4}{7}$ of 56 is $\frac{8}{9}$ of 3 times what number?

ANALYSIS.—$\frac{4}{7}$ of 56 is 32, and 32 is $\frac{8}{9}$ of 36, and 36 is 3 times $\frac{1}{3}$ of 36, which is 12. Hence $\frac{4}{7}$ of 56 is $\frac{8}{9}$ of 3 times 12.

25. $\frac{5}{8}$ of 64 is $\frac{4}{9}$ of 9 times what number?

26. $\frac{2}{3}$ of 21 is $\frac{7}{4}$ of 8 times what number?

27. Paid $60 for a sideboard, which was $\frac{3}{5}$ of the cost of a bookcase. What was the cost of the bookcase?

28. A scarf cost $$1\frac{3}{5}$, which was $\frac{3}{8}$ of the cost of a vest What was the cost of the vest?

29. Paid $100 for a sleigh, which was $\frac{5}{3}$ of 3 times what I paid for a harness. What did I pay for the harness?

Written Exercises of this kind are included in the review examples

REVIEW OF FRACTIONS.

ORAL EXAMPLES.

244. 1. What fraction added to $\frac{3}{7}$ will make $\frac{5}{6}$?

2. What number taken from $25\frac{2}{3}$ will leave $7\frac{3}{4}$?

3. If the sum of two fractions is $\frac{13}{20}$ and one of them is $\frac{1}{5}$, what is the other?

4. From what number must $3\frac{5}{8}$ be taken to leave $5\frac{1}{6}$?

5. A boy spends $\frac{3}{8}$ of his earnings for board, and $\frac{1}{4}$ for clothing. What part has he left?

6. The less of two numbers is $5\frac{7}{10}$, and their difference $\frac{7}{8}$. What is the greater?

7. What number divided by $\frac{4}{5}$ will give a quotient of $1\frac{2}{3}$?

8. The product of two numbers is 4, and one of them is 18? What is the other?

9. If 2 be added to both terms of the fraction $\frac{3}{5}$, will its value be increased, or diminished, and how much?

10. If 2 be added to both terms of the fraction $\frac{5}{3}$, will its value be increased, or diminished, and how much?

11. If a box of tea cost \$$21\frac{7}{8}$, what will $\frac{2}{3}$ of a box cost?

12. A man owning $\frac{7}{8}$ of a steam-mill sold $\frac{4}{5}$ of his share. What part of the whole mill does he still own?

13. A farmer sold 40 acres of land, which was $\frac{8}{15}$ of his whole farm. How many acres were there in his farm?

14. A man sold $\frac{3}{8}$ of his farm, and had 100 acres left. How many acres had he at first?

15. Bought a watch and chain for \$120, the chain costing $\frac{3}{7}$ as much as the watch. What did each cost?

16. A, B, and C together own a yacht. A owns $\frac{3}{7}$ of it, and B. $\frac{4}{9}$ of it. What part does C own?

17. A farmer put all his grain into 4 bins: in the first he put $\frac{3}{8}$ of it, in the second $\frac{1}{4}$, in the third $\frac{1}{6}$, and in the fourth 40 bushels. How many bushels of grain had he?

18. Bought 6 mats at \$$\frac{5}{8}$ each, and had \$5 left. How much money had I at first?

19. How many bushels of grain can be put into 15 bags, if they hold 2$\frac{3}{8}$ bushels each?

20. If 5 men can do a piece of work in 10$\frac{4}{5}$ days, how many days will it take one man to do the same?

21. If a man can build 6 rods of wall in 1 day, how many rods can he build in 7$\frac{3}{4}$ days?

22. How much less than \$10 will 7 pounds of tea cost, at \$$\frac{4}{5}$ a pound?

23. George having \$1$\frac{1}{2}$, gave $\frac{2}{3}$ of it for a knife. What part of a dollar did he give for his knife?

24. At \$12$\frac{1}{2}$ a ton, what will $\frac{5}{6}$ of a ton of hay cost?

25. Bought a cow for \$45$\frac{1}{2}$, and sold her for $\frac{9}{10}$ of what she cost. What did I lose?

26. If a man has 22$\frac{3}{5}$ bushels of clover-seed, and he sells $\frac{3}{4}$ of it, how much has he left?

27. What will 4$\frac{1}{2}$ days' wages come to at \$2$\frac{1}{4}$ a day?

28. A man spent $\frac{2}{3}$ of his money, and then found that \$15 was $\frac{3}{4}$ of what he had left. What had he at first?

29. A man paid \$30 for a cow, $\frac{4}{5}$ of the cost of which was $\frac{2}{3}$ of the cost of a horse. What did the horse cost?

30. How many pounds of tea worth \$$\frac{7}{12}$ a pound, must be given for 9 bushels of apples worth \$$\frac{4}{5}$ a bushel?

31. How many building lots of $\frac{9}{10}$ of an acre each are contained in 1$\frac{4}{5}$ acres of land?

32. At \$$\frac{7}{8}$ each, how many books can be bought for \$3$\frac{1}{2}$?

33. If $\frac{4}{5}$ of a box of figs cost \$1$\frac{7}{8}$, what will 1 box cost?

34. If $5\frac{1}{3}$ dozens of eggs cost $\$1\frac{7}{9}$, what is the cost of 1 dozen? Of $2\frac{1}{2}$ dozens? Of $3\frac{1}{6}$ dozens?

35. If $\frac{4}{7}$ of a barrel of flour cost \$8, what cost 9 barrels?

36. If 3 yards of flannel cost $\$\frac{5}{6}$, what will 8 yards cost?

37. How much tea can be bought for $\$4\frac{1}{2}$, at $\$\frac{5}{8}$ a pound?

38. If $\frac{4}{5}$ of a bushel of quinces cost $\$\frac{5}{6}$, what will 1 bushel cost? $2\frac{1}{3}$ bushels? $3\frac{3}{4}$ bushels?

39. If a gallon of syrup cost $\$\frac{3}{4}$, how many gallons can be bought for $\$\frac{9}{10}$? For $\$1\frac{5}{8}$? For $\$\frac{2}{3}$?

40. A man having \$24, gave $\frac{2}{3}$ of his money for clover-seed at $\$5\frac{1}{3}$ a bushel. How many bushels did he buy?

41. What number taken from $2\frac{1}{3}$ times $12\frac{3}{5}$ leaves $20\frac{3}{4}$?

42. A coal dealer sold $\frac{3}{7}$ of what coal he had on hand for \$90, at the rate of \$6 a ton. How many tons had he?

WRITTEN EXAMPLES.

245. 1. Change $\frac{1}{2}$ of $\frac{4}{9}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{7}{9}$, to equivalent fractions whose denominator shall be 72.

2. Find the least common denominator of $\frac{3}{7}$, $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$.

3. The less of two numbers is $1206\frac{5}{8}$ and their difference $4704\frac{5}{6}$. Find the greater number.

4. Find the value of $(3 \times \frac{7}{8} \times \frac{2}{3} \times 4\frac{5}{6}) - (3\frac{7}{8} \times \frac{2}{3} \times 4 \times \frac{3}{5})$.

5. What number multiplied by $\frac{3}{8}$ will produce $1825\frac{7}{8}$?

6. What number diminished by $\frac{3}{7}$ and $\frac{4}{9}$ of itself leaves a remainder of 144?

7. If $\frac{5}{8}$ of a farm is valued at $\$1729\frac{1}{2}$, what is the value of the whole?

8. A man gave $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{1}{8}$ of his money for different objects and had \$1500 left. How much had he at first?

9. If the dividend is $\frac{7}{9}$, and the quotient $\frac{4}{39}$, what is the divisor?

10. A man owning $\frac{5}{8}$ of a cotton mill, sold $\frac{2}{3}$ of his share for \$4560$\frac{4}{5}$. What was the value of the mill?

11. A stone mason worked 23$\frac{1}{3}$ days, and after paying $\frac{4}{7}$ of his earnings for board and other expenses, had \$53$\frac{1}{3}$ left. What did he receive a day?

12. Gave 6$\frac{3}{4}$ pounds of butter at 36 cents a pound, for 3$\frac{1}{5}$ gallons of oil. What was the oil worth a gallon?

13. A person having 271$\frac{1}{3}$ acres of land, sold $\frac{1}{3}$ of it to one man, and $\frac{3}{8}$ of it to another. What was the value of the remainder at \$57$\frac{3}{8}$ an acre?

14. A man's family expenses are \$2465$\frac{1}{5}$ a year, which is $\frac{5}{7}$ of his income. What does he save?

15. If 7$\frac{1}{2}$ tons of hay cost \$120, how many tons can be bought for \$78?

16. If a man travel 240 miles in 5$\frac{3}{4}$ days, how far would he travel in 3$\frac{1}{2}$ days?

17. A can do a certain piece of work in 8 days, and B can do it in 6 days: in what time can both together do it?

18. A, B, and C can do a piece of work in 5 days; B and C can do it in 8 days: in what time can A do it alone?

19. If $\frac{3}{5}$ of 4 acres of land cost \$205$\frac{3}{5}$, what will $\frac{3}{4}$ of 2 acres cost?

20. Bought $\frac{1}{3}$ of 25$\frac{1}{3}$ yards of cloth for $\frac{1}{5}$ of \$177$\frac{1}{2}$. What was the cost per yard?

21. If $\frac{3}{8}$ of a farm is worth \$9000, what is $\frac{5}{12}$ of it worth?

22. If 8 be added to both terms of the fraction $\frac{11}{12}$, will its value be increased, or diminished, and how much?

23. If 8 be added to both terms of the fraction $\frac{10}{7}$, will its value be increased, or diminished, and how much?

24. How many bushels of oats at \$⅜ a bushel, will pay for ⅝ of a barrel of flour at \$9½ a barrel?

25. A man at his death left his wife \$12500, which was ⅓ of ⅚ of his estate. At her death she left 5/7 of her share to her daughter. What part of the father's estate did the daughter receive from her mother?

26. Paid \$1837½ for 3675 bushels of oats. What was the cost a bushel?

27. A merchant bought a cargo of flour for \$2173½, and sold it for 22/23 of the cost, thereby losing \$.25 on a barrel. How many barrels of flour did he purchase?

28. A man owning ⅘ of 156⅔ acres of land, sold ½ of ¾ of his share. How many acres did he sell, and what was the value of the remainder of his share, at \$42½ an acre?

29. A horse and wagon cost \$360; the horse cost 2⅓ times as much as the wagon. Find the cost of the wagon.

30. If \$7½ will buy 3½ cords of wood, how many cords can be bought for \$31½?

31. A dealer sold 7 barrels of apples for \$32½, which was ⅝ as much as he received for all he had left, at \$4 a barrel. How many barrels in all did he sell?

32. If ¾ of 9 bushels of wheat cost \$13½, what will ⅞ of a bushel cost?

33. A man engaging in trade lost ⅜ of the money he invested, after which he gained \$740, and then, had \$3500. What was his loss?

34. A boy having lost ⅓ of his kite-string, added 45¾ feet; the string was then ⅘ of its original length. What was its original length?

35. There are two numbers the sum of which is 4⅓, and their difference ⅕. What are the numbers?

36. A man invests $\frac{1}{3}$ of his money in cotton, $\frac{1}{4}$ in sugar, $\frac{2}{15}$ in molasses, and the remainder, which is $2542, in dried fruits. What is the amount of each investment, and the total amount?

37. If $\frac{12}{25}$ of $3\frac{1}{2}$ times 1, be multiplied by $\frac{7}{8}$, the product divided by $\frac{3}{5}$, the quotient increased by $4\frac{1}{6}$, and the sum diminished by $\frac{3}{7}$ of itself, what is the remainder?

Reduce to their simplest form:

38. $\dfrac{\frac{5}{9} \text{ of } \frac{3}{7}}{6\frac{1}{5} - 5\frac{4}{15}}$

39. $\dfrac{4\frac{3}{4}+\frac{2}{3}}{3\frac{1}{7} - 1\frac{3}{14}} \times 2.$

40. $\dfrac{\frac{1}{2} \text{ of } \frac{7}{9}}{\frac{2}{3} \text{ of } \frac{1}{4}} - \dfrac{\frac{3}{5} \text{ of } \frac{1}{6}}{\frac{5}{6} \text{ of } \frac{1}{7}}.$

41. $\dfrac{\frac{1}{2} \text{ of } 7\frac{1}{2}}{\frac{3}{8} \text{ of } 15} + \dfrac{4\frac{3}{4} \times \frac{9}{10}}{11 \times 1\frac{1}{2}}.$

Complete the following equations:

42. $\left(\dfrac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{13}{36}} + \dfrac{1}{2\frac{1}{2}}\right) \div \dfrac{5\frac{1}{4}}{8\frac{3}{4}} = ?$

43. $\dfrac{3\frac{1}{9} + 2\frac{5}{6}}{3\frac{1}{9} - 2\frac{5}{6}} + \dfrac{3\frac{1}{2} \times 3\frac{1}{7}}{3\frac{1}{2} \div \frac{7}{11}} - \dfrac{9\frac{1}{3}}{7} = ?$

44. $(\overline{7\frac{1}{2} + 1\frac{17}{22}} \div \overline{8\frac{2}{3} + 3\frac{22}{33}}) - (\overline{3\frac{4}{5} + \frac{22}{45}} \div \overline{3\frac{3}{4} + 14\frac{1}{36}}) = ?$

45. $\dfrac{\dfrac{1+\frac{1}{3}}{1-\frac{1}{3}}}{\dfrac{1+\frac{3}{5}}{1-\frac{3}{5}}} = ?$

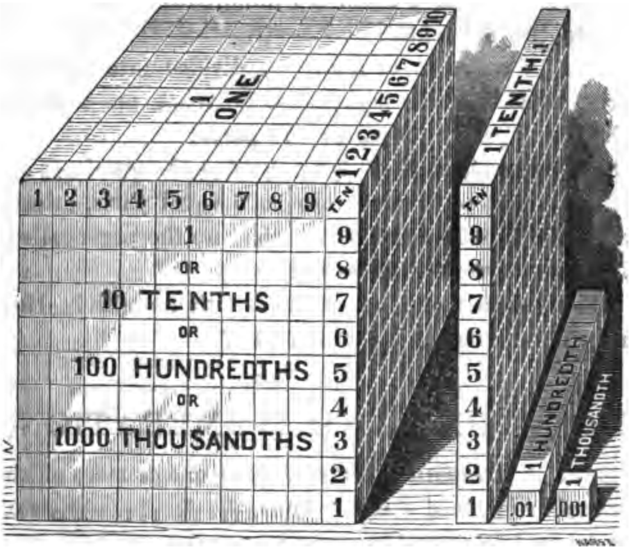
246. SYNOPSIS FOR REVIEW.

FRACTIONS.
- 1. EQUAL PARTS.
- 2. PRINCIPLES, 1 and 2.
- 3. DEFINITIONS.
  - 1. A Fraction.
  - 2. A Fractional Unit.
- 4. EXPRESSION OF FRACTIONS.
- 5. TERMS.
  - 1. Denominator.
  - 2. Numerator.

SYNOPSIS FOR REVIEW.—CONTINUED.

COMMON FRACTIONS.

- 1. CLASSIFICATION.
  - 1. Proper Fractions.
  - 2. Improper Fractions.
- 2. DEFINITIONS.
  - 1. Mixed Numbers.
  - 2. Reciprocal of a Number.
  - 3. " " Fraction.
- 3. VALUE OF A FRACTION.
- 4. GENERAL PRINCIPLES, 1, 2, 3.
- 5. GENERAL LAW.
- 6. REDUCTION.
  - **202.**
    - 1. Defs.
      - 1. *Reduction.*
      - 2. *Higher Terms.*
      - 3. *Lower Terms.*
      - 4. *Lowest Terms.*
    - 2. Principle.
    - 3. Rules, 1, 2.
  - **210.** Rule.
  - **212.** Rule.
  - **214.**
    - 1. Defs.
      - 1. *Common Denominator.*
      - 2. *Least Common Denominator.*
    - 2. Principles, 1, 2.
    - 3. Rule (1); I, II. (2.)
- 7. ADDITION.
  - 1. Principle.
  - 2. Rule, I, II.
- 8. SUBTRACTION.
  - 1. PRINCIPLE.
  - 2. Rule, I, II.
- 9. MULTIPLICATION.
  - **227.** Principles, 1, 2. *Rule I, II* (1).
  - **230.** Principle. *Rule I, II* (2).
- 10. DIVISION.
  - **233.** Principle.
  - **236.** Principle.
  - *Rule I, II, III.*
- 11. RELATION OF NUMBERS. Principle.
  - **241.**
  - **242.**



DECIMALS

ORAL EXERCISES.

247. 1. If a unit be divided into 10 equal parts, what is each part called? What are 2 parts? 3 parts? 4 parts?

2. What is the fractional unit?

3. If 1 *tenth* of a unit be divided into 10 equal parts, what is each part called? What are 2 parts? 4 parts? 5 parts? 7 parts? 12 parts? 25 parts?

4. What is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{2}{10}$ of $\frac{1}{10}$? $\frac{4}{10}$ of $\frac{1}{10}$? $\frac{6}{10}$ of $\frac{1}{10}$?

5. If a unit be divided into 100 equal parts, or each *tenth* into 10 equal parts, what are the parts called?

6. What part of 1 tenth is 1 hundredth? How many hundredths in 1 tenth?

7. If 1 *hundredth* of a unit be divided into 10 equal parts, what is each part called? What are 3 parts? What are 8 parts? 9 parts? 15 parts?

8. What is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{1}{100}$? $\frac{5}{10}$ of $\frac{1}{100}$?

9. If a unit be divided into 1000 equal parts, or each *hundredth* into 10 equal parts, what are the parts called? What are 12 parts? 26 parts? 42 parts?

10. What part of 1 hundredth is 1 thousandth? How many 1 thousandths is 1 hundredth?

NOTATION AND NUMERATION.

248. A ***Decimal Fraction*** is one or more of the *decimal* divisions of a unit. Thus, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{5}{100}$, $\frac{25}{1000}$, etc., are decimal fractions.

Decimal Fractions are commonly called *Decimals.** (**16.**)

249. Decimals are like *other Fractions*, except that their denominators increase and decrease by the *uniform scale* of 10. The fractional units are, therefore, always *tenths, hundredths, thousandths*, etc.

250. The ***Decimal Sign*** (.), called the *decimal point*, is used to distinguish a *decimal* from an *integer*, and *must always* be placed before the numerator of the decimal.

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* The terms *fraction* and *decimal* will hereafter be used to distinguish the common from the decimal *form* of expression. Thus, and .75, are two *forms* of expressing the *same thing*. For nience we shall call the first form a *fraction*, and the other a

251. The position of the decimal sign indicates the *denominator*, and determines the *value* of the decimal expression. Thus,

$\frac{7}{10}$ is expressed .7.
$\frac{36}{100}$ " " .36.
$\frac{126}{1000}$ is expressed .126.
$\frac{1425}{10000}$ " " .1425.

252. The ***Denominator*** of a decimal fraction is always 10, 100, 1000, etc., or 1 with as many ciphers annexed as there are figures in the given decimal. Thus, $.4 = \frac{4}{10}$; $.09 = \frac{9}{100}$; $.007 = \frac{7}{1000}$, etc.

253. The ***Numerator*** of a decimal fraction when expressed alone, must have as many decimal places as there are ciphers in the denominator. Thus, $\frac{8}{10} = .8$; $\frac{12}{100} = .12$; $\frac{125}{1000} = .125$, etc.

If the numerator does not contain as many figures as there are ciphers in the denominator, prefix ciphers until the number of places is equal to the number of ciphers in the denominator, and prefix the decimal point. Thus, $\frac{7}{100} = .07$; $\frac{9}{1000} = .009$, etc.

254. Decimal fractions may be written in two ways; either as *other fractions*, the denominator being expressed, or, in *decimal notation*, the denominator being omitted. Thus,

$\frac{5}{10}$, or .5 is read 5 tenths and is $\frac{1}{10}$ of 5 units.
$\frac{5}{100}$, " .05 " 5 hundredths, " $\frac{1}{10}$ " 5 tenths.
$\frac{5}{1000}$, " .005 " 5 thousandths, " $\frac{1}{10}$ " 5 hundredths.

255. The value of any decimal figure is always $\frac{1}{10}$ of the value of the same figure in the next place to the left.

256. When an integer and decimal are written together, the expression is a ***Mixed Number*** (**195**). Thus, 7.12 and 26.134 are mixed numbers.

257. The relation of *decimals* and *integers* to each other is clearly shown by the following

TABLE.

Hund.-millions.	Ten-millions.	*Millions.*	Hund.-thousands.	Ten-thousands.	*Thousands.*	Hundreds.	Tens.	UNITS.	Tenths.	Hundredths.	*Thousandths.*	Ten-thousandths.	Hund.-thousandths.	*Millionths.*	Ten-millionths.	Hund-millionths.
9	8	7	6	5	4	3	2	1.	2	3	4	5	6	7	8	9
INTEGERS.									DECIMALS.							

The number is read 987 *million* 654 *thousand* 321, *and* 23 million 456 thousand 789 *hundred-millionths.*

A decimal takes the *name* of its right-hand order.

258. In decimals, as in integers, make the order of *units* the starting-point of notation and of numeration, extending the scale to the *left* of the units' place in writing *integers,* and to the *right* of the units' place in writing *decimals.*

The first order to the left of units is *tens,* and the first order to the right of units is *tenths;* the second order to the left of units is *hundreds,* and the second order to the right is *hundredths;* the third order to the left is *thousands,* and the third order to the right is *thousandths,* and so on, the *integers* on the left, and the *decimals* on the right, equally distant from the units' place, corresponding in name.

259. Hence, both in integers and in decimals, the *value* of any figure is determined by the *position* of that figure, and is always *ten times* the value of the same figure in the next *lower order,* or 1 *tenth* the value of the same figure in the *next higher* order. Hence,

260. In writing decimals, *vacant orders* must be filled with ciphers. (**36,** 2.)

Dictation exercises, both *oral* and *written*, should be given, until the pupil can *write* and *read* decimals with rapidity and correctness. *Oral*, thus, *Ques.*, "The denominator of a fraction is 100, the numerator 7; what will express the decimal?" The prompt response should be, "*Point, naught, seven,* read, *seven-hundredths*" (.07). *Ques.* "The denominator is 1000, the numerator 35." *Ans.* "*Point, naught, three, five,* read *thirty-five thousandths*" (.035), etc.

Also the converse; thus, *Ques.* "Point, naught, eight; what will express the fraction?" *Ans.* "The numerator is *eight*, the denominator *one hundred*, and the fraction is *eight-hundredths*" ($\frac{8}{100}$). *Ques.* "Point, naught, one, five?" *Ans.* "The numerator is *fifteen*, the denominator is *one thousand*, and the fraction *fifteen-thousandths*" ($\frac{15}{1000}$), etc.

WRITTEN EXERCISES.

261. Express in the form of a fraction,

1. .12.
2. .16.
3. .138.
4. .003.
5. .2162.
6. .0056.
7. .14036.
8. .00035.

Express in the form of a decimal,

9. $\frac{25}{100}$.
10. $\frac{206}{1000}$.
11. $\frac{7}{1000}$.
12. $\frac{18}{1000}$.
13. $\frac{3027}{10000}$.
14. $\frac{302}{10000}$.
15. $\frac{42}{10000}$.
16. $\frac{145}{100000}$.

262. Prefixing a cipher to a decimal multiplies the denominator by 10, and hence *divides* the decimal by 10 (**200**, 2). Thus, $.5 = \frac{5}{10}$; $.05 = \frac{5}{100}$; $.005 = \frac{5}{1000}$; or, $.5 \div 10 = .05$; $.05 \div 10 = .005$, etc.

263. Rejecting a cipher from the left of a decimal divides the denominator by 10, and hence *multiplies* the decimal by 10 (**200**, 1). Thus, $.007 = \frac{7}{1000}$; $.07 = \frac{7}{100}$; $.7 = \frac{7}{10}$; or, $.007 \times 10 = .07$; $.07 \times 10 = .7$.

264. Annexing a cipher to a decimal multiplies both numerator and denominator by 10, and hence reduces the fraction to *higher terms* (**200**, 3). Thus, $.3 = \frac{3}{10}$; $.30 = \frac{30}{100}$; $.300 = \frac{300}{1000}$.

265. Rejecting a cipher from the right of a decimal divides both numerator and denominator by 10, and hence reduces to *lower terms* (**200,** 3). Thus, $\frac{600}{1000} = .600$; $\frac{60}{100} = .60$; $\frac{6}{10} = .6$.

From the foregoing explanations are deduced the following

266. PRINCIPLES.—1. *Decimals are governed by the same laws of notation as integers.* Hence,

2. *The value of any decimal figure depends upon the place it occupies at the right of the decimal sign.* (**258.**)

3. *Every removal of a decimal figure one place to the right diminishes its value tenfold.* (**262.**)

4. *Every removal of a decimal figure one place to the left increases its value tenfold.* (**263.**)

5. *Ciphers may be annexed or rejected at the right of any decimal, without changing its value.* (**264, 265.**)

WRITTEN EXERCISES.

267. Express in figures and decimally;

1. Seventy-five thousandths. $\frac{75}{1000} = .075$.

2. Fifteen hundredths.
3. Seven thousandths.
4. Fifty-three thousandths.
5. Nine ten-thousandths.
6. 22 ten-thousandths.
7. 245 ten-thousandths.
8. 1042 hundred-thousandths.
9. 14605 millionths.

10. $\frac{128}{1000}$.
11. $7\frac{9}{1000}$.
12. $\frac{3906}{100000}$.
13. $127\frac{12}{100}$.
14. $\frac{572}{1000000}$.
15. $84\frac{308}{10000}$.
16. $\frac{13067}{1000000}$.
17. $60\frac{15}{10000}$.

RULE.—I. *Write the numerator of the decimal as if an integer, writing ciphers in the place of vacant orders to give each significant figure its proper value, and place the decimal point before tenths.*

II. *Read the decimal as if an integer, and give it the name of its right-hand order.*

In like manner express decimally the following fractions and mixed numbers:

18. 596 thousandths.
19. 625 ten-thousandths.
20. 12 ten-thousandths.
21. 74 millionths.
22. 105 ten-millionths.
23. 99010 billionths.

24. Four hundred thirty-seven thousand five hundred 49 millionths.

25. Three million forty thousand 12 ten-millionths.

26. Six hundred and 24 hundred-millionths.

27. Four hundred ninety-five million seven hundred five thousand and 43075 ten-millionths.

28. Four million seven hundred thirty-five thousand and 903624 hundred-millionths.

29. $\frac{17}{10000}$.
30. $\frac{165}{100000}$.
31. $\frac{100354}{1,000,000}$.
32. $\frac{15704}{10000,000}$.
33. $205\frac{65}{100}$.
34. $68\frac{36}{100000}$.
35. $705\frac{1005}{1000,000}$.
36. $300\frac{10781}{10,000,000}$.

Copy and read the following decimals and mixed numbers:

37. .705.
38. .0023.
39. .3607.
40. .00705.
41. .400564.
42. .000256.
43. .0010275.
44. .0000407.
45. 18.0031.
46. 6.306.
47. 49.0703.
48. 10.0064.
49. 22.09042.
50. 1.10106.
51. 14.00370.
52. 70.00063.
53. .00078.
54. .3050040.
55. .0003006.
56. 42.0637.
57. 108.0094.
58. 230.40685.
59. 30.26002015.
60. 8.040103463.

DECIMAL CURRENCY.

268. ***Currency*** is coin, bank-bills, treasury notes, etc., employed in trade and commerce.

269. A ***Decimal Currency*** is a currency whose denominations increase and decrease by the *decimal scale*

270. The ***Legal Currency*** of the United States is a decimal currency; it is sometimes called *Federal Money*, because issued by the Federal Government.

TABLE.

10 mills (*m.*)	make	1 cent.	*c.* or *ct.*
10 cents	"	1 dime.	*d.*
10 dimes or 100 cents	"	1 dollar.	$.
10 dollars	"	1 eagle.	*E.*

271. Since the dollar is the *unit* of United States Money, dimes, cents, and mills are respectively *tenths, hundredths,* and *thousandths* of the unit.

272. Dollars should be written as *integers,* with the sign ($), prefixed; and dimes, cents, and mills, as *decimals,* with the decimal point at their left, or before *tenths.* Thus, 7 dollars 3 dimes 4 cents 5 mills, are written $7.345.

273. The denominations eagles and dimes are not regarded in business operations, eagles being tens of dollars, and dimes tens of cents. Thus, $34.27 is read 34 dollars 27 cents, instead of 3 eagles 4 dollars 2 dimes 7 cents.

274. Since the two places of dimes and cents, or of *tenths* and *hundredths* are appropriated to cents, when the number of cents is less than 10, write a cipher in the place of *tenths.* Thus, 9 cents are written $.09. (**73.**)

275. The *half-cent* may be written, either as a fraction ($\frac{1}{2}$), or as 5 mills. Thus, thirty-seven and a half cents are written $$.37\frac{1}{2}$, or $.375.

276. Cents are often written as fractions of a dollar. Thus, $9.28 may be also written $\$9\frac{28}{100}$.

277. In business transactions, if the mills in the final *result* are 5 or more than 5, they are considered a *cent*, if *less* than 5, they are not regarded. Thus, $5.197, would be called $5.20, and $5.194 would be called $5.19.

278. PRINCIPLES.—1. *Decimal currency is expressed according to the decimal system of notation.*

2. *All the operations in Decimal Currency are the same as the corresponding operations in Decimals.*

REDUCTION OF DECIMALS.

279. To reduce decimals to units of lower or higher orders.

ORAL EXERCISES.

1. How many tenths in 2 units? In 5 units?
2. How many tenths in 20 hundredths? In .40?
3. How many hundredths in 2 units? In 4 units?
4. How many hundredths in 200 thousandths?
5. How many hundredths in 5 tenths? In .6? .7? .8?
6. How many thousandths in .06? In .25? .48? .75?
7. How many hundredths in .150? In .260? In .2500?
8. In 400 thousandths how many hundredths? Tenths?
9. How many tenths of a dollar in $6? Hundredths?
10. Change 4 dollars 50 cents to cents. To mills.
11. How many dollars are 300 cents? 540 cents?
12. How many cents are 2600 mills? Dollars?

13. What is the decimal expression for 5 cents?

Ans. Sign, point, naught, five; read *five hundredths* ($.05).

14. Express decimally 7 cents; 9 cents; 15 cents.

15. Express decimally 7 mills; 5 cents 6 mills.

16. Express decimally 2 dollars 45 cents and 6 mills.

Ans. Sign, two, point, four, five, six; read, *two* and *four hundred fifty-six thousandths* dollars ($2.456).

17. What is the decimal expression for 84 cents 5 mills?

18. Change .3 to hundredths; to thousandths.

19. Change .4 and .05 to thousandths; .07 and .01

20. Change .5, .08, and .023 to equivalent decimals, having a common denominator of 1000. Also, .14, .009, and .6. .7, .007, and .091.

21. Reduce .7, .150 and .600, to equivalent decimals, having the least common denominator. Also, .50, .250, and .1700. .43, .006, and .0214.

280. From the foregoing it appears,

1. That dollars may be reduced to cents by annexing *two* ciphers; and to mills, by annexing *three* ciphers.

Omit the sign $ and write *cts.* or *m.* after the result.

2. That cents may be reduced to mills by annexing *one* cipher.

3. That cents may be reduced to dollars by pointing off *two* figures from the right; and mills to dollars, by pointing off *three* figures from the right, and prefixing the sign ($).

4. That mills may be reduced to cents by pointing off *one* figure from the right.

5. That two or more decimals are reduced to a *common denominator* by annexing or rejecting ciphers at the right until the decimal places of all are equal.

WRITTEN EXERCISES.

281. Reduce

1. \$85 to cents. (**280,** 1.)
2. \$615 to cents.
3. \$24.06 to cents.
4. \$9.206 to mills.
5. \$57 to mills.
6. 86 cents to mills. (**280,** 2.)
7. \$.763 to mills.
8. \$.47½ to mills.

Change

9. 486 cts. to dollars. (**280,** 3.)
10. 32462 cents to dollars.
11. 40327 mills to dollars.
12. 846 mills to cents.
13. 50000 mills to dollars.
14. 61040 cents to dollars.

15. Reduce .7, .05, and .304, each to hundred-thousandths. (**280,** 5.)

16. Reduce 2.5, .107, and .0008, each to ten-thousandths.

17. Change 4, 2.17, .136, and .0408 to equivalent decimals having a common denominator.

18. Reduce 9 tenths, 24 thousandths, 109 hundred-thousandths, and 47 millionths to equivalent decimals having the least common denominator. Also,

19. 100.03, 41.0034, .475, .0753, and 6.00044.

20. .84003, 120.4, 5.00031, and 15.240007.

282. To reduce a decimal to a fraction.

ORAL EXERCISES.

1. How many halves in $\frac{5}{10}$? In $\frac{50}{100}$? In $\frac{500}{1000}$?
2. How many fifths in $\frac{8}{10}$? In $\frac{40}{100}$? $\frac{60}{100}$? .6?
3. How many fourths in $\frac{25}{100}$? In .50? In .75?
4. How many twentieths in $\frac{10}{100}$? In $\frac{15}{100}$? In .20?
5. In .50 how many halves? Fourths? Tenths?

WRITTEN EXERCISES.

283. 1. Change .375 to an equivalent fraction.

OPERATION.

$.375 = \frac{375}{1000} = \frac{3}{8}$

ANALYSIS.—The numerator is 375, the denominator 1000, and the decimal expressed as a fraction is $\frac{375}{1000} = \frac{3}{8}$. Hence $.375 = \frac{3}{8}$.

Change to equivalent fractions,

2. .16.
3. .125.
4. .625.
5. $.75.
6. $.375.
7. $.655.
8. .024.
9. .5625.
10. .3125.
11. $.875.
12. .0008.
13. .9375.

RULE.—*Omit the decimal point, supply the proper denominator, and then reduce the fraction to its lowest terms.*

14. Reduce .13$\frac{1}{3}$ to an equivalent fraction.

OPERATION.—$.13\frac{1}{3} = \frac{13\frac{1}{3}}{100} = \frac{40}{300} = \frac{2}{15}$.

Reduce to fractions in their lowest terms,

15. \$.37$\frac{1}{2}$.
16. \$.62$\frac{1}{2}$.
17. \$.08$\frac{1}{3}$.
18. .06$\frac{1}{4}$.
19. .58$\frac{1}{3}$.
20. .93$\frac{3}{4}$.
21. \$.33$\frac{1}{3}$.
22. \$.66$\frac{2}{3}$.
23. \$.16$\frac{2}{3}$.
24. .1944$\frac{4}{9}$.
25. .4444$\frac{4}{9}$.
26. .0008$\frac{1}{4}$.

Express by an integer and a fraction,

27. $15.4.
28. $36.75.
29. $9.625.
30. $27.375.
31. 24.26$\frac{2}{3}$.
32. 84.05$\frac{5}{9}$.
33. 38.41$\frac{2}{3}$.
34. 104.00$\frac{4}{9}$.

284. To reduce a fraction to a decimal.

ORAL EXERCISES.

1. How many tenths in $\frac{1}{2}$? How many hundredths? How many thousandths?

2. How many tenths in $\frac{1}{5}$? Hundredths in $\frac{2}{5}$? In $\frac{3}{4}$?

3. How many hundredths in $\frac{3}{20}$? In $\frac{4}{25}$? In $\frac{9}{50}$?

WRITTEN EXERCISES.

285. 1. Reduce $\frac{5}{8}$ to an equivalent decimal.

OPERATION.

$\frac{5}{8} = \frac{5000}{8000} = \frac{625}{1000} = .625$

ANALYSIS.—Annex the same number of ciphers to both terms of the fraction and divide the resulting terms by 8, the significant figure of the denominator, to obtain the *decimal* denominator 1000. Then change to the decimal *form*. (**253.**)

2. Reduce $\frac{2}{125}$ to an equivalent decimal.

OPERATION.

$$125\overline{)2.000}$$
$$.016$$

Or,

$\frac{2}{125} = \frac{2000}{125000} = \frac{16}{1000} = .016$

ANALYSIS. — Since $\frac{2}{125} = \frac{1}{125}$ of 2 units, and 2 units equal 2000 *thousandths*, $\frac{1}{125}$ of 2000 *thousandths* is 16 *thousandths*, or .016.

Reduce to equivalent decimals :

3. $\$\frac{3}{4}$. | 5. $\frac{14}{25}$. | 7. $\$\frac{4}{5}$. | 9. $\frac{3}{125}$.
4. $\$\frac{7}{8}$. | 6. $\frac{15}{16}$. | 8. $\$\frac{29}{200}$. | 10. $\frac{13}{16}$.

RULE.—I. *Annex ciphers to the numerator and divide by the denominator.*

II. *Point off as many decimal places in the result as there are ciphers annexed.*

The sign + is sometimes placed after the result to indicate that there is still a remainder. Thus, $\frac{2}{3} = .666+$, or $.666\frac{2}{3}$.

Reduce to five decimal places :

11. $\frac{5}{6}$. | 12. $\frac{7}{27}$. | 13. $\frac{43}{56}$. | 14. $\frac{37}{150}$.

Reduce to equivalent decimals :

15. $\frac{27}{160}$. | 17. $\frac{1}{320}$. | 19. $\frac{7}{8}$ of $\frac{3}{5}$. | 21. $\frac{3}{4}$ of $\$2\frac{1}{2}$.
16. $\frac{13}{256}$. | 18. $\frac{9}{16000}$. | 20. $\frac{1}{25}$ of $\frac{16}{25}$. | 22. $\$2\frac{3}{4} \times \frac{3}{125}$.

Change to the decimal form :

23. $101\frac{3}{4}$. | 25. $11\frac{1}{8}$. | 27. $\$.93\frac{1}{16}$. | 29. $\frac{31\frac{1}{5}}{8}$ of $\frac{6\frac{3}{4}}{2\frac{1}{12}}$.
24. $\$225\frac{3}{5}$. | 26. $8.6\frac{4}{5}$. | 28. $\$4.0\frac{3}{25}$.

ADDITION.

ORAL EXERCISES.

286. 1. What is the sum of $\frac{5}{10}$ and $\frac{7}{10}$? .6 and .4?

2. What is the sum of $\frac{9}{100}$ and $\frac{13}{100}$? .11 and .15?

3. What is the sum of .12 and .20? .15 and .25?

4. Find the sum of 6 mills and 9 mills. .008 and .021.

5. What is the sum of .4 and .09? Of .04 and .009?

How many decimal figures in the sum of *tenths* and *tenths?* Of *tenths* and *hundredths?* Of *hundredths* and *thousandths?* Of *tenths* and *thousandths?* In adding several decimals, each having a different number of decimal places, how many places will there be in the sum?

287. Since decimals and integers increase and decrease uniformly by the scale of ten, decimals expressing like parts of a unit may be *added, subtracted, multiplied,* and *divided* in the same manner as integers.

The pupil should obtain and express all results in *decimal form.*

WRITTEN EXERCISES.

288. 1. Find the sum of 12.07, 326.2086, .768, and 1.9.

OPERATION.

$$\begin{array}{r} 12.0700 \\ 326.2086 \\ .7680 \\ 1.9000 \\ \hline 340.9466 \end{array}$$

ANALYSIS.—Write the numbers so that units of the same order stand in the same column. After reducing the decimals to a common denominator by annexing ciphers (**280**, 5), or supposing them to be annexed, add as in integers, placing the decimal point before *tenths* in the sum.

In like manner find the sum

2. Of .375, .24, .536, .0437, .50039, and .008236.

3. Of 405.327, 64.03, .84673, 121.8, and 7.00327.

4. Of \$18.19, \$142.095, \$.964, \$5.125, and \$40.50.

Rule.—I. *Write the numbers so that units of the same order stand in the same column and the decimal points in the same vertical line.*

II. *Add as in addition of integers, and place the decimal point before the order of tenths in the sum.*

5. What is the sum of 37 thousandths, 54 ten-thousandths, 407 hundred-thousandths, and 12345 millionths?

6. Find the sum of 45 units, 25 tenths, 360 hundredths, 75 thousandths, 52 ten-thousandths, and 406 millionths.

Find the sum

7. Of $\$25\frac{3}{4}$, $81.09, $\$16\frac{1}{8}$, $\$.87\frac{1}{2}$, $\$150\frac{1}{2}$, and $\$\frac{7}{8}$.

8. Of $103.60\frac{1}{3}$, $6.0\frac{7}{8}$, .37012, and $40.0034\frac{1}{8}$.

9. Of $24.6\frac{1}{2}$, $47.32\frac{7}{35}$, $5.3784\frac{1}{8}$, and $2.64878\frac{3}{4}$.

10. Of 61.843 acres, $8\frac{9}{10}$ acres, 21.04 acres, $15\frac{13}{20}$ acres, and $3\frac{4}{5}$ acres.

11. Bought a ton of coal for $\$7\frac{3}{8}$, a barrel of sugar for $\$28\frac{7}{10}$, a chest of tea for $23.08, and a barrel of flour for $\$10.87\frac{1}{2}$. What was the cost of all?

In the reduction of each fraction, carry the decimal to at least *five* places, to insure accuracy in the *fourth*.

12. Find the sum of $\frac{3}{80}$, $\frac{2}{7}$, $\frac{43}{56}$, $\frac{7}{24}$, and $\frac{75}{436}$, in decimals, correct to the fourth place.

13. A man bought a farm for $6736.75, which was $\$325\frac{3}{8}$ less than he sold it for. What did he sell it for?

14. How many rods of fence will enclose a field, the sides of which are respectively 34.72 rods, $48\frac{11}{25}$ rods, 152.17 rods, $95\frac{3}{8}$ rods, and $56\frac{5}{8}$ rods?

15. Paid for building a house $3450.75, for painting the same $\$518\frac{7}{8}$, for furniture $\$1204.37\frac{1}{2}$, and for carpets $\$810\frac{4}{5}$. What was the cost of the whole?

SUBTRACTION.

ORAL EXERCISES.

289. 1. From $\frac{8}{10}$ take $\frac{5}{10}$. From .9 take .7.

2. From $\frac{25}{100}$ take $\frac{15}{100}$. From .36 take .12.

3. From $\frac{17}{1000}$ take $\frac{8}{1000}$. From .028 take .010.

4. From $\frac{35}{100}$ take $\frac{2}{10}$. From 45 cents take 20 cents.

5. Find the difference between $\frac{3}{5}$ and $\frac{6}{10}$. $\frac{1}{4}$ and .25.

6. Find the value of $\frac{7}{10}$—.3 ; of .5—$\frac{1}{4}$; .65—.5.

7. Find the value of \$$\frac{1}{2}$—30 cents ; 80 cents—\$.6.

How many decimal places in the remainder, if there are *three* in the minuend and *one* in the subtrahend? If *two* in the minuend and *four* in the subtrahend? If *none* in the minuend and *three* in the subtrahend?

WRITTEN EXERCISES.

290. 1. From 3.16 subtract .2453.

OPERATION.

3.1600
.2453
2.9147

ANALYSIS.—Write the given numbers as in Addition, the subtrahend under the minuend, reducing the decimals to a common denominator, by annexing ciphers (**280**, 5), or supposing them to be annexed, and then subtract as in integers.

2. From 324.07 take 70.20681.

3. From \$1034 take \$500.94.

RULE.—I. *Write the subtrahend under the minuend, so that units of the same order stand in the same column.*

II. *Subtract as in subtraction of integers, and place the decimal point before the orders of tenths in the remainder.*

Find the difference, decimally, between

4. \$16$\frac{1}{2}$ and \$43$\frac{3}{10}$.

5. 1.0066 and .630482.

6. \$143$\frac{3}{8}$ and \$304.96.

7. 2 and .00345.

8. 10.0402 and 26 millionths.
9. 115 and 115 tenths.
10. 5 and 125 ten-millionths.
11. $\$.875$ and $\$\frac{7}{8}$.
12. $\$\frac{11}{16}$ and $\$.75$.
13. 7.005 and .7005.
14. $.93\frac{1}{25}$ and $1.169\frac{3}{8}$.
15. $1\frac{7}{8}$ and 1875 millionths.
16. $\$200$ and $\$70\frac{11}{16}$.
17. .4 and $.04\frac{1}{5}$.
18. $\frac{11}{32}$ and $\frac{44}{125}$.
19. $.1\frac{1}{16}$ and $.01\frac{5}{8}$.

20. A speculator having 7346 acres of land, sold at different times $364\frac{1}{2}$ acres, 1235.125 acres, $2700\frac{3}{8}$ acres, and 850.65 acres. How much had he left?

21. A man bought an overcoat for $\$36\frac{3}{4}$, a sack for $\$18\frac{5}{8}$, and pants for $\$8.12\frac{1}{2}$, and gave in payment one fifty, and two ten-dollar bills. What change should he receive?

Find the decimal value

22. Of $\$350 - \$38\frac{1}{2} + \$100\frac{3}{25}$.
23. Of $2\frac{3}{4} - 1\frac{4}{5} + (.9 - \frac{3}{16})$.
24. Of $.37\frac{1}{2} + \frac{7}{8} + 4.2 - (2 - .68\frac{3}{4})$.
25. Of $\$250 - (\$170\frac{29}{100} - \$14\frac{1}{4}) + \$\frac{13}{20}$.
26. Of $\$48\frac{15}{100} + \$.97 - (\$\frac{4}{5} + \$.62\frac{1}{2} + \$\frac{5}{8})$.

MULTIPLICATION.

ORAL EXERCISES.

291. 1. What is 5 times $\frac{1}{10}$? 6 times .3? 4 times .5?
2. What is 7 times $\frac{4}{100}$? 5 times .08? 6 times .09?
3. What is $\frac{7}{10} \times 3$? $3 \times .7$? $4 \times .6$? $.5 \times 7$?
4. What is $\frac{3}{100} \times 5$? $5 \times .04$? $.05 \times 7$? $8 \times .06$?
5. What is $\frac{5}{10} \times \frac{4}{10}$? $.4 \times .3$? $.8 \times .7$? $.6 \times .9$?
6. What is $\frac{5}{100} \times \frac{5}{10}$? $.5 \times .05$? $.12 \times .6$? $.7 \times .11$?
7. What is $\frac{8}{100} \times \frac{7}{100}$? $.03 \times .07$? $.15 \times .06$?
8. What is 8 times $\$.6$? 7 times $\frac{8}{100}$ of a dollar?
9. What is $8 \times .5$? $8 \times .05$? $8 \times .005$? $8 \times .0005$?

How many decimal places in the product of *units* multiplied by *tenths?* *Tenths* by *tenths?* *Tenths* by *hundredths?* *Hundredths* by *hundredths?*

If there are *two* decimal figures in the multiplicand, and *two* in the multiplier, how many are there in the product? If *three* in the multiplicand and *one* in the multiplier? How many decimal places are there always in the product?

292. PRINCIPLE.—*The number of decimal places in any product is equal to the decimal places in both factors.*

WRITTEN EXERCISES.

293. 1. Multiply .64 by .8.

OPERATION.

.64
.8
.512

ANALYSIS.—Multiply as in fractions. (232.) Thus, .64 × .8 = $\frac{64}{100} \times \frac{8}{10} = \frac{512}{1000}$ = .512. Or,

Multiply as in integers, and since *hundredths* multiplied by *tenths* produces *thousandths*, the product must contain *three* decimal places. (PRIN.)

Multiply
2. 1.245 by .27.
3. .4056 by 35.05.

Multiply
4. 7.25 by .00012.
5. $506⅛ by .048½.

RULE.—*Multiply as in multiplication of integers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.*

1. If there are not as many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers.

2. To multiply by 10, 100, 1000, etc., remove the decimal point in the multiplicand as many places toward the *right* as *there are ciphers* in the multiplier. (266, 4.)

Multiply and express the product decimally:

6. $324½ by .324.
7. $175.64 by .205.
8. 5.728 by 100.
9. .6207 by 1000.
10. 5½ hundredths by 25.
11. 26000 by 26 thousandths.
12. 84 tenths by 244 hundredths.
13. 7⅝ tenths by .06¼.

Find the value

14. Of $3.126 \times .046 \times .3$.
15. Of $9\frac{3}{8} \times .07\frac{1}{2} \times 10$.
16. Of $18.75 \times 1.001 \times \frac{4}{5}$.
17. Of $.25$ of $\frac{2}{15} \times .04\frac{1}{2}$.
18. Of $327\frac{1}{3} \times .9 \times 4\frac{1}{4}$.
19. Of $\$8.56 \times .06\frac{1}{4} \times 100$.
20. Of $18\frac{2}{25} \times .0062\frac{1}{2} \times 1000$.
21. Of $.01$ of $\frac{3}{4} \times 100 \times .08\frac{1}{3}$.

22. Bought 156 pounds of cheese at $\$.12\frac{1}{2}$ a pound, 327 pounds of coffee at $\$.26\frac{3}{4}$ a pound, and 17 barrels of apples at $\$2.87\frac{1}{2}$ a barrel. What was the cost of the whole?

23. If an acre of land produce 127.25 bushels of potatoes, how many bushels will 4.375 acres produce?

What is the value

24. Of 170 barrels of apples, at $\$2\frac{5}{8}$ a barrel?
25. Of 100 cords of wood, at $4.38 a cord?
26. Of $204\frac{7}{10}$ acres of land, at $\$72\frac{3}{4}$ an acre?
27. Of $580\frac{1}{2}$ pounds of sugar, at $9\frac{1}{4}$ cents a pound?
28. Of 126 mules, at $\$97\frac{5}{8}$ each?

29. What is the cost of $3\frac{5}{8}$ bales of cloth, each bale containing 36.75 yards, at $.85 a yard?

30. A farmer sold 300 bushels of oats at $.45 a bushel, $16\frac{3}{8}$ cords of wood at $\$3\frac{7}{8}$ a cord. He received in payment 125 pounds of sugar at $\$.12\frac{1}{2}$ a pound, 36 pounds of tea at $\$\frac{7}{8}$ a pound, 6 barrels of flour at $\$8.37\frac{1}{2}$ a barrel, and the remainder in cash. How much cash did he receive?

Complete the following equations:

31. $\$450.75 - \$24\frac{1}{2} \times 3.24 + \$18\frac{3}{25} = ?$
32. $(\$200 - \$125\frac{1}{4}) \times (\frac{3}{4} + 2.5) = ?$
33. $3.0065 \times .304 + 40\frac{3}{16} \times 10 = ?$
34. $.00493 \times 1000 \times (1 - \frac{1}{8} + .025) = ?$
35. $(4 - .00036 + .316) - (.75 + \overline{3\frac{1}{2} - 1\frac{5}{8}}) = ?$
36. $(\frac{99}{200} \times .08\frac{1}{4} + .03685 \times \frac{4}{5}) \times 100 = ?$

DIVISION.

ORAL EXERCISES.

294. 1. What is $\frac{1}{3}$ of $\frac{9}{10}$? $\frac{1}{4}$ of $\frac{24}{100}$? $\frac{1}{5}$ of $\frac{45}{1000}$?

2. What is $\frac{1}{2}$ of .8? $\frac{1}{6}$ of .42? $\frac{1}{8}$ of .072?

3. Divide .8 by 4; .56 by 7; .120 by 10; .0048 by 12.

4. Divide $\frac{16}{10}$ by $\frac{4}{10}$. $\frac{45}{100}$ by $\frac{15}{100}$.

5. Divide 4.8 by 6.

ANALYSIS.—4.8 equals 48 *tenths*, and $\frac{1}{6}$ of 48 *tenths* is 8 *tenths*, or .8.

6. Divide .48 by 6; .48 by .06; .048 by .006.

7. Divide $\frac{6}{10}$ by $\frac{12}{100}$ ($.6 \div .12$); 7.5 by 2.5.

8. Multiply $\frac{8}{10}$ by $\frac{9}{10}$ ($.8 \times .9$). Divide .72 by .9.

9. Multiply $\frac{8}{100}$ by $\frac{9}{100}$ ($.08 \times .09$). Divide .0072 by .09.

10. The product of two factors is .096, one of which is .8; what is the other?

How many decimal places in the quotient when *tenths* are divided by *units*? *Tenths* by *tenths*? *Hundredths* by *tenths*? *Thousandths* by *hundredths*?

If there are *two* decimal figures in the divisor and *three* in the dividend, how many are there in the quotient? If *three* in the divisor and *three* in the dividend? If *none* in the divisor and *three* in the dividend? If *two* in the divisor and *none* in the dividend?

295. PRINCIPLES.—1. *The dividend must contain at least as many decimal places as the divisor, before division is possible.*

2. *Since the dividend is the product of the divisor and quotient, it contains as many decimal places as both divisor and quotient.* Hence,

3. *The quotient must contain as many decimal places as the number of decimal places in the dividend exceeds those in the divisor.*

WRITTEN EXERCISES.

296. 1. Divide .952 by .7.

OPERATION.

$$\begin{array}{r} .7\,)\,.952 \\ \hline 1.36 \end{array}$$

ANALYSIS.—Divide as in fractions. (**238.**) Thus, $.952 \div .7 = \frac{952}{1000} \div \frac{7}{10} = \frac{952}{1000} \times \frac{10}{7} = \frac{136}{100} = 1.36$. Or,

Divide as in integers, and since the dividend contains *three* decimal places, and the divisor *one*, the quotient must have *two* decimal places. (PRIN. 3.)

Divide

2. 81.6 by 3.6.
3. 675 by .15.
4. .952 by 4.76.

Divide

5. $41.25 by 33.
6. $518.70 by $14.25.
7. 345.15 by .075.

RULE.—*Divide as in division of integers, and from the right of the quotient point off as many figures as the decimal places in the dividend exceed those in the divisor.*

1. If the number of figures in the quotient be *less* than the excess of the decimal places in the dividend over those in the divisor, the deficiency must be supplied by *prefixing* ciphers.

2. If there be a remainder after dividing the dividend, annex ciphers, and continue the division: the ciphers annexed are decimals of the dividend.

3. In most business transactions, the division is considered sufficiently exact when the quotient is carried to 4 decimal places, unless great accuracy is required.

4. To divide by 10, 100, 1000, etc., remove the decimal point in the dividend as many places to the *left* as *there are ciphers* in the divisor. (**266**, 3.)

8. Divide 88.476 by 1.2 ; by 3.6 ; by .01½ ; by 1.04.

9. Divide $56.05 by .59; $408.37½ by 27.

10. Divide $6.45 by $.45; $52 by $.65 ; 293.75 by 45¾.

11. Divide .0026 by .003 ; 3 by .450 ; 75 by 1000.

12. What is the quotient of 75.15208 divided by 24? by .24? by .024? by .0024? by .00024 ?

13. Divide $3875 by 10; by 100 ; by 1000; by 10000.

What is the value of

14. $645.5 \div 1000$.
15. $\$1000 \div \$.02$.
16. $\$56 \div .007$.
17. $1.904 \div 4.76$.
18. $3 \div 18\frac{3}{4}$.
19. $4.2 \div 31\frac{1}{4}$.
20. $17\frac{1}{2} \div 1000$.
21. $.73\frac{3}{8} \div 100$.
22. $\$27 \div 37\frac{1}{2}$.
23. $.001 \div 100$.
24. $100 \div .001$.
25. $\$48\frac{3}{4} \div \$\frac{3}{8}$.

26. Divide .24 by 72 ; $\frac{5}{8}$ of .24 by $\frac{3}{14}$ of .042.

27. If 64 tons of iron cost $4816, how many tons can be bought for $1730.75 ?

28. How many coats can be made from 32.4 yards of cloth, allowing 2.7 yards for each coat ?

29. At $\$287\frac{3}{8}$ each, how many horses can be bought for $4885.80 ?

30. If 125 bushels of potatoes cost $\$82\frac{1}{2}$, how many barrels, each containing $2\frac{1}{2}$ bushels, can be bought for $224.40 ?

31. If $3\frac{1}{2}$ cords of wood cost $\$11.37\frac{1}{2}$, what will $20\frac{1}{8}$ cords cost ?

32. How much sugar can be bought for $46.75, if $\frac{3}{4}$ of a hundred pounds cost $\$6\frac{3}{8}$?

33. Gave $10\frac{3}{4}$ cords of wood, worth $\$4\frac{1}{2}$ a cord, for 7.74 barrels of flour. What was the flour worth a barrel ?

34. A man sold a horse for $125, and received in payment $12\frac{1}{2}$ yards of cloth at $\$3\frac{1}{4}$ a yard, and the balance in tea at $\$.62\frac{1}{2}$. How many pounds of tea did he receive ?

Find the second member in each of the following equations :

35. Of $(1.008 \div 18 + \overline{63 \div 4000} \times 100) - \frac{1}{8} = ?$
36. Of $\overline{714 - .714} \div (.\overline{34 - .034} \times .25 \text{ of } 6) = ?$
37. Of $(\overline{.48 \div 800} \times 10000 + \overline{6.4 \div .08}) \div .125 = ?$
38. Of $(34 \times .193 + 2.7 \times .4\frac{1}{8}) \div (4.81 - \frac{3}{8} \text{ of } 1.662) = ?$
39. Of $(\$262.90 \div \$.56) \times .\overline{0084 + .02\frac{7}{8}} \times 100 = ?$
40. Of $(\$1260 \times 3.49) \div \$10.47 - \$850 \div \$6.80 = ?$

CIRCULATING DECIMALS.

ORAL EXERCISES.

297. 1. What are the prime factors of 10? Of 100?

2. Change to the decimal form $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{3}{8}$; $\frac{7}{16}$. (**285.**)

What are the prime factors of each of the denominators of these fractions?

Are they the same as the prime factors of 10?

Can these fractions be reduced to perfect decimals?

3. Change to the decimal form, extending to four places, $\frac{1}{3}$; $\frac{2}{3}$; $\frac{7}{9}$; $\frac{8}{11}$.

Can these fractions be reduced to perfect decimals?

What are the prime factors of their denominators?

4. Change to the decimal form, extending to three places, $\frac{1}{6}$; $\frac{1}{12}$; $\frac{2}{15}$.

Can these fractions be reduced to perfect decimals?

What are the prime factors of their denominators?

How do the decimals produced by these fractions differ from the decimals produced by the fractions in examples 2 and 3?

What kind of decimals are all fractions equivalent to, that in their lowest terms have denominators containing the factors 2 or 5?

5. What figure is constantly repeated in reducing to a decimal $\frac{1}{3}$? $\frac{2}{3}$? $\frac{4}{9}$? $\frac{5}{11}$?

6. If a decimal consists of 3 repeated indefinitely, what fraction is it equal to?

7. Is there any difference between $\frac{1}{3}$ and $\frac{3}{9}$? $\frac{2}{3}$ and $\frac{6}{9}$? $\frac{6}{9}$ and $\frac{66}{99}$? $\frac{66}{99}$ and $\frac{6666}{9999}$?

8. Is there any difference between $\frac{6}{11}$ and $\frac{54}{99}$? $\frac{54}{99}$ and $\frac{5454}{9999}$?

9. If the numerator is 4444, what must be its denominator so that the fraction may equal $\frac{4}{9}$?

To change a repeating decimal number to an exact fraction, what figures must be used in the denominator?

DEFINITIONS AND PRINCIPLES.

298. A ***Finite Decimal*** is a perfect decimal, or one that terminates with the figures written; as, .25, .375.

299. A ***Circulating Decimal*** is a decimal in which a figure, or set of figures, is constantly repeated in the same order; as, .333+, .727272+.

300. A ***Repetend*** is the figure or set of figures, continually repeated.

The repetend is written but once, and when it consists of a single figure a point is placed over it; when it consists of more than one figure, points are placed over the first, and over the last figure. Thus, the circulating decimal .666+, and .297297+, are written $.\dot{6}$, and $\dot{2}9\dot{7}$.

301. A ***Pure Circulating Decimal*** is a decimal which commences with a repetend; as $.\dot{7}$, or $.\dot{2}7\dot{9}$.

302. A ***Mixed Circulating Decimal*** is a decimal in which the repetend is preceded by one or more decimal places called the *finite part* of the decimal; as, $.2\dot{7}$, or $.04\dot{6}48$, in which .2 or .04 is called the finite part.

303. The law for the formation of repetends will be apparent from the following:

1.	$\frac{1}{9}$	$=.1111+$	$=.\dot{1}.$	5.	$\frac{4}{9}$	$=.4444+$	$=.\dot{4}.$
2.	$\frac{1}{99}$	$=.01010+$	$=.\dot{0}\dot{1}.$	6.	$\frac{23}{99}$	$=.2323+$	$=.\dot{2}\dot{3}.$
3.	$\frac{1}{999}$	$=.001001+$	$=.\dot{0}0\dot{1}.$	7.	$\frac{135}{999}$	$=.135135+$	$=.\dot{1}3\dot{5}.$
4.	$\frac{1}{9999}$	$=.00010001+$	$=.\dot{0}00\dot{1}$	8.	$\frac{1728}{9999}$	$=.17281728+$	$=.\dot{1}72\dot{8}$

304. **Principles.**—1. ***Every fraction in its lowest terms, whose denominator contains no other prime factors than* 2 *or* 5 *is equivalent to a finite decimal.***

2. ***Every fraction in its lowest terms, whose denominator contains other prime factors than* 2 *or* 5 *is equivalent to a circulating decimal.***

3. *Every fraction in its lowest terms, whose denominator contains 2 or 5 with other prime factors is equivalent to a mixed circulating decimal.*

4. ***Every pure circulating decimal is equal to a fraction whose numerator is the repetend, and whose denominator consists of as many 9's as there are places in the repetend.***

WRITTEN EXERCISES.

305. To change a fraction to a finite or to a circulating decimal.

1. Change $\frac{7}{16}$ to a finite decimal. **(285.)**

2. Change to finite decimals, $\frac{4}{5}$; $\frac{7}{8}$; $\frac{9}{25}$; $\frac{13}{16}$; $\frac{23}{40}$; $\frac{11}{125}$; and $\frac{77}{200}$. (PRIN. 1.)

3. Change to a pure circulating decimal $\frac{7}{27}$.

OPERATION.—$\frac{7}{27} = 7.000000 \div 27 = .259259 + = .\dot{2}5\dot{9}$. (PRIN. 2.)

4. Change to pure circulating decimals, $\frac{6}{7}$; $\frac{7}{9}$; $\frac{9}{11}$; $\frac{12}{37}$; $\frac{10}{21}$; and $\frac{7}{41}$.

5. Change to a mixed circulating decimal $\frac{5}{6}$.

OPERATION.—$\frac{5}{6} = 5.0000 \div 6 = .8333 + = .8\dot{3}$. (PRIN. 3.)

6. Change to mixed circulating decimals $\frac{5}{12}$; $\frac{8}{15}$; $\frac{13}{22}$; $\frac{11}{30}$; and $\frac{47}{150}$.

7. Change to finite, or to circulating decimals the following fractions: $\frac{3}{25}$; $\frac{1}{8}$; $\frac{5}{17}$; $\frac{31}{64}$; $\frac{18}{27}$; $\frac{9}{32}$; $\frac{11}{125}$; $\frac{5}{21}$; and $\frac{143}{625}$.

306. To change a pure circulating decimal to a fraction.

1. Change $.\dot{2}1\dot{6}$ to a fraction.

OPERATION.

$$\dot{2}\,1\,\dot{6} = \frac{216}{999} = \frac{8}{37}$$

ANALYSIS.—Since $.\dot{0}0\dot{1} = \frac{1}{999}$ **(303)**, $\dot{2}1\dot{6}$ is equal to $\frac{216}{999}$, which reduced to its lowest terms equals $\frac{8}{37}$. Hence $.\dot{2}1\dot{6} = \frac{8}{37}$.

Change to fractions,

2. $.\dot{4}\dot{5}$.
3. $.\dot{6}\dot{6}$.
4. $.\dot{2}9\dot{7}$.
5. $.\dot{6}7\dot{5}$.
6. $.\dot{3}2\dot{4}$.
7. $.\dot{4}15\dot{8}$.

RULE.—*Write the figures of the repetend for the numerator of a fraction, and as many 9's as there are places in the repetend for the denominator, and reduce to its lowest terms.*

In like manner change to fractions,

8. $.\dot{2}7\dot{9}$.
9. $.\dot{3}2\dot{1}$.
10. $.\dot{6}43\dot{5}$.
11. $.\dot{1}06\dot{7}$.
12. $.\dot{9}512\dot{1}$.
13. $.\dot{9}2307\dot{6}$.
14. Reduce $2.\dot{2}9\dot{7}$ to an improper fraction.
15. Reduce $12.\dot{0}8\dot{1}$ to an improper fraction.

307. To change a mixed circulating decimal to a fraction.

1. Change $.2\dot{2}\dot{7}$ to a fraction.

OPERATION.

1st. $.2\dot{2}\dot{7} = \frac{2}{10} + \frac{27}{990} = \frac{5}{22}$

Or 2d. $.2\dot{2}\dot{7} = .2\frac{27}{99} = \dfrac{2\frac{27}{99}}{10} = \dfrac{225}{990} = \dfrac{5}{22}$

Or 3d. $2\dot{2}\dot{7}$ given decimal.
$\underline{2}$ finite part.
$.225$ $\quad \frac{225}{990} = \frac{5}{22}$

ANALYSIS.—Since the repetend is not $\frac{27}{99}$, but $\frac{27}{99}$ of $\frac{1}{10} = \frac{27}{990}$, write the finite part and the repetend each as fractions and add them, the reasons for which will appear more clearly in the second solution.

Or, by an abbreviated method of reducing the fractions to a common denominator, 2 × 99 = 2 × 100 − 2; hence, 2 × 100 + 27 − 2 = 225 is the numerator of the equivalent common fraction.

2. Change to fractions, $.5\dot{7}$; $.04\dot{8}$; $.100\dot{4}$; $.64\dot{7}\dot{2}$.
3. Change to mixed numbers, $7.5\dot{4}\dot{3}$; $2.5\dot{6}\dot{4}$; $7.01\dot{2}\dot{6}$.

RULE.—*Reduce the finite part and the repetend of the given decimal each to the form of a fraction. Then add them, and reduce to lowest terms.* Or,

From the given decimal subtract the finite part for a numerator, and for a denominator write as many 9's as there are figures in the repetend, with as many ciphers annexed as there are figures in the finite part.

Change to fractions,

4. $.04\dot{6}4\dot{8}$.
5. $.\dot{7}85\dot{2}$.
6. $.9\dot{2}8571\dot{4}$.
7. $.35\dot{1}3\dot{5}$.
8. $.01\dot{2}\dot{6}$.
9. $5.2\dot{7}$.

To *add, subtract, multiply*, or *divide* circulating decimals, reduce them to fractions, and then perform the required operation.

For a fuller development of "Circulating Decimals" and "Continued Fractions," see "Robinson's Higher Arithmetic."

SHORT METHODS.

ORAL EXERCISES.

308. 1. What part of $1 are 8⅓ cents? 16⅔ cents? 12½ cents? 25 cents? 50 cents?

2. At 25 cents a pound, what cost 22 pounds of coffee?

ANALYSIS.—Since 25 cents are $¼, 22 pounds will cost 22 times $¼, or $$\frac{22}{4}$, equal to $5½, or $5.50. Or,

At $1 a pound, 22 pounds will cost $22, and at $¼ a pound, ¼ of $22, which is $5½, or $5.50.

3. What is the cost of 80 pounds of beef at 12½ cents a pound? At 16⅔ cents? At 20 cents? At 25 cents?

4. At 33⅓ cents a can, what will be the cost of 25 cans of sweet corn? Of 37 cans? Of 54 cans? Of 60 cans?

5. What is the cost of 160 pounds of sugar at 6¼ cents a pound? At 8⅓ cents? 10 cents? 12½ cents?

6. How many pounds of raisins, at 16⅔ cents a pound, can be bought for $5?

ANALYSIS.—Since 16⅔ cents are $⅙, $5 will buy as many pounds of raisins as $⅙ is contained times in $5, which are 30 times. Hence, etc.

7. At $.50 a bushel, how many bushels of oats can be bought for $15? For $16½? For $25?

8. At 12½ cents a yard, how many yards of calico can I buy for 27 pounds of butter, at 33⅓ cents a pound?

9. What is the cost of 40 pairs of shoes, at $1.25 a pair?

ANALYSIS.—At $1 a pair, the cost would be $40; but since the price is $1 + $¼, the whole cost is $40 + ¼ of $40, or $50.

10. At $1.50 each, what is the cost of 48 chairs?

11. What is the cost of 60 yards of cloth, at $1.12½ a yard? At $1.16⅔? At $1.25? At $1.33⅓? At $2.50?

12. At $2.25 a pair, what is the cost of 12 pairs of shoes? Of 16 pairs? Of 18 pairs? 20 pairs? 25 pairs?

DEFINITIONS.

309. ***Quantity***, in commercial transactions, is the amount of anything bought or sold, and is estimated by the number of times it contains the *measuring unit*.

310. ***Price*** is the value in money of each *measuring unit* of any commodity.

311. ***Cost*** is the value of the *entire* quantity.

312. An ***Aliquot Part*** or ***Even Part***, of a number is such a part as will exactly divide that number. Thus, 2, 2½, 3⅓, and 5, are aliquot parts of 10.

An *aliquot part* may be either an integer or a mixed number, while a *component factor* must be an integer.

ALIQUOT PARTS OF ONE DOLLAR.

5 cents = $\frac{1}{20}$ of $1.
10 cents = $\frac{1}{10}$ of $1.
20 cents = $\frac{1}{5}$ of $1.
25 cents = $\frac{1}{4}$ of $1.
50 cents = $\frac{1}{2}$ of $1.

6¼ cents = $\frac{1}{16}$ of $1.
8⅓ cents = $\frac{1}{12}$ of $1.
12½ cents = $\frac{1}{8}$ of $1.
16⅔ cents = $\frac{1}{6}$ of $1.
33⅓ cents = $\frac{1}{3}$ of $1.

WRITTEN EXERCISES.

313. To find the cost of a quantity when the price is an aliquot part of one dollar.

1. What cost 951 bushels of oats, at $.33⅓ a bushel?

OPERATION.

```
3)951
  317
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ANALYSIS.—At $1 a bushel, the cost would be $951; but since the price is ⅓ of $1 a bushel, the cost is ⅓ of $951, which is $317. Or, the cost is ⅓ as many dollars as there are bushels, and $\frac{951}{3}$=317. Hence, etc.

2. What cost 750 slates, at 33⅓ cents each? At 25 cents?

3. At $.50 each, what cost 631 shad? 1250? 1605?

RULE.—*Take such a fractional part of the given number or quantity as the price is of one dollar.*

4. What is the cost of 12 sacks of coffee, each sack containing 43 pounds, at 33⅓ cents a pound?

5. A merchant sold 5 pieces of prints, each containing 28 yards, at 16⅔ cents per yard, 6 pieces of sheeting, each containing 34 yards, at 8⅓ cents per yard, and received in payment 41 bushels of oats at $.50 a bushel, and the balance in money. How much money did he receive?

6. At $1.12½ a foot, what cost 324 feet of wire fence?

OPERATION.

8) 324
40.5
$364.5

ANALYSIS.—At $1 a foot, the cost would be $324; but since the cost is $1 + $⅛, the entire cost is $324 + ⅛ of $324, which is $364.50.

7. At $1.33⅓ each, what will 642 steel shovels cost?

8. What cost 320 cloth caps, at $1.20 each?

314. To find the quantity when the cost is given, and the price is an aliquot part of one dollar.

1. How many barrels, at $.50 each, can be bought for $213?

OPERATION.

$\$213 \div \$\frac{1}{2} = 426$

Or, $213 \times 2 = 426$

ANALYSIS.—Since $½ will pay for 1 barrel, $213 will pay for as many barrels as $½ is contained times in $213, or 426 barrels. Or, since $1 will pay for 2 barrels, $213 will pay for 213 times 2, or 426 barrels.

2. How many baskets of pears can be bought for $318, at $.33⅓ each? At $.50 each?

3. How many pine-apples can be bought for $240, at 16⅔ cents each? At 20 cents? At 25 cents?

RULE.—*Divide the cost by such a fraction as will express the price as an aliquot part of one dollar.*

4. How many pounds of cheese can be bought for $350, at 6¼ cents a pound? At 8⅓? 10 cents? 12½ cents?

5. How many cocoa-nuts, at $.25, can be bought for $150.75?

315. To find the cost when the quantity and the price of 100, or 1000 are given.

1. What cost 564 cedar posts, at $12.25 for 100 posts?

1st OPERATION.

```
      $12.25
         564
 ───────────
100)6909.00
     $69.09
```

2d OPERATION.

$12.25 × 5.64 = $69.09

Analysis.—At $12.25 a post, the cost would be $12.25 × 564 = $6909. But since $12.25 is the price of 100 posts, $6909 is 100 times the cost. Hence divide by 100 (**296**, Note 4), and the result is $69.09. Or,

Since 564 is 5 and 64 hundredths (5.64), if 1 hundred cost $12.25, 5.64 will cost 5.64 times $12.25, or $69.09.

If the price is by the *thousand*, divide the product by 1000, or reduce the quantity to thousands and decimals of a thousand before multiplying.

2. What is the cost of 1684 pounds of beef, at $9.37½ a hundred pounds?

3. What cost 22840 railroad ties, at $174.55 a thousand?

4. How much is the freight on 4575 pounds of merchandise from New York to Baltimore, at $.98 for 100 pounds?

Rule.—*Multiply the price by the quantity reduced to hundreds and decimals of a hundred, or to thousands and decimals of a thousand, and point off in the product as in multiplication of decimals.*

In business transactions, the letter C is sometimes used for *hundreds*, and M for *thousands*, when the price is by the 100, or 1000.

What is the cost,

5. Of 536720 bricks, at $8.75 per M.?

6. Of 2108 feet of pine boards, at $3.12½ per C.?

7. Of 2700 pine-apples, at $16¼ per 100?

8. Of 875 feet of scantling, at $10½ per M.?

9. Of 2160 oysters, at $1.86 per 100?

10. Of 3080 fence pickets, at $5¾ per 1000?

11. Of 28642 feet of timber, at $11⅜ per M.?

12. Of 1480 pounds of maple sugar, at $12.37½ per 100?

13. What is the value of 3700 cedar rails, at $5¾ per C.?

14. What is the value of 12500 shingles, at $6⅞ per M.?

15. Find the cost of 527 feet of boards at $15⅛ per M. and of 972 feet of siding at $1.62½ per C.

316. To find the cost, when the quantity and the price of a ton of 2000 pounds are given.

1. What is the cost of a load of hay, weighing 2280 pounds, at $18.50 a ton?

OPERATION.

$$\begin{array}{r} 2\,)\underline{18.50} \\ \$9.25 \\ \underline{2.28} \\ \$21.09 \end{array}$$

ANALYSIS.—Since $18.50 is the cost of 2000, ½ of $18.50, or $9.25 is the cost of 1000 pounds; and 2280 pounds will cost 2.280 times $9.25, or $21.09.

2. At $4.75 a ton, what will a load of plaster weighing 2806 pounds cost?

3. What is the freight on 21672 pounds of iron, at $2.80 a ton?

RULE.—*Multiply one-half the price of a ton by the number of thousands and decimals of a thousand in the given quantity, as in* **315.**

4. What is the value of 150 sacks of guano, each sack containing 162½ pounds, at $51¾ a ton?

5. Find the value of 6340 pounds of Lehigh coal, at $7½ a ton, and 5080 pounds of soft coal at $6¼ a ton.

6. At $26.44 a ton, what will be the cost of 1526 pounds of bone dust?

LEDGER ACCOUNTS.

317. A ***Ledger*** is the *principal book* of accounts kept by business men. Into it are transferred, in a condensed form, all the items of the *Journal*, or *Day Book*, for convenient reference and preservation.

318. The *debits* (marked *Dr.*) are placed on the left, and the *credits* (marked *Cr.*) are placed on the right.

319. The ***Balance of an Account*** is the *difference* between the debit and credit sides. When this is settled, or paid, the account is said to be *balanced.*

320. Find the balance of the following Ledger Accounts:

(1.) Dr.	(1.) Cr.	(2.) Dr.	(2.) Cr.
$506.76	$42.17	$2371.67	$4763.84
194.32	36.24	571.84	7061.39
173.26	8.42	90.50	8242.76
71.32	10.71	2037.69	364.96
39.46	94.30	94.46	410.31
152.60	347.16	876.54	5724.27
71.78	40.00	679.81	6317.66
320.00	12.94	4930.71	2431.27
48.50	271.19	104.13	163.55
63.41	500.50	1987.67	7063.21
56.00	11.44	142.84	451.09
410.10	81.92	522.71	200.00
72.22	10.10	3114.60	1867.36
137.89	107.09	152.91	768.72
276.44	207.16	9328.42	3024.27

ACCOUNTS AND BILLS.

321. An ***Account,*** in commercial transactions, is a record of *debits and credits.*

322. A ***Debtor*** is a person who owes another money, goods, or services.

323. A ***Creditor*** is a person to whom money, goods, or services are due from another.

324. A ***Bill*** is a written statement of money paid, of goods sold or delivered, or of services rendered. It is sometimes called an *Invoice.*

An account or bill should always state the place and the time of each transaction, the names of both the parties, the price or value of each item, and the entire cost.

325. A ***Bill is receipted*** when the words "Received Payment" are written at the bottom, and the creditor's name is signed either by himself, or by some authorized person.

326. The following abbreviations are in general use:

@	At.	Disc't	Discount.	Net	Without disc't.
% or Acc't	Account.	Do.	The same.	No.	Number.
Am't	Amount.	Doz.	Dozen.	Pay't	Payment.
Bal.	Balance.	Dr.	Debtor.	Pd.	Paid.
Bbl.	Barrel.	Exch.	Exchange.	Per	By.
Bo't	Bought.	Fol.	Folio.	Prem.	Premium.
B. L.	Bill of Lading.	Fwd.	Forward.	Prox.	Next month.
%	Per cent.	Fr't	Freight.	Rec'd	Received.
Co.	Company.	Ins.	Insurance.	Sund's	Sundries.
Cr.	Creditor.	Inst.	This month.	Ult.	Last month.
Com.	Commission.	Int.	Interest.	Yd.	Yard.
Dft.	Draft.	Mdse.	Merchandise.	Yr.	Year.

The character @ is always followed by the *price of a unit.* Thus, 5 yd. of cloth @ $3.25, signifies, 5 yards of cloth at $3.25 *a yard;* ½ lb. of tea @ $.90, signifies ½ a pound of tea at $.90 *per pound.*

327. Required the footings and balances of the following bills and accounts:

(1.)

New York, May 10, 1875.

A. S. Mann & Co.,

Bought of Halsted, Haynes & Co.

336 yd.	Muslin, ...	@ 26¢	. .	$87.56
98½ "	Canton Flannel, . .	" 18¢	. .	17.73
162 "	Victoria Gingham, .	" 16½¢	. .	26.7
110 "	Cassimere, ...	" $2.87½	.	316.25
				$448.07

Find the footing of this bill.

(2.)

Boston, June 20, 1876.

Messrs. C. P. Mead & Son,

Bo't of Belknap, Bro.

216 pairs Boys' Kip Boots, ... @	$2.25		
160 " " Brogans, ... "	1.12½		
75 " Women's Fox'd Gaiters, . "	1.25		
110 " " Enameled Boots, "	1.37½		
6 cases Men's Calf Boots, ... "	75.50		
1 case Drill, 648 yd., ... "	.14½		
36 gross Silk Buttons, ... "	.87½		
		$	

Received Payment,
Belknap, Bro.

(3.)

Charleston, S. C., Oct. 4, 1874.

Mr. Chas. Elliott,

Bo't of Wm. J. Aikin.

8 bales, ea. 485 lb., Ordinary Tex. Cotton, @ 18½¢		
6 " " 506 " Upland, Middlings, . " 21½¢		
3 hhd., 215 gal., N. O. Molasses (N. Crop), " 60¢		
	$	

Rec'd Payment by draft on N. Y.,
Wm. J. Aikin.

(4.)

Chicago, Sept. 10, 1876.

Messrs. Cook & Cheney,

Bo't of **Baker & Ellis.**

275 bbl. Flour, State Superfine, . .	@ $7.10		
146 " " Minnesota Ex., . .	" 7.87½		
94 " " Wisconsin XX, . .	" 8.12½		
650 bu. Wheat, No. 1, Red Winter,	" 1.75		
400 " " Illinois, No. 1, . .	" 1.82		
368 " Corn, Southern White, . .	" .87½		
		$	

Rec'd Paym't by note at 4 mo.,

Baker & Ellis.

(5.)

San Francisco, Jan. 1, 1875.

Mr. James Wilde,

To **Hodge and Son,** *Dr.*

1874					
Sept.	10	To 75 lb. Sugar, ...	@ 12½¢		
"	"	" 1 caddy Japan Tea, 22 lb., .	" 98¢		
Oct.	16	" 1 sack Rio Coffee, 116 lb., .	" 21¢		
"	"	" 1 " Rice, 75 lb., ...	" 9¢		
"	"	" 1 box P. & G. Soap, 60 lb., .	" 10¢		
Nov.	1	" 25 lb. Mackerel, ...	" 8¢		
"	"	" 9 gal. Molasses, ...	" 62½¢		
"	15	" 18 lb. Soda Crackers, ...	" 9¢		
Dec.	20	" 12 " Dried Beef, ...	" 12½¢		
"	26	" 1 box S. G. Starch, 28 lb., .	" 10¾¢		
				$	

Rec'd Paym't,

Hodge & Son,

Per **Henry Scott.**

(6.)

DETROIT, May 28, 1877.

MR. JACOB R. KENT,

To GEORGE W. PARKER, *Dr.*

Jan.	6	For Building Out-house as per contract,	$150	00
"	"	" Extra Labor, ...	$14	50
Mch.	20	" 15 days' work of self, @ $3½	$52	50
"	"	" 7 " " of son, " 1.50	$10	50
"	"	" 784 ft. Boards, ... 2¼ per C.	17	60
April	16	" 2 days' work, . . " 3.50	7	00
"	"	" Nails, Hinges, and Sundries, . . .	$4	75
			$254	85

(7.)

Statement of Account.

ST. LOUIS, Nov. 6, 1875.

MESSRS. WOOD & COLE,

To PHELPS & DODGE, *Dr.*

April	15	To 30 tons Eng. Iron, . . .	@ $34.30	$	
"	"	" 12 cwt. Eng. Blister Steel,	" 15.25		
June	21	" 6 doz. Hoes (Trowel Steel),	" 9.78		
Aug.	10	" 30 Buckeye Plows, . . .	" 10.45		
Oct.	3	" 12 Cross-cut Saws, . . .	" 12.12½		
"	"	" 37 cwt. Bar Lead, . . .	" 6.90		
				$	
		Cr.			
May	25	By 22 M. feet of Boards, . .	@ $27.60		
July	14	" 36 M. " Plank, . .	" 13.37½		
"	"	" 45 M. Shingles, ...	" 3.62½		
Sept.	5	" Draft on New York, ...		$500	
"	12	" 46 C. feet Scantling, . .	@ 1.38		
				$	
		Bal. due PHELPS & DODGE,		$	

(8.)

Account Current; Balanced by Note.

Geo. B. Damon & Co.,

In % with **Gray & Banks.**

Dr. *Cr.*

1876					1876				
Aug.	2	To 796 lb. Butter @ $.28			Nov.	3	By 27 bbl. Pears @ $9.25		
Sept.	17	" 972 " Cheese " .09			"	24	" 56 " Apples " 1.87		
"	24	" 481½ " Lard " .12			Dec.	1	" 70¾ bu. Corn " .70		
Oct.	4	" 509¾ " Tallow " .16			"	22	" 31¼ " Peas " 1.95		
"	18	" 81 doz. Eggs " .26			1877				
"	31	" 15 bbl. Salt " 2.40			Jan.	2	" Note at 3 mo. to Bal.		
Dec.	15	" 963 lb. Hams " .14							

Gray & Banks.

Philadelphia, Jan. 2, 1877.

REVIEW.

WRITTEN EXAMPLES.

328. What is the cost,

1. Of 7½ barrels of flour, if 4¾ barrels cost $38 ?
2. Of 9¼ tons of coal, if .875 of a ton cost $5.635 ?
3. Of 14.25 yards of cloth, if 36.48 yards cost $54.72 ?
4. Of 100 pounds of pork, if .93 cwt. cost $6.975 ?
5. Of 25.42 acres of land, if .125 of an acre cost $15⅞ ?
6. Of 1 ton of plaster, if 1680 pounds cost $2.856 ?
7. Of .8 of a pound of tea, if 1 pound cost $.62½ ?
8. Of 18640 feet of timber, at $6¼ per C. ?
9. Of 1375 pounds of potash, at $121½ a ton ?
10. Of 19600 bricks, at $9¾ per M. ?
11. Of .625 of a ton of coal, at $7½ a ton ?
12. Of 35 yards of cloth, if 29 yards cost $101½ ?
13. Of 1 bushel of potatoes, if 28.8 bushels cost $9.60 ?

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14. If 36 boxes of raisins, each containing 36 pounds, cost $194.40, what is the price per pound?

15. What will be the freight on 10860 pounds of merchandise from New York to St. Louis, at $1.62½ per C.?

16. How much must be paid for 1220 feet of boards, at $25½ per M.; 1866 feet of scantling at $2.12½ per C.; and 9525 feet of lath at $3½ per M.?

17. If I pay $1.37 a bushel for wheat, $.95 for rye, and $.73 a bushel for corn, how much, of each an equal number of bushels, can I purchase for $70.15?

18. Bought 27½ barrels of sugar for $453.75, and sold it at a profit of $4.62½ a barrel. At what price was it sold?

19. Three persons bought 645 tons of coal, and divided so that the first had .375 of it, the second $\frac{5}{12}$, and the third the remainder. How much did the third receive?

20. What is $814\frac{9}{20} \times 26\frac{15}{32}$ correct to 5 decimal places?

10 21. A person having $55.92, wished to purchase a
equal number of pounds of tea, coffee, and sugar
tea at $.87½, the coffee at $.18¾, and the sugar
How many pounds of each could he buy?

10 22. A dealer bought 240000 feet of lu
per M., and retailed it out at $2¼ per
whole gain?

10 23. Three hundred seventy-fi
dry goods, valued at $8000,
would a man lose who ov

10 24. Bought 150 bar
of wheat @ $1.44
@ $8¼, and all
rel must th
on the

10 25. Sold 20900 feet of timber for \$339.62½, and gained thereby \$78.37½. What did it cost per C.?

10 26. Reduce $\left(\frac{1\frac{3}{4}}{4\frac{1}{3}} \div \frac{2\frac{1}{3}}{2\frac{1}{4}}\right) \times \frac{4}{5}$ of $\frac{1}{2}$ to a decimal.

11 27. A farmer exchanged 28½ bushels of oats worth \$.75 per bushel, and 453 pounds of middlings worth \$1½ per hundred, for 12520 pounds of plaster. What was the plaster worth per ton?

1 28. A merchant tailor bought 27 pieces of broadcloth, each piece containing 19⅓ yards, at \$4.31¼ a yard; and sold it so as to gain \$381.87½, after deducting \$9.62½ for freight. For what was the cloth sold per yard?

10 29. If 10½ cords of wood cost \$34.12½, what cost 60⅜ cords?

12 30. If 1½ hundred pounds of sugar cost \$12¾, how many pounds can be bought for \$93½, at the same rate?

31. Paid \$108 for grain, $\frac{3}{10}$ of it being barley at \$.62½ per bushel, and ⅜ of it wheat at \$1.87½ per bushel; the rest of the money was paid for oats at \$.37½ per bushel. How many bushels of grain were bought?

at is the value of $\left(\frac{31\frac{1}{3}}{8} + \frac{6\frac{3}{4}}{2\frac{1}{12}}\right) \div 4.23$?

er sold to a merchant 3 loads of hay weigh-
1826, 1478, and 1921 pounds, at \$17.60
pounds of pork at \$5.25 per C. He
yards of sheeting @ \$.18, 11½
balance in money. How

purchase of rye at
d corn at \$.73
h kind; how

135. A farmer had 150 acres of land, which he could have sold at one time for $100 an acre, and thereby have gained $3900; but after keeping it for a time he was obliged to sell it at a loss of $2250. What did the land cost him an acre, and for how much an acre did he sell it?

136. Bought 2500 bushels of wheat @ $1.40, and 735 bushels of oats @ $.54; I had 1470 bushels of the wheat floured, and sold it at a profit of $435.87½, and I sold 528 bushels of the oats at a loss of $30. Afterward I sold the remainder of the wheat at $1.25 per bushel, and of the oats at $.45 per bushel. Did I gain or lose, and how much?

329. SYNOPSIS FOR REVIEW.

DECIMALS.

- 1. NOTATION AND NUMERATION.
    - 1. Defs.
        - 1. *Decimal Fractions.*
        - 2. " *Sign.*
    - 2. Denominator—how composed.
    - 3. Numerator—decimal places in.
    - 4. Two ways of writing decimals.
    - 5. Value of decimal figures—how determined.
    - 6. Starting point in notation and numeration.
    - 7. Principles, 1, 2, 3, 4, 5.
    - 8. Rule, I, II.
- 2. DECIMAL CURRENCY.
    - 1. Defs.
        - 1. *Currency.*
        - 2. *Decimal Currency.*
        - 3. *Federal Money.*
    - 2. **271, 272, 274, 275, 277.**
    - 3. Principles, 1, 2.
- 3. REDUCTION.
    - 1. Art. **279.**
    - 2. **280,** 1, 2, 3, 4, 5.
    - 3. **282. 283.**
    - 4. **284. 285,** Rule, I, II.

SYNOPSIS FOR REVIEW—CONTINUED.

DECIMALS.

- 4. ADDITION. Rule, I, II.
- 5. SUBTRACTION. Rule, I, II.
- 6. MULTIPLICATION.
  - 1. Principle.
  - 2. Rule.
- 7. DIVISION.
  - 1. Principles, 1, 2, 3.
  - 2. Rule.
- 8. CIRCULATING DECIMALS.
  - 1. Definitions.
    - 1. *Finite Decimal.*
    - 2. *Circ. Decimal.*
    - 3. *Repetend.*
    - 4. *Pure Circ. Dec.*
    - 5. *Mixed Circ. Dec.*
  - 2. Principles, 1, 2, 3, 4.
  - 3. **306.** *Rule.*
  - 4. **307.** *Rule*, 1, 2.
- SHORT METHODS.
  - 1. Definitions.
    - 1. *Quantity.*
    - 2. *Price.*
    - 3. *Cost.*
    - 4. *Aliquot Part.*
  - 2. **313.** *Rule.*
  - 3. **314.** *Rule.*
  - 4. **315.** *Rule.*
  - 5. **316.** *Rule.*
- 10. LEDGER ACCOUNTS.
  - 1. Definitions.
    - 1. *Ledger.*
    - 2. *Bal. of Account.*
  - 2. Position of Debits and Credits.
- 11. ACCOUNTS AND BILLS.
  - 1. Definitions.
    - 1. *Account.*
    - 2. *Debtor.*
    - 3. *Creditor.*
    - 4. *Bill.*
  - 2. Receipt of a Bill.
  - 3. Mercantile Abbreviations.

DENOMINATE NUMBERS

DEFINITIONS.

330. A ***Denominate Number*** is a concrete number, and may be either simple or compound; as, 8 quarts, 5 feet 10 inches, etc.

331. A ***Simple Denominate Number*** consists of a unit or units of but one denomination; as, 16 cents, 24 hours, 30 barrels, etc.

332. A ***Compound Denominate Number*** consists of units of two or more denominations of the same nature; as, 10 pounds 6 ounces, 5 yards 2 feet 8 inches, etc.

333. In integral numbers, and in decimals, the law of increase and decrease is by the *uniform* scale of 10; but in Compound Numbers, the scale varies.

MEASURES.

334. A ***Measure*** is a *standard unit* established by law or custom, by which *quantity*, such as extent, dimension, capacity, amount, or value, is measured or estimated.

Thus, the *standard unit* of Measures of Extension is the *yard;* of Liquid Measure, the *wine gallon;* of Dry Measure, the *Winchester bushel;* of Weight, the *Troy pound*, etc. Hence the length of a piece of cloth is ascertained by applying the *yard* measure; the capacity of a cask, by the use of the *gallon* measure; of a bin, by the use of the *bushel* measure; the weight of a body, by the *pound* weight, etc.

335. Measures may be classified into *six kinds:*

1. Extension.
2. Capacity.
3. Weight.
4. Time.
5. Angles or Arcs.
6. Money or Value.

MEASURES OF EXTENSION.

336. ***Extension*** is that which has one or more of the dimensions *length, breadth,* and *thickness.* It may be a *line,* a *surface,* or a *solid.*

337. The ***Standard Unit*** of measures of *extension,* whether linear, surface, or solid, is the *yard.*

LINEAR MEASURE.

338. ***Linear*** or ***Long Measure*** is used in measuring lines and distances.

339. A ***Line*** has only *one* dimension—*length.*

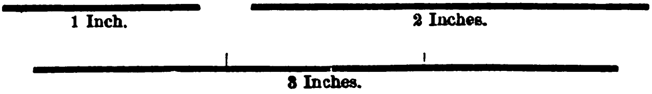


TABLE.

12	Inches (*in.*)	= 1 Foot	. .	*ft.*
3	Feet	= 1 Yard	. .	*yd.*
5½ 16½	Yards, or Feet	= 1 Rod	. .	*rd.*
320	Rods	= 1 Mile	. .	*mi.*

mi.	*rd.*	*ft.*	*in.*
1 =	320 =	5280 =	63360
	1 =	16½ =	198
		1 =	12

1. The *Inch* is generally divided into *halves, quarters, eighths, sixteenths,* and sometimes into *tenths* or *twelfths.*

2. Civil and mechanical engineers, and others, use *decimal* divisions of the *foot* and *inch.*

OTHER DENOMINATIONS.

3 Barley-corns, or sizes	=1 Inch.	Used by shoemakers.
4 Inches	=1 Hand.	" to measure the height of horses at the shoulder.
9 Inches	=1 Span.	Among sailors, 8 spans = 1 fathom.
~~21.888 Inches~~	=1 Sacred Cubit.	
6 Feet	=1 Fathom.	Used to measure depths at sea.
120 Fathoms	=1 Cable's Length.	
3 Feet	=1 Pace.	
1.152⅔ Common Miles	=1 Geog. Mi.	Used to meas. distances at sea.
3 Geographic Miles	=1 League.	
60 Geographic, or 69.16 Statute Miles	=1 Degree	of Latitude on a Meridian, or of Longitude on the Equator.
360 Degrees	=the Circumference of the Earth.	

1. A *Knot* is 1 geographical or nautical mile, used to measure the speed of vessels.

2. The geographic mile is $\frac{1}{60}$ of $\frac{1}{360}$, or $\frac{1}{21600}$ of the circumference of the earth. It is a little more than 1.15 common miles.

340. ***Cloth Measure*** is practically out of use. In measuring goods sold by the yard, the yard is divided into *halves, fourths, eighths,* and *sixteenths.*

At custom houses, in estimating duties, the yard is divided into *tenths* and *hundredths.*

341. ***Surveyors' Linear Measure*** is used by land surveyors in measuring roads and boundaries of land.

TABLE.

		mi.	*ch.*	*rd.*	*l.*	*in.*
7.92 Inches = 1 Link . . .	*l.*	1 =	80 =	320 =	8000 =	63360
25 Links = 1 Rod . . .	*rd.*		1 =	4 =	100 =	792
4 Rods = 1 Chain . .	*ch.*			1 =	25 =	198
80 Chains = 1 Mile . . .	*mi.*				1 =	7.92

1. A *Gunter's Chain* is the *unit* of measure, and is 4 rods, or 66 feet long, and consists of 100 links.

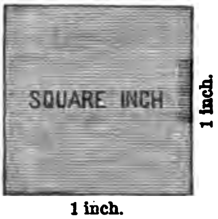
2. Engineers commonly use a chain, or measuring tape, 100 feet long.

3. Measurements are recorded in *chains* and *hundredths.*

SURFACE OR SQUARE MEASURE.

342. ***Surface*** or ***Square Measure*** is used in computing areas or surfaces.

343. A ***Surface*** has *two* dimensions—*length* and *breadth*.



344. The ***Area*** of a surface is expressed by the product of the numbers that represent these *two* dimensions.

345. A ***Square*** is a plane figure bounded by four equal sides, and having four right angles.

A *Square Inch* is a square each side of which is 1 *inch* in length.

Table.

144	Square Inches (*sq. in.*)	=	1 Square Foot	*sq. ft.*
9	Square Feet	=	1 Square Yard	*sq. yd.*
30¼	Square Yards	=	1 Square Rod or Perch .	*sq rd., P.*
160	Square Rods	=	1 Acre	*A.*

sq. mi.		*A.*		*sq. rd.*		*sq. yd.*		*sq. ft.*		*sq. in.*
1	=	640	=	102400	=	3097600	=	27878400	=	4014489600

346. ***Surveyors' Square Measure*** is used by surveyors in computing the area or contents of *land*.

Table.

625	Square Links (*sq. l.*)	=	1 Pole	*P.*
16	Poles	=	1 Square Chain . .	*sq. ch.*
10	Square Chains	=	1 Acre	*A.*
640	Acres	=	1 Square Mile . .	*sq. mi.*
36	Square Miles (6 miles square)	=	1 Township . . .	*Tp.*

Tp.		*sq. mi.*		*A.*		*sq. ch.*		*P.*		*sq. l.*
1	=	36	=	23040	=	230400	=	3686400	=	2304000000

1. The *Acre* is the *unit* of land measure.
2. Measurements of land are commonly recorded in *square miles*, *acres*, and *hundredths* of an acre.

For Notes and Applications, see "Measurements" (**467, 468**).

CUBIC OR SOLID MEASURE.

347. ***Cubic*** or ***Solid Measure*** is used in computing the contents or volume of solids.

348. A ***Solid*** or ***Body*** has *three* dimensions—*length*, *breadth*, and *thickness*.

349. The ***Volume*** of a body is expressed by the product of the numbers that represent these dimensions.

350. A ***Cube*** is a body bounded by six equal squares, called *faces*.

The sides of these squares are called the *edges* of the cube.

A *Cubic Inch* is a cube each side of which is 1 *inch* in length.

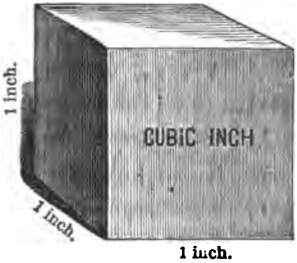

1 inch.
CUBIC INCH
1 inch.
1 inch.

TABLE.

1728 Cubic In. (*cu. in.*)	= 1 Cubic Ft.	*cu. ft.*	*cu. yd.*	*cu. ft.*	*cu. in.*
27 Cubic Ft.	= 1 Cubic Yd.	*cu. yd.*	1 =	27 =	46656

351. ***Wood Measure*** is used to measure wood and rough stone.

TABLE.

16	Cubic Feet	= 1 Cord Foot	*cd. ft*
8	Cord Feet, or	= 1 Cord	*Cd.*
128	Cubic Feet		
24¾	Cubic Feet	= 1 Perch of Stone, or of Masonry . . .	*Pch.*

For Notes and Applications, see "Measurements" (**474–477**).

ORAL EXERCISES.

352. 1. How many inches in 3 feet? In 2 ft. 6 in.?

2. How many feet in 48 in.? In 67 in.? In 75 in.?

3. In 5 yd., how many feet? In 6⅓ yd.? In 7½ yd.?

4. How many quarters in 3 yd. 2 qr.? Eighths in 5 qr.?

5. At 6 cents a quarter, what cost 3 yd. 3 qr. of cord?

6. How many yards in 96 in.? In 25 ft.? In 108 in.?

7. In 22 yd., how many rods? In 3 rd., how many ft.?

8. If a vessel sail 4 leagues an hour, how many hours will she be in sailing 75 miles?

9. How high is a horse that measures 16 hands?

10. How many fathoms deep is a body of water that requires 45 ft. of line to measure it?

11. A vessel sunk in 9½ fathoms of water: what was the depth of the water in feet?

12. What part of a foot are 9 in.? Of a yard are 12 in.?

13. How many rods is ⅛ of a mile? ¼? ⅛? ¾?

14. What part of a mile are 80 rods? 32 rd.? 64 rd.?

15. At $½ a foot, what will 6 yd. 1 ft. of lead pipe cost?

16. What part of a mile are 20 ch.? Are 60 ch.?

17. At $¾ a rod, what will it cost to dig a trench ¼ of a mile long?

18. How many square yards in 54 sq. ft.? In 84 sq. ft.?

19. In a piece of zinc 12 in. long and 9 in. wide, how many square inches?

20. Find the difference of 6 ft. square, and 6 sq. ft.?

21. In a lot 12 rd. long and 10 rd. wide, how many square rods? What part of an acre?

22. How many yards of carpeting a yard wide, will cover a floor 15 ft. long and 12 ft. wide?

23. What will it cost to pave a court 10 ft. by 15 ft., at $.50 a square foot?

24. At 20 cents a square yard, what will it cost to paint a ceiling 18 ft. by 10 ft.?

25. How many cubic feet in 2 cu. yd.? In 3 cu. yd.?

26. How many cubic inches in 1 cu. ft. 20 cu. in.?

27. What part of a cubic yard are 9 cu. ft.? Are 12 cu. ft.?

28. How many cubic feet in 3 cd. ft.? In 4 cd. ft.?

29. In ½ of a cord, how many cord feet? Cubic feet?

30. In 2 perch of stone, how many cubic feet?

31. How many cubic inches in a 10 inch cube?

32. What is the difference between 4 cubic inches, and a 4 inch cube?

33. How many blocks, each containing 1 cu. ft., are equal to a block 6 ft. long, 5 ft. wide, and 3 ft. thick?

MEASURES OF CAPACITY.

353. ***Capacity*** signifies extent of *room* or *space.*

354. Measures of capacity are divided into two classes; Measures of Liquids and Measures of Dry Substances.

355. The ***Units*** of Capacity are the *Gallon* for Liquid, and the *Bushel* for Dry Measure.

LIQUID MEASURE.

356. ***Liquid Measure*** is used in measuring *liquids.*

Table.

4 Gills (*gi.*) = 1 Pint . . . *pt.*
2 Pints = 1 Quart . . *qt.*
4 Quarts = 1 Gallon . . *gal.*

gal.	*qt.*	*pt.*	*gi.*
1 =	4 =	8 =	32
	1 =	2 =	8
		1 =	4

In estimating the capacity of cisterns, reservoirs, etc.

	hhd.	*bbl.*	*gal.*	*qt.*	*pt.*
31½ Gal. = 1 Barrel . . . *bbl.*	1	2	63	252	504
63 Gal. = 1 Hogshead . . *hhd.*		1	31½	126	252

1. The barrel and hogshead are not fixed measures, but vary when used for commercial purposes.

2. The tierce, hogshead, pipe, butt, and tun are the names of casks, and do not express any fixed measures. They are usually *gauged*, and have their capacities in gallons marked on them.

357. ***Apothecaries' Fluid Measure*** is used in prescribing and in compounding *liquid* medicines.

TABLE.

60 Minims, or drops (♏) = 1 Fluidrachm . . *f*ʒ.
8 Fluidrachms = 1 Fluidounce . . *f*℥.
16 Fluidounces = 1 Pint . . . . . *O.*
8 Pints = 1 Gallon . . . . *Cong.*

Cong. 1 = *O.* 8 = *f*℥ 128 = *f*ʒ 1024 = ♏ 61440.

1. *Cong.*, for *congius*, is the Latin for *gallon; O.*, for *octarius*, is the Latin for *one-eighth.*

The *minim* is equivalent to a drop of water. A *pint* of water weighs a pound.

Drops are indicated in a physician's prescription by *gtt.*

The symbols, as in Apothecaries' Weight, *precede* the numbers to which they refer; thus, *O.* 3 *f*℥ 6, is 3 pints 6 fluid ounces.

DRY MEASURE.

358. ***Dry Measure*** is used in measuring dry articles, such as grain, fruit, roots, salt, etc.

TABLE.

	bu.	*pk.*	*qt.*	*pt.*
2 Pints (*pt.*) = 1 Quart . . *qt.*	1	4	32	64
8 Quarts = 1 Peck . . *pk.*		1	8	16
4 Pecks = 1 Bushel . . *bu.*			1	2

For Notes and Applications, see "Measurements" (**482**).

ORAL EXERCISES.

359. 1. How many gills in 3 pints? In 2 qt. 1 pt.?

2. How many pints in 1 gal.? In 1 gal. 2 qt. 1 pt.?

3. In 36 pints, how many quarts? How many gallons?

4. What part of a quart are 6 gi.? What part of a gallon?

5. What part of 2 gal. are 4 pints? Are 8 pt.? 2 qt.?

6. How many gills in ¼ of a quart? In ⅛ of a gallon?

7. How many pints in 64 gills? How many quarts? Gallons?

8. How many fluidrachms in 5 fluidounces?

9. How many pint bottles will be required to hold 3 gal. 1 qt. of syrup? 2 gal. 3 qt.?

10. At 5 cents a pint, what will 2 gal. of milk cost?

11. If 10 gal. 2 qt. are drawn from a barrel of vinegar, how many gallons remain?

12. If a gallon of wine cost $6, what will 3 pt. cost?

13. How many barrels can be filled from 20 hogsheads?

14. At 20 cents a quart, how many gallons of molasses will $4 buy? $6? $5.60?

15. How many pints in 6 quarts? In 2 pk. 1 qt.?

16. How many quarts in 3 pk. 6 qt.? In 1 bu. 2 pk.?

17. In 96 qt., how many pecks? How many bushels?

18. What part of 5 bu. are 5 pk.? Of 1 bu. are 12 qt.?

19. How many quart boxes will 1 bu. 2 pk. 6 qt. fill?

20. At 20 cents a quart, what will ½ bu. of plums cost?

21. At 5 cts. a pt., what is a bushel of chestnuts worth?

22. At $3.20 a bushel, how many quarts of peanuts can be bought for $2?

23. Bought ½ bu. of chestnuts for $1½, and sold them for 8 cents a pint? What was the gain?

MEASURES OF WEIGHT.

360. ***Weight,*** on the earth, is the measure of gravity, and varies according to the quantity of matter a body contains.

361. The ***Standard Unit*** of weight is the *Troy pound of the Mint,* and contains 5760 grains.

TROY WEIGHT.

362. ***Troy Weight*** is used in weighing gold, silver, and jewels, and in philosophical experiments.

TABLE.

			lb.	*oz.*	*pwt.*	*gr.*
24 Grains (*gr.*)	= 1 Pennyweight.	*pwt.*	1 =	12 =	240 =	5760
20 Pennyweights	= 1 Ounce . . .	*oz.*		1 =	20 =	480
12 Ounces	= 1 Pound . . .	*lb.*			1 =	24

A *Carat* is a weight of about 3.2 Troy grains, and is used to weigh diamonds and precious stones.

The term *carat* is also used to express the *fineness* of gold, and means a *twenty-fourth* part. Thus, gold is said to be 18 carats fine, when it contains 18 parts of pure gold, and 6 parts of alloy, or baser metal.

APOTHECARIES' WEIGHT.

363. ***Apothecaries' Weight*** is used by physicians and apothecaries in prescribing and mixing *dry* medicines.

TABLE.

20 Grains (*gr.* xx) = 1 Scruple . . . . . . *sc.*, or ℈.
3 Scruples (℈ iij) = 1 Dram . . . . . . *dr.*, or ʒ.
8 Drams (ʒ viij) = 1 Ounce . . . . . . *oz.*, or ℥.
12 Ounces (℥ xij) = 1 Pound . . . . . . *lb.*, or ℔.
℔ 1 = ℥ 12 = ʒ 96 = ℈ 288 = *gr.* 5760.

1. *Medicines* are bought and sold by Avoirdupois Weight.

2. The pound, ounce, and grain are the same as those of Troy Weight, the ounce being *differently divided.*

3. Physicians write prescriptions according to the Roman notation, using small letters, preceded by the symbols, writing j for i, when it terminates a number. Thus, 6 ounces is written, ℥ vj; 8 dr., ʒ viij; 14 sc., ℈ xiv., etc.

4. ℞ is an abbreviation for *recipe*, or take; ā, āā., for equal quantities; ij. for 2; *ss.* for *semi*, or half; gr. for grain; P. for *particula*, or little part; P. æq. for equal parts; q. p., as much as you please.

AVOIRDUPOIS WEIGHT.

364. ***Avoirdupois Weight*** is used for weighing all coarse and heavy articles.

TABLE.

		T.	*cwt.*	*lb.*	*oz.*
16 Ounces (*oz.*)	=1 Pound . . . . *lb.*	1=	20=	2000=	32000
100 Pounds	=1 Hundred-weight *cwt.*		1=	100=	1600
20 Cwt., or 2000 lb.	=1 Ton . . . . . *T.*			1=	16

1. The *Ounce* is often divided into *halves*, *quarters*, etc.

2. The *long*, or *gross ton*, *hundred-weight*, and *quarter* were formerly in common use; but they are now seldom used, except in estimating duties at the U. S. Custom Houses, and in weighing a few of the coarser articles, such as coal at the mines, etc.

LONG TON TABLE.

		T.	*cwt.*	*qr.*	*lb.*	*oz.*
16 Ounces	= 1 Pound . *lb.*	1=	20=	80=	2240=	35840
28 Pounds	= 1 Quarter . *qr.*		1=	4=	112=	1792
4 Quarters	= 1 Hund. . *cwt.*			1=	28=	448
20 Cwt., or 2240 lb.	= 1 Ton . . *T.*				1=	16

3. Both *custom* and the *law* of most of the States make 100 pounds a *hundred-weight*.

365. The following denominations are also in use:

100	Pounds of Grain or Flour	make	1 Cental.
100	" Dry Fish	"	1 Quintal.
100	" Nails	"	1 Keg.
196	" Flour	"	1 Barrel.
200	" Pork or Beef	"	1 Barrel.
280	" Salt at N. Y. S. works	"	1 Barrel.
240	" Lime	"	1 Cask.

366. The weight of the bushel of certain grains and roots has been fixed by statute in many of the States; and these statute weights must govern in buying and selling, unless specific agreements to the contrary are made.

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL,

As prescribed by statute in the several States named.

COMMODITIES.	*California.*	*Connecticut.*	*Delaware.*	*Illinois.*	*Indiana.*	*Iowa.*	*Kentucky.*	*Louisiana.*	*Maine.*	*Massachusetts.*	*Michigan.*	*Minnesota.*	*Missouri.*	*N. Hampshire.*	*New Jersey.*	*New York.*	*Ohio.*	*Oregon.*	*Pennsylvania.*	*Rhode Island.*	*Vermont.*	*Washington T.*	*Wisconsin.*
Barley	50			48	48	48	48	32		46	48	48	48		48	48	48	46	47		46	45	48
Beans				60	60	60	60						60			60							
Blue Grass Seed				14	14	14	14						14										
Buckwheat	40	45		40	50	52	52			46	42	42	52		50	48		42	48		46	42	42
Castor Beans				46	46	46							46										
Clover Seed				60	60	60	60				60	60	60		64	60	60	60				60	60
Dried Apples				24	25	24					28	28	24					28				28	28
Dried Peaches				33	33	33					28	28	33					28				28	28
Flax Seed				55	56	56	56						56		55	55	56						56
Hemp Seed				44	44	44	44						44										
Indian Corn	52	56	56	52	56	50	56	56		56	56	56	52		56	58	56	56	56		56	56	56
Indian Corn in ear				70	68	68																	
Indian Corn Meal				48	50		50		50	50										50			
Mineral Coal				80	70	80							80										
Oats	32	28		32	32	35	33⅓	32	30	32	32	32	35	30	30	32	32	34	32		32	36	32
Onions				57	48	57	57			52			57							50		50	
Potatoes		60		60	60	60	60		60				60	60	60	60		60		60	60	60	60
Rye	54	56		54	56	56	56	32		56	56	56	56		56	56	56	56	56		56	56	56
Rye Meal									50	50										50			
Salt					50	50	50						50			56							
Timothy Seed				45	45	45	45						45			44							46
Wheat	60	56	60	60	60	60	60	60		60	60	60	60		60	60	60	60	60		60	60	60

1. In Pennsylvania 80 lb. coarse, 70 lb. ground, or 62 lb. fine salt make 1 bushel; and in Illinois, 50 lb. common, or 55 lb. fine salt make 1 bushel.

2. In Maine 64 lb. of ruta-baga turnips, or of beets make 1 bushel.

ORAL EXERCISES.

367. 1. How many grains in 3 pwt.? In ℈5?

2. How many ounces in 60 pwt.? In 100 pwt.? 120 pwt.?

3. How many ounces in 5 lb.? In 3 lb. 10 oz.? 4½ lb.?

4. How many ounces in 40 drams? In 64 dr.? 120 dr.?

5. How many pounds in 36 oz.? In 70 oz.? 110 oz.?

6. How many scruples in 10 drams? In 80 grains?

7. What will a gold chain, weighing 1 oz. 12 pwt., cost at $1 a pennyweight?

8. What part of a pound Troy are 4 oz.? 6 oz.? 8 oz.?

9. How many parts of pure gold in a ring 16 carats fine?

10. How many powders of 8 grains each, can be made from half an ounce of medicine?

11. How many tablespoons, each weighing 2 oz., can be made from 2 lb. 10 oz. of silver?

12. How many pills of gr. 5 each can be made from ʒ1 ℈2 of calomel?

13. What is the value of a gold bracelet weighing 3 oz. 15 pwt., at $20 an ounce?

14. How many ounces in 4 lb. Avoir.? In 5 lb. 6 oz.?

15. How many pounds in 7 cwt.? In 8¼ cwt.?

16. How many cwt. in 600 lb.? In 350 lb.? In 875 lb.?

17. In 3 T., how many hundred-weight? How many lb.?

18. What part of a cwt. are 25 lb.? 50 lb.? 75 lb.?

19. How many cwt. in ½ of a ton? In ¼ of a ton?

20. How many tons are 50 cwt.? 80 cwt.? 95 cwt.?

21. What will a ton of hay cost, at 1 cent a pound?

22. At 8 cents an ounce, what will 2½ lb. of licorice cost?

23. What will ¾ lb. of candy cost, at 3 cents an oz.?

24. At $2 a bushel, what must be paid for 2 bags of wheat, each containing 120 lb.?

368. SYNOPSIS FOR REVIEW.

DENOMINATE NUMBERS.

- 1. **Definitions.**
  - 1. Denominate Number. 2. Simple Number. 3. Comp. Denom. Number
- 2. **Measures.**
  - 1. **Definition of Measure.**
  - 2. **Classification.**
  - 3. **Measures of Extension.**
    - 1. **Linear Measure.**
      - 1. For what used.
      - 2. Table.
      - 3. Other Denominations.
      - 4. Surveyors' Linear Meas.
    - 2. **Square Measure.**
      - 1. For what used.
      - 2. Defs.
        - 1. *Surface.*
        - 2. *Area.*
        - 3. *Square.*
      - 3. Table.
      - 4. Surveyors' Square Meas.
    - 3. **Cubic Measure.**
      - 1. For what used.
      - 2. Defs.
        - 1. *Solid.* 2. *Volume.* 3. *Cube.*
      - 3. Table.
    - 4. **Wood Measure.**
      - 1. For what used.
      - 2. Table.
  - 4. **Meas. of Capac'y.**
    - 1. **Definition of Capacity.**
    - 2. **Units of Capacity.**
    - 3. **Liquid Measure.**
      - 1. For what used.
      - 2. Table.
      - 3. Apoth. Fluid Measure
    - 4. **Dry Measure.**
      - 1. For what used.
      - 2. Table.
  - 5. **Meas. of W'ght.**
    - 1. **Definition of Weight.**
    - 2. **Standard Unit.**
    - 3. **Troy Weight.**
      - 1. For what used.
      - 2. Table.
    - 4. **Apothecaries Weight.**
      - 1. For what used.
      - 2. Table.
    - 5. **Avoirdupois Weight.**
      - 1. For what used.
      - 2. Table.

MEASURES OF TIME.

369. ***Time*** **is a measured portion of duration.**

370. The ***Unit*** of measure is the *mean solar day.*

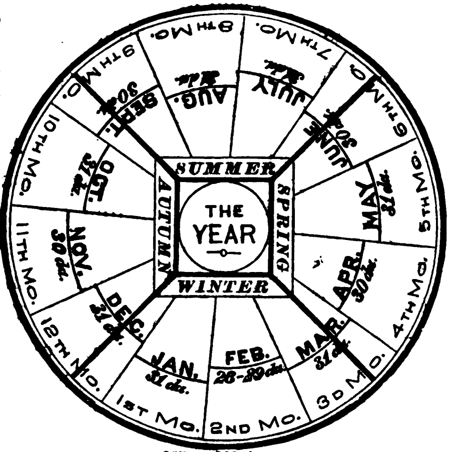
TABLE.

60 Seconds (*sec.*)	= 1 Minute	. . . . . .	*min.*
60 Minutes	= 1 Hour	. . . . . .	*hr.*
24 Hours	= 1 Day	. . . .	*da.*
7 Days	= 1 Week	. . . .	*wk.*
365 Days, or 12 Calendar Months	= 1 Common Year	. .	*yr.*
366 Days	= 1 Leap Year	. . .	*yr.*
100 Years	= 1 Century	. . . .	*C.*

$$\begin{array}{ccccccccccc} \textit{yr.} & & \textit{mo.} & & \textit{da.} & & \textit{hr.} & & \textit{min.} & & \textit{sec.} \\ 1 & = & 12 & = & \left\{\begin{array}{c} 365 \\ 336 \end{array}\right. & \begin{array}{c} = \\ = \end{array} & \begin{array}{c} 8760 \\ 8784 \end{array} & \begin{array}{c} = \\ = \end{array} & \begin{array}{c} 525600 \\ 527040 \end{array} & \begin{array}{c} = \\ = \end{array} & \begin{array}{c} 31536000 \\ 31622400 \end{array} \end{array}$$

In most business transactions 30 days are considered *a month*, and 12 months a *year*. Four weeks are sometimes called a *lunar month*.

The calendar year is divided as shown in the diagram:



365 or 366 days.

1. The *Solar Day* is the interval of time between two successive passages of the sun across the meridian of any place.

2. The *Mean Solar Day* is the mean or average length of all the solar days in the year.

3. The *Civil Day*, used for business purposes and which corresponds with the mean solar day, begins and ends at 12 o'clock, midnight. A.M. denotes the time before noon; M., at noon; and P.M., afternoon.

4. The *Solar Year* is exactly 365 da. 5 hr. 48 min. 49.7 sec.

5. The *Common Year* consists of 365 da. for 3 successive years, every *fourth* year containing 366 da., one day being added for the excess of the solar year over 365 da. This day is added to the month of February, which then has 29 da., and the year is called *Leap-year.*

371. The following rule for leap year will render the calendar correct to within 1 day, for a period of 4000 years

I. *Every year exactly divisible by* 4 *is a leap year, the centennial years excepted; the other years are common years.*

Thus, 1876 is a leap year, but 1877 is a common year.

II. *Every centennial year exactly divisible by* 400 *is a leap year; the other centennial years are common years.*

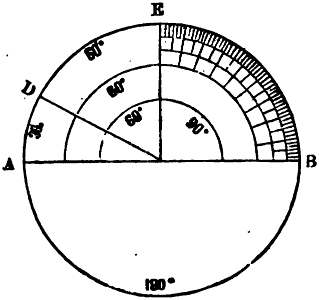
Thus, the year 2000 is a L. year, but 1800 and 1900 are com. years.

CIRCULAR MEASURE.

372. ***Circular*** or ***Angular Measure*** is used in measuring angles and arcs of circles, in determining latitude and longitude, the location of places and vessels, etc.

373. The ***Unit*** is the *Degree,* which is $\frac{1}{360}$ part of the circumference of any circle.

FIG. 1.



374. A ***Circle*** is a plane figure bounded by a curved line every point of which is equally distant from a point within called the *Center.*

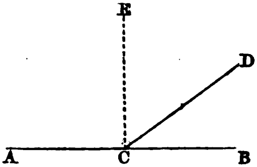
375. The ***Circumference*** of a circle is the line that bounds it.

376. An ***Arc*** is any part of the circumference; as A D, D E.

377. An ***Angle*** is the difference in the direction of two lines proceeding from a common point called the *vertex*. Thus, A C D and D C B are *angles*, and C is their vertex.

378. A ***Right Angle*** is formed by drawing one line perpendicular to another. Thus, A C E and E C B are right angles.

FIG. 2.



379. A ***Degree*** is one of the 360 equal parts into which the circumference of a circle is supposed to be divided. Thus, E and B (Fig. 1) are at the distance of 90°, or a *right angle* from each other, the vertex being at the center of the circle.

380. The ***Measure of an Angle*** is the arc of the circle included between its sides. Thus, the arc D B (Fig. 3) is the measure of the angle D C B.

FIG. 3.

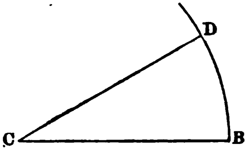


TABLE.

			Cir.	*S.*	°	′	″
60 Seconds (″)	= 1 Minute .	′	1=	12=	360=	21600=	1296000
60 Minutes	= 1 Degree .	°		1=	30=	1800=	108000
30 Degrees	= 1 Sign . .	*S.*			1=	60=	360
12 Signs, or 360°	= 1 Circle .	*Cir.*				1=	60

1. A ***Semi-Circum.*** is *one-half* of a circumference, or 180°.

2. A ***Quadrant*** is *one-fourth* of a circumference, or 90°.

3. A ***Sextant*** is *one-sixth* of a circumference, or 60°.

4. A ***Sign*** is *one-twelfth* of a circumference, or 30°.

5. A *degree* varies with the size of the circle; thus, a degree of long. at the Equator is 69.16 statute miles, at 30° of latitude it is 59.81 mi., at 60° of latitude it is 34.53 mi., and at 90°, it is nothing.

6. A *minute* of the earth's circumference is called a geographic, or nautical mile, and is a small fraction less than 1.16 common miles.

COUNTING.

381. The following table is used in counting certain classes of articles:

12 Units or things = 1 Dozen . . . *doz.*
12 Dozen = 1 Gross . . . *gro.*
12 Gross = 1 Great Gross . *G. gro.*
20 Units or things = 1 Score . . . *Sc.*

1. *Two* things of a kind are often called a *pair*, and *six* things a *set*; as a *pair* of horses, a *set* of chairs, spoons, etc.

PAPER.

382. The denominations of the following table are used in the paper trade:

24 Sheets = 1 Quire.
20 Quires = 1 Ream.
2 Reams = 1 Bundle.
5 Bundles = 1 Bale.

1 Bale = 5 Bundles.
1 Bundle = 2 Reams.
1 Ream = 20 Quires.
1 Quire = 24 Sheets.

BOOKS.

383. The terms *folio, quarto, octavo,* etc., indicate the number of leaves into which a sheet of paper is folded.

When a sheet is folded into	The book is called	And 1 sheet of paper makes
2 leaves	a Folio,	4 pp. (pages).
4 "	a Quarto or 4to,	8 "
8 "	an Octavo or 8vo,	16 "
12 "	a Duodecimo or 12mo,	24 "
16 "	a 16mo,	32 "
18 "	an 18mo,	36 "

Clerks and copyists are usually paid by the *folio* for making copies of legal papers, records, and documents.

72 words make 1 folio, or sheet of common law.
90 " 1 " chancery.

ORAL EXERCISES.

384. 1. How many seconds in ½ min.? In ⅔ min.?

2. How many minutes in 120 sec.? In 180 sec.?

3. How many hours in 90 min.? In 200 min.?

4. How many hours in 2 da.? In 3⅔ da.? In 5½ da.?

5. How many hours from 6 A.M. to 5 P.M.?

6. In 4 wk. 3 da., how many days? In 5 wk. 4 da.?

7. How many minutes from 10 min. past 9 o'clock to 25 min. past 10 A.M.?

8. How much time from 20 minutes before 11 A.M. to half past 10 o'clock P.M.?

9. Which of the months have 30 da. each? 31 da. each?

10. How many days from Jan. 1 to March 5, inclusive?

11. How many days from May 10 to July 16, inclusive?

12. Which of the following are leap years, and which are common years: 1874? 1876? 1880? 1886? 1900?

13. How many centuries and years since the birth of Christ?

14. How many leap years in every century?

15. How many degrees in ½ a circle? In ⅓? In ¼? In ⅙?

16. How many geographic miles in 2°? In 3°? In 4°?

17. How many common miles in 6 geographic miles?

18. How many degrees in 360 nautical miles?

19. How many degrees in ½ a quadrant? In ⅓ a sextant?

20. How many degrees in ¼ of a circumference?

21. What part of a circumference are 60°? 90°? 180°?

22. How many dozens in 2½ gross? In 3¾ gro.?

23. How many dozens in ⅓ of a great gross? In ¾?

24. How many score in 100? Pairs in 50? Sets in 75?

25. In 1 B. of paper, how many reams? How many quires?

26. How many eggs in 5¾ dozen? In 12 doz. and 7?

27. How many quires of paper in ½ a ream? In 3¼ rm.?

28. How many years in 4 score years and 10?

29. How many sheets of paper will be required to make a 12mo book of 320 pages? Of 480 pages?

30. How many sheets will be required to make a quarto book of 144 pages? Of 240 pp.? Of 360 pp.?

31. How many 16mo books will the paper for 1 quarto book make?

MEASURES OF VALUE.

385. ***Money*** is the measure of the *value* of things, and is used as a medium of exchange in trade.

386. ***Specie*** or ***Coin*** is metal struck, stamped, or pressed with a die, to give it a fixed legal value, and authorized by Government to be used as money.

387. ***Paper Money*** consists of bills and notes duly authorized by Government to circulate as substitutes for, or representatives of, money.

388. ***Currency*** is a term applied to all kinds of money employed in trade and commerce, both of coin and paper.

389. A ***Mint*** is a place in which the coin of a country or government is manufactured.

390. An ***Alloy*** is a metal compounded with another of greater value. In coinage, the less valuable or *baser metal* is not reckoned of any value.

Gold and silver, in a pure state, are too soft for coinage; hence they are hardened by compounding them with an alloy of baser metal, while their color and other valuable qualities are not impaired.

UNITED STATES MONEY.

391. ***United States Money*** is the legal currency of the United States, and is sometimes called *Federal Money*.

392. The ***Unit*** of U. S. Money is the *Gold Dollar*.

TABLE.

	E.	**$**	*d.*	*ct.*	*m.*
10 Mills (*m.*) = 1 Cent . . *ct.*	1 =	10 =	100 =	1000 =	10000
10 Cents = 1 Dime . . *d.*		1 =	10 =	100 =	1000
10 Dimes = 1 Dollar. . **$.**			1 =	10 =	100
10 Dollars = 1 Eagle . . *E.*				1 =	10

Federal money was adopted by Congress in 1786. Previous to this, pounds, shillings, and pence were in use. There is no coin for the *mill*.

393. The ***Coin*** of the United States consists of *gold, silver, nickel,* and *bronze,* and is as follows:

394. ***Gold.*** The double-eagle, eagle, half-eagle, quarter-eagle, three-dollar and one-dollar pieces.

395. ***Silver.*** The *Trade*-dollar, one-dollar, half-dollar, quarter-dollar, twenty-cent, and the ten-cent pieces.

396. ***Nickel.*** The five-cent, and three-cent pieces.

397. ***Bronze.*** The one-cent piece.

1. The half-dime and three-cent pieces, the bronze two-cent, and the nickel one-cent pieces are no longer coined.

2. The *Trade-dollar* weighs 420 grains, and is designed solely for purposes of commerce and not for currency. The *legal-tender* dollar weighs 412½ grains.

3. The *Standard purity* of the *gold* and *silver* coins is, 9 parts (.9) pure metal, and 1 part (.1) alloy. The alloy for *gold* coins is *silver* and *copper*, the silver, by law, not to exceed $\frac{1}{10}$ of the whole alloy. The alloy of *silver* coins is *pure copper*.

4. The *five* and *three* cent pieces consist of 3 parts (.75) copper, and 1 part (.25) nickel.

5. The *one* cent piece consists of .95 copper, and .05 of zinc and tin.

CANADA MONEY.

398. ***Canada Money*** is the legal currency of the *Dominion* of Canada, and is a *decimal* currency.

The denominations are *dollars, cents,* and *mills,* and have the same *nominal* value as the corresponding denominations of U. S. Money.

399. The *Coin* of the Dominion of Canada is *silver* and *bronze*

400. The ***Silver Coins*** are the fifty-cent, twenty-five-cent, ten-cent, and five-cent pieces.

401. The ***Bronze Coin*** is the one-cent piece.

1. The *gold coins* in use are the *Sovereign* and the *Half-Sovereign.*

2. The intrinsic value of the 50-cent piece in United States coin is about 46½ cents, of the 25-cent piece $23\frac{1}{10}$ cents. In ordinary business transactions, they pass the same as United States coin.

ENGLISH MONEY.

402. ***English*** or ***Sterling Money*** is the legal currency of Great Britain.

403. The ***Unit*** of Eng. Money is the *Pound Sterling.*

TABLE.

4 Farthings (*far.*)	=	1 Penny	*d.*
12 Pence	=	1 Shilling	*s.*
20 Shillings	=	1 Sovereign, or . .	*sov.*
		1 Pound	£.

£.	*s.*	*d.*	*far.*
1	=20	=240	=960
	1	= 12	= 48
		1	= 4

The value of a Sovereign in United States money is $4.8665.

The character for pound (£) is written before integers; 5 pounds = £5.

OTHER DENOMINATIONS.

2 Shillings (*s.*) = 1 Florin . . . . *fl.*
5 Shillings = 1 Crown . . . . *cr.*

404. The ***Coin*** of Great Britain in general use consists of *gold, silver,* and *copper,* as follows:

405. ***Gold.*** The sovereign and half-sovereign.

406. ***Silver.*** The crown, half-crown, florin, shilling, sixpenny, and three-penny piece.

407. ***Copper.*** The penny, half-penny, and farthing.

FRENCH MONEY.

408. ***French Money*** is the legal money of France and is a *decimal* currency.

409. The ***Unit*** of French Money is the *Silver Franc.*

TABLE.

10 Millimes (*m.*) = 1 Centime . . . *ct.*
10 Centimes = 1 Decime . . . *dc.*
10 Decimes = 1 Franc . . . *fr.*

fr.	*dc.*	*ct.*	*m.*
1 =	10 =	100 =	1000
	1 =	10 =	100
		1 =	10

The value of a franc in U. S. money of account is $.193.

410. The ***Coin*** of France consists of *gold, silver,* and *bronze.*

411. ***Gold.*** The 100, 40, 20, 10, and 5 franc pieces.

412. ***Silver.*** The 5, 2, and 1 franc, the 50 and the 25 centime pieces.

413. ***Bronze.*** The 10, 5, 2, and 1 centime pieces.

GERMAN MONEY.

414. The ***Empire of Germany*** has adopted a new and uniform system of coinage.

415. The ***Unit*** is the "*Mark*" (*Reichsmark*), equal to 23.85 cents, U. S. Money.

A pound of gold .900 fine is divided into 139½ pieces, and the $\frac{1}{10}$ part of this gold coin is called a "Mark," and this is subdivided into 100 pennies (*Pfennige*).

416. The ***Coin*** of the New Empire consists of *gold, silver,* and *nickel,* and is as follows:

417. ***Gold.*** The 20, the 10, and the 5-mark pieces.

418. ***Silver.*** The 2, and the 1-mark, and the 20-penny pieces

419. ***Nickel.*** The 10, and the 5-penny, and pieces of less valuation.

The 10-mark piece (gold) is equal to 3⅓ P. Thalers (old).

The 1-mark (silver) is equal to 10 S. Groschen, or 100 pennies.

The 20-penny (silver) is equal to 2 S. Groschen, or ⅕ of a mark.

The 10-penny (nickel) is equal to 1 S. Groschen, or $\frac{1}{10}$ of a mark.

19. How many minutes in 5 hr. 40 min.? In 4½ hr.?
20. In ½ a sign, how many degrees? Geographic miles?
21. In 10 gross 9 doz., how many dozen? In 7½ gro.?
22. In 2 reams of paper how many quires? Sheets?

23. In £5 10s., how many shillings? In 3½ sov.?
24. In 5 francs how many centimes? In 10⅕ francs?
25. How many pence in 1½ crowns? In 12 florins?
26. How many crowns in £5? How many florins?
27. How many pennies in 3 marks? In 5½ marks?

424. Principle.—*Denominate Numbers are changed to lower denominations by* MULTIPLICATION.

WRITTEN EXERCISES.

425. 1. Reduce 28 rd. 4 yd. 2 ft. 10 in. to inches.

OPERATION.

28 rd. 4 yd. 2 ft. 10 in.
5½
158 yd.
3
476 ft.
12
5722 in.

ANALYSIS.—Since 1 rod equals 5½ yards, 28 rd. 4 yd. equal 28 times 5½ yd., plus 4 yd.; 5½ yd. × 28 + 4 yd. = 158 yd., the number of *yards* in 28 rd. 4 yd.

Since 1 yard equals 3 feet, 158 yd. 2 ft. equal 158 times 3 ft., plus 2 ft.; 3 ft. × 158 + 2 ft. = 476 ft., the number of *feet* in 28 rd. 4 yd. 2 ft.

Since 1 foot equals 12 inches, 476 ft. 10 in. equal 476 times 12 in., plus 10 in.; 12 in. × 476 + 10 in. = 5722 in., the number of *inches* in 28 rd. 4 yd. 2 ft. 10 in.

2. Reduce 7 lb. 10 oz. 16 pwt. 11 gr. to grains.
3. In 3 T. 6 cwt. 21 lb. 12 oz., how many ounces?
4. How many inches in 12 fathoms 3 ft. 10 in.?
5. Change 6 wk. 5 da. 9 hr. 25 min. to minutes.

Rule.—I. ***Multiply the units of the highest denomination of the given number, by that number of the scale that will reduce it to the next lower denomination, and to the product add the number of that denomination given.***

II. ***Proceed in like manner with this and each successive denomination obtained, until the number is reduced to the required denomination.***

Reduce

6. 12 mi. 36 rd. 10 ft. to ft.
7. 10 rd. 5½ ft. to inches.
8. 27¾ yd. to eighths.
9. 1 A. 15 sq. yd. to sq. ft.
10. 2 sq. mi. 125 A. to P.
11. 14 sq. mi. to acres
12. 3 mi. 51 ch. 6 l. to links.
13. 75 Cd. 6 cd. ft. to cu. ft.
14. 12 hhd. 21 gal. to pt.
15. 24 bu. 3 pk. to quarts.
16. Cong. 4, O. 5, f℥ 8 to fʒ.
17. 31½ gal. to gills.

Change

18. 7 T. 9 cwt. 18 lb. to lb.
19. 22 lb. 10 oz. to pwt.
20. ℔ 16, ℥ 7, ʒ 3, to ℈.
21. 1 common year to min.
22. The summer mos. to sec.
23. 1 leap year to hours.
24. 10 S. 22° 5′ to min.
25. 5 bundles to quires.
26. 6 G. gro. to dozens.
27. 326¼ sov. to pence.
28. 26½ fr. to centimes.
29. £34½ to pence.

30. How much is 5 lb. 9 oz. 14 pwt. of gold dust worth, at $.75 a pwt.?

31. How many rods of fence will enclose a farm ¾ of a mile square?

32. If 1 barrel will hold 2 bu. 3 pk., how many barrels will be required to hold 1548 bu. 1 pk.?

33. How many boxes, each containing 12 lb., can be filled from a hogshead containing 9 cwt. 60 lb. of sugar?

34. If I buy 9 bu. of chestnuts at $4¾ a bushel, and retail them at 12½ cents a pint, what is my whole gain?

35. How many times will a wheel 16½ ft. in circumference revolve in running 42 miles?

36. How many minutes less in every autumn of a common year than in either spring or summer?

37. If it require 4 reams 10 quires of paper to print a book, how many sheets are required?

38. At 12½ cents each, what will be the cost of 2 great gross of writing books?

39. If a clock tick seconds, how many times will it tick during February, 1877?

40. If your age is 21 yr. 26 da., how many minutes old are you, if 5 leap years have occurred in that time?

41. If a vessel sail 120 leagues in a day, how many statute miles does she sail?

42. How many pint, quart, and 2-quart bottles, of each an equal number, can be filled from a barrel of 31½ gallons?

43. In the eighteenth century, how many hours?

44. How large an edition of a 12mo book can be printed from 2 bales, 2 bundles, 15 quires of paper, allowing 8 sheets to the volume?

45. How many pages, 2 pages to each leaf, will there be in an 8vo book, containing 16 fully printed sheets?

How many pounds

46. In 36¼ centals of grain?
47. In 42¾ bbl. of flour?
48. In 29.5 quintals of fish?
49. In 116½ bbl. of salt?
50. In 63.25 kegs of nails?
51. In .75 of 75 bu. of salt?
52. In 125¾ bu. of wheat?
53. In ⅜ of 21648 bu. oats?
54. In .7 of 40 bu. corn meal? X
55. In 7.5 casks of lime? X

What is the value in U. S. Money

56. Of 28 sovereigns?
57. Of £25 10s.?
58. Of 25 francs?
59. Of 42½ marks?

426. To reduce denominate numbers from lower to higher denominations.

ORAL EXERCISES.

1. How many feet in 108 in.? How many yards?
2. How many square yards in 63 sq. ft.? In 85 sq. ft.?
3. How many chains in 200 l.? In 425 l.? In 674 l.?
4. In 81 cu. ft., how many cu. yd.? How many cd. ft.?
5. How many cords in 100 cd. ft.? In 256 cu. feet?
6. Change 120 sq. ch. to A. 80 P. to square chains.
7. In 162 in., how many hands? Spans? Feet?

8. In 112 pt., how many quarts? Pecks? Bushels?
9. How many gallons in 46 qt.? 96 pt.? 128 gi.?
10. Reduce O. 160 to Cong.; f ʒ 90 to f ℥.
11. Change 96 oz. to Troy pounds; to Avoir.
12. Reduce ℈45 to oz.; ℥ 75 to pounds.
13. In 400 pwt., how many oz.? How many lb.?
14. In 508 lb., how many cwt.? In 1276 lb.?
15. In 630 lb. of wheat, how many bushels?

16. In 140 da. how many wk.? Months, of 30 da. each?
17. Change 1200 min. to hours. 84 hr. to days.
18. How many doz. are 240 eggs? How many gross?
19. How many degrees in 180′? Minutes, are 240″?
20. In 90 units, how many score? Sets? Pairs?
21. In 120 quires of paper, how many reams? Bundles?
22. In 120d. how many shillings? Crowns? Florins?
23. In 500 pennies, how many marks?

427. PRINCIPLE.—*Denominate numbers are changed to higher denominations by* DIVISION.

WRITTEN EXERCISES.

428. 1. Change 5722 inches to rods.

OPERATION.

12)5722 in.

3)476 ft. + 10 in.

$5\frac{1}{2}$)158 yd. + 2 ft.

2 2

11)316 half-yd.

28 rd. + 4 yd.

5722 in. = 28 rd. 4 yd. 2 ft. 10 in.

ANALYSIS.—Since 12 in. make 1 ft., in 5722 in. there are as many feet as 12 in. are contained times in 5722 in., or 476 ft. and 10 in. more.

Since 3 ft. make 1 yd., in 476 ft. there are 158 yd. and 2 ft. more.

And since $5\frac{1}{2}$ yd. make 1 rd., in 158 yd. there are 28 rd. and 4 yd. more.

In order to divide by $5\frac{1}{2}$, both dividend and divisor may be reduced to *halves* before dividing. In this case the remainder, if any, is *halves*, which may be reduced to *integers*.

2. Reduce 157540 minutes to weeks.
3. Reduce 80820 links to miles.
4. Change 487630 pwt. to pounds.

RULE.—I. *Divide the units of the given denomination by that number of the scale which is equal to a unit of the next higher denomination, and write the remainder as a part of the answer.*

II. *In like manner, divide this and each successive quotient until reduced to the denomination required. The last quotient, with the remainders annexed, is the required result.*

How many

5. Miles are 3168000 in.?
6. Acres are 256800 P.?
7. Sq. mi. are 27878400 sq. ft.?
8. Cu. ft. are 216840 cu. in.?
9. Cords are 38042 cu. ft.?

Reduce

10. 30876 gills to hhd.
11. 27072 qt. to bushels.
12. 66742 pt. to barrels.
13. 103720 pt. to gallons.
14. *f*℥ 8106 to Cong.

How many

15. Pounds Troy are 85894 gr.?
16. Tons are 51570 pounds?
17. Cwt. are 40607 ounces?
18. Pounds are 3000 pwt.?
19. Bu. are 12060 lb. of wheat?
20. Bbl. are 3038 lb. of flour?
21. Bu. are 6496 lb. of oats?
22. Quin. are 3172 lb. of fish?

23. Weeks are 3114061 sec.
24. Months are 8263420 min.?
25. Degrees are 2007200″?
26. Deg. are 5270 Naut. mi.?

Reduce

27. 120400 pens to gro.
28. 2734 eggs to dozens.
29. 5020 balls to scores.
30. 10738 sheets to rm.
31. 6048 quires to bun.

32. 24684d. to crowns.
33. 4076s. to florins.
34. $104.66 to half-sov.
35. 42346 far. to £.
36. $86.85 to francs.
37. $225.40 to sov.
38. $47.70 to marks.

39. If the Atlantic Cable is 3200 mi. in length, and cost 10 cents a foot, what was its entire cost?

40. If a cubic foot of gray limestone weigh 175 lb., what is the weight of a cubic yard?

41. What is the cost of a load of oats weighing 1860 lb., at $.56 a bushel?

42. In a storm at sea, a ship changed her longitude 423 geographic mi. How many degrees and minutes?

43. How much time will a person gain in 40 yr., by rising 25 min. earlier and retiring 20 min. later every day, counting 9 leap years in the time?

44. What will a peck of clover-seed cost, at $.12½ a lb.?

45. What will a ton of corn-meal cost, at $1.20 a bu.?

46. An Illinois farmer sold a load of corn weighing 2496 lb., and a load of oats weighing 1920 lb.; for the corn he received $.62 a bushel, and for the oats $.44 a bushel. What did he receive for both loads?

REDUCTION OF DENOMINATE FRACTIONS.

429. A ***Denominate Fraction*** is a fraction whose integral *unit* is a denominate number. Thus, $\frac{5}{7}$ of a week, .7 of an acre, are denominate fractions.

The Principles, Operations, and Analyses of the reduction of *denominate fractions* are essentially the same as those of *denominate integers.*

430. To reduce denominate fractions from higher to fractions of lower denominations.

ORAL EXERCISES.

1. Reduce $\frac{1}{16}$ of a gallon to the fraction of a pint.

ANALYSIS.—Since in 1 gal. there are 4 qt., in $\frac{1}{16}$ gal. there are $\frac{1}{16}$ of 4 qt., or $\frac{1}{4}$ qt.; and since in 1 qt. there are 2 pt., in $\frac{1}{4}$ qt. there are $\frac{1}{4}$ of 2 pt., or $\frac{1}{2}$ pt. Hence $\frac{1}{16}$ gal. equals $\frac{1}{2}$ pt.

2. Reduce $\frac{1}{24}$ lb. Troy to the fraction of an oz.

3. What part of a pint is $\frac{3}{10}$ of a qt.? $\frac{1}{18}$ of a pk.?

4. What decimal part of a day is .12 of a week?

ANALYSIS.—Since in 1 wk. there are 7 days, in .12 wk. there are .12 of 7 da., or .84 da.

5. What part of a peck is .02 of a bu.? .07 bu.? .25 bu.?

6. Reduce .5 gal. to the fraction of a quart? Of a pint?

7. What part of an inch is $\frac{1}{16}$ of a foot? $\frac{1}{40}$ of a yard?

8. Change .04 of a pound to the decimal of an ounce.

WRITTEN EXERCISES.

431. 1. Reduce $\frac{1}{112}$ of a bushel to the fraction of a pint

OPERATION.

$\frac{1}{112}$ bu. $\times 4 = \frac{1}{28}$ pk.

$\frac{1}{28}$ pk. $\times 8 = \frac{2}{7}$ qt.

$\frac{2}{7}$ qt. $\times 2 = \frac{4}{7}$ pt.

Or, $\frac{1}{\underset{7}{\cancel{112}}} \times \frac{4}{1} \times \frac{8}{1} \times \frac{2}{1} = \frac{4}{7}$ pt.

ANALYSIS.—Same as for oral questions. (**430.**)

Multiply successively by 4, 8, and 2, the numbers in the descending scale required to reduce bushels to pints. (**425.**)

2. Reduce $\frac{1}{132}$ of a rod to the fraction of a foot.
3. Change $\frac{1}{960}$ of an ounce to the fraction of a grain.
4. What part of a pint is $\frac{1}{672}$ of a hogshead?
5. What part of a shilling is .012 of a £?

Rule.—*Multiply the fraction of the higher denomination by the numbers as factors in the descending scale successively between the given and the required denomination.* (425.)

6. What part of an ounce is $\frac{3}{1280}$ of a pound Avoir.?
7. Reduce $\frac{7}{1440}$ of an acre to the fraction of a sq. rd.
8. Reduce .005 of a bushel to the decimal of a pint.
9. How many yards is $\frac{5}{7}$ of $\frac{4}{11}$ of a rod?
10. Change .0000625 mi. to the decimal of a foot.
11. What part of an ounce Troy is $\frac{3}{5}$ of $\frac{1}{6}$ of 2 pounds?
12. What part of a yard is $\frac{1}{2112}$ of a mile?
33. What fraction of a link is $\frac{1}{45}$ of a rod?
14. What part of a minute is .000175 of a day?
15. What part of a sq. rd. is $\frac{3}{1360}$ of $4\frac{1}{2}$ times $\frac{2}{19}$ A.?

432. To reduce denominate fractions to integers of lower denominations.

ORAL EXERCISES.

1. How many hours in $\frac{2}{3}$ of a day?

Analysis.—Since in 1 da. there are 24 hr., in $\frac{2}{3}$ of a day there are $\frac{2}{3}$ of 24 hr., or 16 hr. Hence $\frac{2}{3}$ da. equals 16 hr.

2. How many minutes in $\frac{5}{12}$ hr.? In $\frac{4}{15}$ hr.? In $\frac{3}{4}$ hr.?
3. How many quarts in $\frac{5}{8}$ pk.? In $\frac{1}{4}$ bu.? In $\frac{7}{16}$ bu.?
4. How many ounces in .5 of a pound?

Analysis.—Since in 1 lb. there are 16 oz., in .5 lb. there are .5 of 16 oz., or 8 oz.

5. How much is .7 hr.? .25 hr.? .15 hr.? .8 hr.?
6. How many yards in $\frac{3}{11}$ of a rod? In $\frac{5}{8}$ of a rod?
7. How many cwt. in $\frac{2}{5}$ of a ton? How many pounds?
8. Change to pints $\frac{1}{2}$ gal. $\frac{1}{4}$ pk. $\frac{3}{16}$ bu. $\frac{3}{4}$ of 2 pk.
9. Change $\frac{1}{4}$ of an acre to sq. rd. $\frac{4}{7}$ sq. yd. to sq. ft.
10. How many pecks in .75 bu.? Quarts in 1.25 pk.?
11. Change $\frac{3}{4}$ lb. to oz. .45 oz. to pwt. .53 cwt. to lb.

WRITTEN EXERCISES.

433. 1. Reduce $\frac{5}{6}$ bu. and .645 da., each to integers of lower denominations.

1ST OPERATION.	1ST OPERATION.
4 pk. × $\frac{5}{6}$ = $\frac{20}{6}$ pk. = $3\frac{1}{3}$ pk.	24 hr. × .645 = 15.48 hr.
8 qt. × $\frac{1}{3}$ = $\frac{8}{3}$ qt. = $2\frac{2}{3}$ qt.	60 min. × .48 = 28.8 min.
2 pt. × $\frac{2}{3}$ = $\frac{4}{3}$ pt. = $1\frac{1}{3}$ pt.	60 sec. × .8 = 48 sec.
$\frac{5}{6}$ bu. = 3 pk. 2 qt. $1\frac{1}{3}$ pt.	.645 da. = 15 hr. 28 min. 48 sec.

1. The analyses of the above are the same as in (425) and (430).

2. The following methods may be regarded as most convenient in practice, since the operations are performed without *rewriting the fractional part of each product.*

2D OPERATION.

```
          5
          4
         ——
     6)20(3 pk. 2 qt. 1⅓ pt.
4        18
         ——
2         2
—
6)8 pt.   8   ⅚ bu. = 3 pk. 2 qt. 1⅓ pt.
  6    6)16 qt.
  —
  2      12
         ——
          4
```

2D OPERATION.

```
 .645 da.
   24
 ————
 2580
1290
————
15.480 hr.
    60
 ————
28.800 min.
     60
  ————
48.000 sec.
```

.645 da. = 15 hr. 28 min. 48 sec.

Reduce to integers of lower denominations,

2. $\frac{11}{24}$ of a £.
3. .35 lb. Apoth.
4. $\frac{4}{15}$ of a mi.
5. .75 lb. Troy.
6. .625 of a fath.
7. .55 lb. Avoir.

Rule.—I. *Multiply the given fraction or decimal by that number in the scale that will reduce it to the next lower denomination.* (425.)

II. *Proceed in like manner with the fractional part of each successive product, until it is reduced to the denomination required.*

III. *The integral parts of the several products, arranged in their proper order, is the required result.*

Find the value in *integers* of lower denominations,

8. Of $\frac{4}{7}$ mo.
9. Of .555 £.
10. Of $\frac{7}{18}$ A.
11. Of $\frac{2}{3}$ of $\frac{5}{8}$ lb.
12. Of $\frac{8}{15}$ cu. yd.
13. Of .1934 S.
14. Of f ℥ .7.
15. Of $\frac{327}{640}$ T.
16. Of .875 hhd.
17. Of $\frac{5}{8}$ sq. rd.
18. Of ℔ $\frac{15}{32}$.
19. Of $\frac{4}{7}$ G. gro.
20. Of .67 lea.
21. Of .125 bbl.
22. Of .578125 bu.
23. Of .6625 mi.
24. Of $\frac{7}{9}$ of $5\frac{1}{2}$ T.
25. Of $\frac{3}{8}$ of $3\frac{2}{3}$ A.
26. Of $\frac{4}{5}$ of $3\frac{3}{8}$ Cd.
27. Of $\frac{4}{5}$ of .225 mi.
28. Of .3125 ream.
29. At $8\frac{1}{3}$ cents a pound, what will $\frac{13}{40}$ T. of cheese cost?

434. **To reduce denominate fractions from lower to fractions of higher denominations.**

ORAL EXERCISES.

1. Reduce $\frac{2}{3}$ of a peck to the fraction of a bushel.

Analysis.—Since 4 pk. make 1 bu., there are $\frac{1}{4}$ as many bushels as pecks; $\frac{1}{4}$ of $\frac{2}{3}$ pk. $= \frac{2}{12}$ or $\frac{1}{6}$ bu.

2. Reduce $\frac{4}{5}$ of a pint to the fraction of a gallon.
3. What part of a pound Avoir. is $\frac{3}{5}$ oz.?
4. What part of a week is $\frac{2}{3}$ da.? $\frac{7}{8}$ da.? $\frac{5}{8}$ da.? $\frac{4}{5}$ da.?

5. What decimal part of a gallon is .28 of a quart?

Analysis.—Since 4 qt. make 1 gal., there are $\frac{1}{4}$ as many gallons as quarts; $\frac{1}{4}$ of .28 qt. is .07 gal.

6. Change .32 of a pint to the decimal of a quart.

7. What decimal of a pound Troy is .48 oz.? .84 oz.?

8. What decimal of a week is .35 da.? .63 da.?

9. Change .72 in. to the decimal of a foot. Of a yd.

WRITTEN EXERCISES.

435. 1. What fraction of a bushel is $\frac{4}{5}$ of a pint?

OPERATION.

$\frac{4}{5}$ pt. $\div 2 = \frac{2}{5}$ qt.
$\frac{2}{5}$ qt. $\div 8 = \frac{1}{20}$ pk.
$\frac{1}{20}$ pk. $\div 4 = \frac{1}{80}$ bu.
Or, $\frac{4}{5} \times \frac{1}{2} \times \frac{1}{8} \times \frac{1}{4} = \frac{1}{80}$ bu.

Analysis.—Divide successively by 2, 8, and 4, the numbers in the ascending scale, required to reduce pints to bushels. (**428.**) Hence $\frac{4}{5}$ pt. $= \frac{1}{80}$ bu.

2. Reduce $\frac{8}{11}$ of a gill to the fraction of a gallon.

3. Change $\frac{8}{9}$ of a shilling to the fraction of a £.

4. Reduce .64 of a pint to the decimal of a bushel.

5. What part of a pound Troy is .576 of a grain?

Rule.—*Divide the fraction of the lower denomination by the numbers as factors in the ascending scale successively between the given and the required denomination.* (**428.**)

6. What decimal of a ton is .8 lb.? .36 of a cwt.?

7. Reduce $\frac{12}{49}$ of a cord foot to the fraction of a cord?

8. Reduce .216 gr. to the decimal of an ounce Troy.

9. What part of a ton is $\frac{5}{7}$ of a pound?

10. What part of a day is $\frac{2}{4}$ of a minute? .12 hr.?

11. What decimal of a rod are 3.96 in.?

12. How much less is $\frac{3}{5}$ of a pint than $\frac{1}{630}$ of a hhd.?

13. What part of an acre is $\frac{7}{9}$ of a square rod?

436. To reduce a compound denominate number to a fraction of a higher denomination.

ORAL EXERCISES.

1. What part of a pound are 4 oz.? 8 oz.? 10 oz.?
2. What part of a foot are 9 in.? What part of a yard?
3. What part of a bushel are 2 pk. 4 qt.?

ANALYSIS.—1 bu.=32 qt., and 2 pk. 4 qt.=20 qt.; 20 qt.=$\frac{20}{32}$ bu. =$\frac{5}{8}$ bu., or .625 bu. Hence 2 pk. 4 qt.=$\frac{5}{8}$ bu., or .625 bu.

4. What part of a yard are 2 ft. 6 in.? Are 18 in.?
5. What part of 3 lb. Troy are 1 lb. 6 oz.? Are 9 oz.?
6. What part of 5 gal. are 2 gal. 2 qt.? 3 gal. 1 qt.?
7. Reduce 12 oz. to the decimal of 3 lb. Avoir.
8. What fraction of 3 Cd. 6 cd. ft. are 2 Cd. 4 cd. ft.?
9. What part of 3 pk. are 1 pk. 4 qt.? Are $2\frac{1}{2}$ pk.?

WRITTEN EXERCISES.

437. 1. What decimal of a pound Troy are 2 oz. 14 pwt.?

1ST OPERATION.

$$\begin{array}{r} 20\,\underline{)\,14\text{ pwt.}} \\ 12\,\underline{)\,2.7\text{ oz.}} \\ .225\text{ lb.}=\tfrac{9}{40}\text{ lb.} \end{array}$$

2D OPERATION.

$$2\text{ oz. }14\text{ pwt.} = 54\text{ pwt.}$$
$$1\text{ lb.} = 240\text{ pwt.}$$
$$\tfrac{54}{240} = \tfrac{9}{40}\text{ lb.} = .225\text{ lb.}$$

ANALYSIS.—Since 20 pwt. make 1 oz., there are $\frac{1}{20}$ as many ounces as pennyweights; and $\frac{1}{12}$ as many pounds as ounces (**428**). Hence 2 oz. 14 pwt.=.225 lb., or changed to a *fraction*, by **283**, is $\frac{9}{40}$ lb.

ANALYSIS.—In order to find *what part* one compound number is of another, *both* must be *like numbers*, and must be reduced to the lowest denomination in *either*. Thus, 2 oz. 14 pwt. are equal to 54 pwt., and 1 lb. is equal to 240 pwt. Hence 2 oz. 14 pwt.=$\frac{54}{240}$ lb =$\frac{9}{40}$ lb., or, reduced to a *decimal*, by **285**, .225 lb.

2. Reduce 3 gal. 3 qt. $1\frac{1}{2}$ pt. to the fraction of a bbl.
3. Reduce 3 cd. ft. 8 cu. ft. to the decimal of a cord.

RULE.—I. *Divide the units of the lowest denomination given by that number in the scale which is equal to a unit of the next higher denomination, and annex the quotient as a decimal to the number given of that denomination.*

II. *Proceed in like manner until the whole is reduced to the denomination required.* Or,

Reduce the given number to its lowest denomination for the numerator of the required fraction, and a unit of the required denomination to the same denomination for the denominator, and reduce the fraction to its lowest terms, or to a decimal.

1. If the given number contain a fraction, the *denominator* of this fraction must be regarded as the *lowest* denomination.

2. The pupil may be required to give the answers either in the form of a fraction, or of a decimal, or both.

4. Reduce 13 gal. 3 qt. 3.62 gi. to the decimal of a hhd.

5. What part of a pound Troy are 10 oz. 13 pwt. 9 gr.?

6. What fraction of 2 T. 7 cwt. 28 lb. are 5 cwt. 91 lb.?

7. What part of 3 A. 80 P. are 51.52 P.?

8. What part of a f ℥ are f ʒ 5 ♏36?

9. Change 126 A. 4 sq. ch. 12 P. to the decimal of a Tp.

10. What decimal part of 25° 42′ 40″ are 7° 42′ 48″?

11. From a hhd. of molasses 28 gal. 2 qt. were drawn. What part of the whole remained?

12. What decimal of a league are 2 mi. 3 rd. 1 yd. 3⅜ in.?

13. What part of 3 bbl. of flour are 110 lb. 4 oz.?

14. What decimal part of a ton is $\frac{1}{7}$ of 22¾ lb.?

15. Reduce .45 pk. to the decimal of 1¼ bu.

16. What part of 54 cords of wood are 4800 cu. ft.?

17. Change 18s. 5d. $2\frac{2}{13}$ far. to the fraction of a £.

REVIEW.

WRITTEN EXAMPLES.

438. 1. How many steps of 30 in. each must a person take in walking 21 miles?

2. How long will it take one of the heavenly bodies to move through a sextant, at the rate of 3′ 12″ a minute?

3. Reduce £10 18s. 6d. to United States Money.

4. Paid $425.75 for 2¼ tons of cheese, and retailed it at 12½ cents a pound. What was the whole gain?

5. Reduce 580 francs to United States Money.

6. Change $291.99 to Sterling Money.

7. What cost 30 bu. 2 pk. 1 qt. of beans, at $4.20 a bu.?

8. Bought 15 cwt. 22 lb. of rice at $4.25 a cwt., and 6 cwt. 36 lb. of pearl-barley at $5.60 a cwt. What would be gained by selling the whole at 6¼ cents a pound?

9. How many bushels of corn in 36824 lb., Illinois standard? Louisiana? New York?

10. 5000 bu. of oats in Ohio are equal to how many bushels in Connecticut, by weight? In New Jersey?

11. If I buy 16 T. 3 cwt. 3 qr. 24 lb. of Eng. iron, by long ton weight, at 3d. a lb., and sell the same at $140, by the short ton, what do I gain by the transaction?

12. How many carats fine is a piece of gold ⅝ pure?

13. How many acres in a piece of land 105 ch. 85 l. long, and 40 ch. 15 l. wide?

14. If 10 lb. of milk make 1 lb. of cheese, what will it cost at 1 cent a pound to manufacture the cheese that can be made from 90000 lb. of milk?

15. At $75¾ an acre, what is the value of a farm 189.5 rd. long and 150 rd. wide?

16. What cost 2 bu. 3 pk. 6 qt. of green peas, at $.30 a peck?

17. What cost 3 T. 17 cwt. 20 lb. of hay, at $22¾ a ton?

18. If a grocer's scales give ¼ oz. short of true weight on every pound, of how much money does he defraud his customers, in the sale of 3 bbl. of sugar, each weighing 2 cwt. 10 lb., at 12½ cents a pound?

19. If 37 A. 128 P. are sold from a farm containing 170 A. 16 P., what part of the whole remains?

20. Paid $526.05 for 3½ tons of cheese, and retailed it at 9½ cents a pound. How much was the whole gain?

21. How many bushels of oats in Connecticut are equivalent in weight to 2500 bushels in Iowa?

22. How many centals of barley in California are equivalent to 1500 bushels in Missouri?

23. A man sold 12 bu. 3 pk. 6 qt. of cranberries at $3½ a bushel, and took his pay in flour at 4 cents a pound. How many barrels did he receive?

24. If 3 T. 12 cwt. 20 lb. of ground plaster cost $15.75, what will be the cost of 5 T. 80 lb. at the same rate?

25. Bought 37 Cd. 48 cu. ft. of wood for $129.81, and there was but 13 Cd. 59 cu. ft. delivered. What part of the money should be paid?

26. If a grocer's gallon measure is too small by 1 gi., what does he make dishonestly in selling 2 hhd. of molasses, averaging 58 gal. 2 qt. 1 pt. each, worth $.80 a gal.?

27. How many reams of paper are required to supply 4500 subscribers with a weekly newspaper for 1 year?

28. A publisher printed an edition of 10000 copies of a 12mo book of 336 pp. How much paper did he use, allowing 1 quire to each ream for waste?

ADDITION.

439. Denominate numbers are *added, subtracted, multiplied,* and *divided* by the same general methods as are employed for like operations in Simple Numbers.

The corresponding processes are based upon the *same principles.* The only modification of the rules needed is that which is required by a *varying scale* instead of a *uniform scale* of 10.

The *principles* will be made sufficiently plain in the *operations* and *analyses* to render *special rules unnecessary.*

WRITTEN EXERCISES.

440. 1. Find the sum of 4 cwt. 46 lb. 12 oz., 12 cwt. $9\frac{1}{2}$ lb., $2\frac{1}{4}$ cwt., and $21\frac{5}{8}$ lb.

OPERATION.

cwt.	lb.	oz.
4	46	12
12	9	8
2	25	0
	21	10
19	2	14

Analysis.—Write the numbers so that units of the same denomination stand in the same column, and begin at the right to add.

The sum of the ounces is 30, equal to 1 lb. 14 oz. Write the 14 oz. under the column of ounces, and add the 1 lb. to the pounds of the next column.

The sum of the pounds is 102 lb., equal to 1 cwt. and 2 lb. Write the 2 lb. under the column of pounds, and add the 1 cwt. to the cwt. of the next column.

The sum of the cwt. is 19 cwt., which write under the column of cwt. Hence the entire sum is 19 cwt. 2 lb. 14 oz.

2. What is the sum of $\frac{1}{10}$ wk., $\frac{3}{5}$ da., and $\frac{3}{8}$ hr.?

1st OPERATION.

	da.	hr.	min.	sec.
$\frac{1}{10}$ wk. =	4	21	36	00
$\frac{3}{5}$ da. =		14	24	00
$\frac{3}{8}$ hr. =			22	30
	5	12	22	30

Analysis.—First find the value of each denominate fraction in integers of lower denominations (**433**), and then add the resulting compound numbers. Or, 2D OPERATION.

$\frac{3}{8}$ da. $= \frac{3}{56}$ wk.
$\frac{3}{8}$ hr. $= \frac{1}{448}$ wk.
$\frac{7}{10}$ wk. $+ \frac{3}{56} + \frac{1}{448} = \frac{353}{448}$ wk.
$\frac{353}{448}$ wk. $=$ 5 da. 12 hr. 22 min. 30 sec.

Reduce the given fractions to fractions of the same denomination (434), then add the results and find the value of their sum in integers of lower denominations.

If denominate fractions occur in the given numbers, they should be reduced to integers of lower denominations (433) before adding.

3. Add 7 yd. 2 ft., 5 yd. $1\frac{1}{4}$ ft., 2 ft. $9\frac{1}{3}$ in., 3 yd. 1 ft. $6\frac{1}{3}$ in., $2\frac{3}{4}$ ft., $4\frac{1}{3}$ yd.

4. Add 5 Cd. 7 cd. ft., 2 Cd. 2 cd. ft. 12 cu. ft., 6 cd. ft. 15 cu. ft., $7\frac{3}{8}$ Cd., and 3 Cd. 2 cu. ft.

5. What is the sum of $1\frac{2}{3}$ hhd., 36 gal. 3 qt. $1\frac{1}{4}$ pt., $\frac{7}{8}$ gal., 2 qt. $\frac{3}{4}$ pt., and 1.75 pt.?

6. What is the sum of $\frac{5}{8}$ of a day added to $\frac{7}{15}$ of an hour?

7. To $\frac{6}{7}$ of a hhd. add $\frac{5}{8}$ of 10 gal.

8. What is the sum of $22\frac{4}{7}$ cwt., $26\frac{7}{8}$ lb., and 14 oz.?

9. Add $5\frac{1}{3}$ Pch., 18 cu. ft., 86.6 cu. ft., and $\frac{5}{6}$ Pch.

10. Find the sum of ℔ 4 ℥ 6 ʒ 5, and ℔ 6 ℥ $4\frac{1}{2}$ ʒ $9\frac{3}{4}$.

11. A Missouri farmer received $.75 a bushel for 4 loads of corn; the first contained 48.4 bu., the second 2626 lb., the third $36\frac{3}{4}$ bu., and the fourth 41 bu. 52 lb. What did he receive for the whole?

12. Bought three loads of hay at $15 a ton. The first weighed 1.125 T., the second $1\frac{3}{5}$ T., and the third 2750 lb. What did the whole cost?

13. When B was born, A's age was 3 yr. 9 mo. 24 da.; when C was born, B's age was 12 yr. 19 da.; when D was born, C's age was 5 yr. 11 mo., and when E was born, D's age was 10 yr. 1 mo. 20 da. What was A's age when E was born?

SUBTRACTION.

WRITTEN EXERCISES.

441. 1. From 25 rd. 2 yd. 2 ft. 6.3 in., subtract 12 rd. 4 yd. 11.6 in.

OPERATION.

rd.	yd.	ft.	in.
25	2	2	6.3
12	4	0	11.6
12	3 ($\frac{1}{2}$)	1	6.7
	$\frac{1}{2}$ =	1	6
12	4	0	.7

ANALYSIS.—Write the numbers so that units of the same denomination stand in the same column, and begin at the right to subtract.

Since 11.6 in. cannot be subtracted from 6.3, take 1 ft., equal to 12 in., from the 2 ft., leaving 1 ft. and add it to the 6.3 in., making 18.3 in. Subtract 11.6 in., and write the remainder, 6.7 in., under the inches.

Since 1 ft. has been taken from 2 ft., subtract 0 ft. from 1 ft., and write the remainder 1 ft. under the feet.

Since 4 yd. cannot be taken from 2 yd., take 1 rd., equal to 5$\frac{1}{2}$ yd., from 25 rd., leaving 24 rd., and add it to the 2 yd., making 7$\frac{1}{2}$ yd. Subtract 4 yd. from 7$\frac{1}{2}$ yd., and write the remainder, 3$\frac{1}{2}$ yd., under the yards.

Since 1 rod has been taken from 25 rd., subtract 12 rd. from 24 rd., and write the remainder, 12 rd., under the rods.

The $\frac{1}{2}$ yd., reduced to feet and inches, and added to 1 ft. 6.7 in. of the remainder, gives 12 rd. 4 yd. .7 in.

2. From 1$\frac{5}{8}$ bu. subtract $\frac{4}{5}$ bu.

OPERATION.

1$\frac{5}{8}$ bu. =	1 bu.	2 pk.	4 qt.	0 pt.
$\frac{4}{5}$ bu. =		3	1	1$\frac{1}{5}$
		3	2	$\frac{4}{5}$

Or,

1$\frac{5}{8}$ bu. = $\frac{13}{8}$ bu.; $\frac{13}{8}$ bu. − $\frac{4}{5}$ bu. = $\frac{33}{40}$ bu.

$\frac{33}{40}$ bu. = 3 pk. 2 qt. $\frac{4}{5}$ pt.

ANALYSIS. — First find the value of each denominate fraction in integers of lower denominations (**433**), and subtract the less value from the greater.

Or, reduce the given fractions to fractions of the same denomination (**434**), then subtract the less from the greater, and find the value of their difference in integers of lower denominations.

3. From a pile of wood containing 42 Cd. 5 cd. ft., take 16 Cd. 6 cd. ft. 12 cu. ft., and how much remains?

4. From the sum of $\frac{2}{7}$ of $3\frac{3}{4}$ mi. and $17\frac{1}{7}$ rd., take $120\frac{1}{3}$ rd.

Find the difference between

5. $8\frac{9}{10}$ cwt. and $48\frac{3}{8}$ lb.
6. £$\frac{5}{8}$ and $\frac{2}{3}$ of $\frac{3}{4}$s.
7. $\frac{3}{40}$ lb. and 5 lb. 4 oz. 8 pwt.
8. .659 wk. and 2 wk. $3\frac{5}{6}$ da.
9. $\frac{117}{224}$ hhd. and 3.625 gal.
10. $\frac{3}{8}$ lea. and $\frac{7}{10}$ mi.
11. $\frac{5}{8}$ gross and $\frac{2}{3}$ doz.
12. ʒ 2 ℈ 1 and ℥ 4.
13. .9 da. and $\frac{1}{4}$ wk.
14. $\frac{15}{16}$ A. and 84.56 P.

15. If from a hhd. of molasses 14 gal. 1 qt. 1 pt. be drawn at one time, 10 gal. 3 qt. at another, and 29 gal. 1 pt. at another, how much will remain?

16. Of a farm containing 250 A., two lots were reserved, one containing 75 A. 136.4 P., and the other 56 A. 123.3 P.; the remainder was sold at \$$62\frac{1}{4}$ an acre. What did it sell for?

17. From 1 T. 11 cwt. 30 lb., take $\frac{4}{5}$ of a long ton.

18. From a pile of wood containing $125\frac{3}{4}$ Cd., was sold at one time 26 Cd. 7 cd. ft.; at another, 30 Cd. $4\frac{5}{8}$ cd. ft.; at another, $37\frac{9}{16}$ Cd. How much remained?

442. To find the interval of time between two dates.

1. How many yr., mo., da., and hr., from 3 o'clock P. M. of May 16, 1864, to 9 o'clock A. M. of Sept. 25, 1875?

OPERATION.

yr.	mo.	da.	hr.
1875	9	25	9
1864	5	16	15
11	4	8	18

ANALYSIS.—Since the later date expresses the greater period of time, write it as the minuend, and the earlier date as the subtrahend, writing the denominations in the order of the scale, then subtract.

1. When hours are to be obtained, reckon from 12 at night, and if minutes and seconds, write them still at the right of hours.

2. In finding the difference of time between two dates, 12 mo. are usually considered a year, and 30 days a month.

3. When the time is less than a year, the *true* number of days in each month and parts of a month is added.

4. The day on which a note, draft, or contract is *dated*, and that on which they *mature*, are *not both* included. The *former* is generally omitted.

2. The war between England and America was commenced April 19, 1775, and peace was restored Jan. 20, 1783. What length of time did the war continue?

3. The American Civil War began April 11, 1861, and closed April 9, 1865. What time did it continue?

4. How long has a note to run that is dated Jan. 16, 1873, and made payable July 10, 1875?

5. A note dated May 28, 1875, was paid Feb. 10, 1876. What length of time did it run?

6. A person started on a tour of the world at 9 o'clock A.M., Sept. 3, 1874, and returned to the same depot at 3 P.M., July 15, 1876. What time was he absent?

7. How many years, months, and days from your birthday to this date; or, what is your age?

8. How many days from June 20th to the 10th of Jan. following?

9. What length of time elapsed from 12 o'clock M., Jan. 10. 1876, to June 16, 9 o'clock A.M.?

10. What length of time elapsed from 16 min. past 10 o'clock A.M., July 4, 1873, to 22 min. before 8 o'clock P.M., Dec. 12, 1875?

11. What length of time will elapse from 40 min. 25 sec. past 12 o'clock M., April 21, 1875, to 4 min. 36 sec. before 5 o'clock P.M., Jan. 1, 1878?

MULTIPLICATION.

WRITTEN EXERCISES.

443. 1. Multiply 28 rd. 2 yd. 2 ft. by 7.

OPERATION.

28 rd.	2 yd.	2 ft.	
		7	
199	1 (½)	2	
	½ =	1	6 in.
199	2	0	6

Analysis.—Write the multiplier under the lowest denomination of the multiplicand, and multiply.

7 times 2 ft. are 14 ft., equal to 4 yd. 2 ft. Write the 2 ft. under the feet, and reserve the 4 yd. to add to the product of yards.

7 times 2 yd. are 14 yd., and 4 yd. added make 18 yd., equal to 3 rd. 1½ yd. Write the 1½ yd. under the yards, and reserve the 3 rd. to add to the product of rods.

7 times 28 rd. are 196 rd., and 3 rd. added make 199 rd., which write under the same denomination.

The ½ yd. is equal to 1 ft. 6 in., which added to the product, gives 199 rd. 2 yd. 6 in. for the entire product.

1. The multiplier must be an abstract number. (**103.**)

2. When the multiplier is large and is a composite number, the work may be shortened by multiplying successively by its factors. (**109.**)

2. In 9 bbl. of walnuts, each containing 2 bu. 3 pk. 6 qt., how many bushels?

3. If a man cut 3 Cd. 36 cu. ft. of wood in 1 da., how many cords can he cut in 12 days?

4. Multiply 8 gal. 3 qt. 1 pt. 3.25 gi. by 96.

5. If 1 A. produce 42 bu. 1 pk. 5 qt. 1 pt. of corn, how many bushels will 64 A. produce?

6. Multiply O. 8 f℥ 9 fʒ 6 ♏ 34 by 24.

7. What will 84 yd. of cloth cost, at £1 8s. 9½d. a yd.?

8. If $80 will buy 3 A. 24 P. 20 sq. yd. 4 sq. ft. of land, how much will $4800 buy?

9. How many bushels of grain in 47 bags, each containing 2 bu. 2 pk. 6 qt.?

OPERATION.

$$
\begin{array}{llll}
\multicolumn{4}{c}{47 = (9 \times 5) + 2} \\
2 \text{ bu.} & 2 \text{ pk.} & 6 \text{ qt.} & \\
 & & 9 & \\
\hline
24 \text{ bu.} & 0 \text{ pk.} & 6 \text{ qt.} & \text{in } 9 \text{ bags.} \\
 & & 5 & \\
\hline
120 \text{ bu.} & 3 \text{ pk.} & 6 \text{ qt.} & \text{`` } 45 \text{ ``} \\
5 & 1 & 4 & \text{`` } 2 \text{ ``} \\
\hline
126 \text{ bu.} & 1 \text{ pk.} & 2 \text{ qt.} & \text{`` } 47 \text{ ``}
\end{array}
$$

ANALYSIS.—Multiplying the contents of 1 bag by 9, and the resulting product by 5, gives the contents of 45 bags, which is the composite number *next less* than the given prime number, 47. Next find the contents of 2 bags, which, added to the contents of 45 bags, gives the contents of 45+2, or 47 bags.

10. If a load of coal by the long ton weigh 1 T. 6 cwt. 2 qr. 26 lb. 10 oz., what will be the weight of 67 loads?

11. Multiply 4 yd. 1 ft. 4.7 in. by 125.

12. Multiply 7 T. 15 cwt. 10.5 lb. by 1.7.

13. At $1.37½ a gallon, what will be the cost of 5 casks of wine, each containing 28 gal. 2 qt. 1 pt.?

14. A farmer sold 4 loads of oats, averaging 41 bu. 3 pk. each, at $.75 a bushel. What did he receive for the whole?

DIVISION.

444. 1. Divide 56 lb. 9 oz. 12 pwt. by 6.

OPERATION.

$$
\begin{array}{r|rrr}
 & \text{lb.} & \text{oz.} & \text{pwt.} \\
6 & 56 & 9 & 12 \\
\hline
 & 9 & 5 & 12
\end{array}
$$

ANALYSIS.—Write the divisor at the left of the dividend. The object is to find 1 *sixth* of a compound number.

$\frac{1}{6}$ of 56 lb. is 9 lb. and a remainder of 2 lb. Write the 9 lb. in the quotient, and reduce the 2 lb. to ounces, which, added to 9 oz., make 33 oz.

$\frac{1}{6}$ of 33 oz. is 5 oz. and a remainder of 3 oz. Write the 5 oz. in the quotient, and reduce the 3 oz. to pwt., which added to 12 pwt., make 72 pwt. $\frac{1}{6}$ of 72 pwt. is 12 pwt., which write in the quotient.

2. Divide 358 A. 57 P. 6 sq. yd. 2 sq. ft. by 7.

3. Divide £35 9s. 7d. by 5; by 7; by 8.

4. Divide 282 bu. 3 pk. 1 qt. 1 pt. by 9; by 10; by 12.

When the divisor is large, and is a *composite* number, the work may be shortened by dividing successively by its factors.

5. Divide 254 yd. 4 ft. 3⅓ in. by 21; by 42.

6. Divide 196 Cd. 4 cd. ft. 12 cu. ft. by 72.

7. How many iron rails, each 16 ft. long, are required to lay a railroad track 26 mi. long?

8. Divide 24 sq. mi. 140 P., by 22½.

9. Divide 202 yd. 1 ft. 6¾ in. by ⅗.

10. Divide 336 bu. 3 pk. 4 qt. by 4 bu. 3 pk. 2 qt.

Reduce both dividend and divisor to the same denomination, and divide as in simple numbers.

11. How many boxes, each holding 1 bu. 1 pk. 7 qt., can be filled from 356 bu. 3 pk. 5 qt. of cranberries?

12. Divide 311 gal. 1 qt. 1 pt. by 53.

OPERATION.

53)311 gal. 1 qt. 1 pt. (5 gal. 3 qt. 1 pt.
265
46 gal. rem.
4
185 qt. in 46 gal. 1 qt.
159
26 qt. rem.

26 qt. rem.
2
53 pt. in 26 qt. 1 pt.
53

13. The aggregate weight of 41 hhd. of sugar is 19 T. 6 cwt. 22 lb. What is the average weight?

14. If a town 4 mi. square be equally divided into 62 farms, how much land will each farm contain?

LONGITUDE AND TIME.

445. The ***Longitude*** of a place is its distance east or west from a given meridian, measured on the equator.

The meridian from which longitude is reckoned is called the *first meridian*, and is marked 0°. All places east of this, within 180°, are in east longitude, and all places west, within 180°, are in west longitude.

The English and Americans usually reckon longitude from the meridian of Greenwich, England; the French, from Paris.

446. Since the earth revolves on its axis once in 24 hours, the sun *appears to pass* from east to west around the earth, or through 360° of *longitude* once in 24 hours of time. Hence in 1 hour the sun appears to pass through $\frac{1}{24}$ of 360°, or 15°; in 1 minute, through $\frac{1}{60}$ of 15°, or 15′; and in 1 second, through $\frac{1}{60}$ of 15′, or 15″.

COMPARISON OF LONGITUDE AND TIME.

For a difference of	There is a difference of
15° in Long.	1 hr. in Time.
15′ " "	1 min. " "
15″ " "	1 sec. " "
1° " "	4 min. " "
1′ " "	4 sec. " "
1″ " "	$\frac{1}{15}$ sec. " "

1. Since the sun appears to move from east to west, when it is 12 o'clock at one place, it will be *past* 12 o'clock at all places east, and *before* 12 at all places west. Hence, knowing the difference of time between two places, and the exact time at one of them, the exact time at the other is found by *adding* their difference to the given time, if it is *east*, and by *subtracting*, if it is *west*.

2. If one place is in east and the other in west longitude, the difference of longitude is found by *adding* them, and if the sum is greater than 180°, by *subtracting* it from 360°.

ORAL EXERCISES.

447. 1. The earth revolves on its axis once in every 24 hr. What part of a revolution does it make in 12 hr.?

2. How many degrees of the earth's surface pass under the sun's rays in 24 hr.? In 12 hr.? In 4 hr.? In 1 hr.?

3. How many degrees of longitude cause a difference of 1 hr. in time? 2 hr.? 3 hr.?

4. When it is 6 o'clock at Chicago, what is the hour 15° east of Chicago? 15° west of Chicago?

5. When it is noon in New York, what is the hour 15° east of New York? 30° west of N. Y.?

6. When it is 3 o'clock at Washington, what is the time 15° 15′ east of Washington? 30° 30′ west?

7. What difference of longitude causes a difference of 1 hr. in time? Of 1 minute? Of 1 second?

8. If the difference in the time of Boston and of St. Louis is 1 hr. 15 min., what is the difference in their longitude?

9. A man left New Orleans and traveled until his watch was 1 hr. 2 min. too fast. How far had he traveled, and in what direction?

10. Two persons, at different points, observe an eclipse of the moon; one seeing it at 9½ P. M., and the other at midnight. What is the difference in their longitude?

11. A tourist leaves home at 12 M. on Monday, and on Saturday finds his watch 1 hr. 15 min. slow. In what direction has he been traveling? How far?

12. A and B start from opposite points and travel towards each other. When they meet, A's watch is 40 min. slow and B's 1 hr. fast. How far apart are the two points of starting, and in what direction did each travel?

WRITTEN EXERCISES.

448. To find the difference of longitude between two places, when the difference of time is known.

1. When it is 9 o'clock at Washington, it is 7 min. 4 sec. past 8 o'clock at St. Louis. Find the diff. of longitude.

OPERATION.

$$\begin{array}{llll} 9\text{ hr.} & 0\text{ min.} & 0\text{ sec.} & \\ \underline{8} & \underline{7} & \underline{4} & \\ & 52 & 56 & \text{Diff. in Time.} \\ & & \underline{15} & \\ 13^\circ & 14' & 00'' & \text{Diff. in Long.} \end{array}$$

$$\text{Or,}\quad 4\overline{)\,52\text{ min.}\quad 56\text{ sec.}}$$

$$\begin{array}{ll} 13^\circ & 14' \end{array}$$

Analysis.—Since every hour of time corresponds to 15° of long., and every minute of time to 15′ of long., and every second of time to 15″ of long. (**446**), there are 15 times as many deg., min., and sec. in the difference of longitude, as there are hr., min., and sec. in the difference of time. Or,

Since 4 min. of time make a difference of 1° of long., and 4 sec. of time a difference of 1′ of long., there will be $\frac{1}{4}$ as many degrees of long. as there are minutes of time, and $\frac{1}{4}$ as many minutes of long. as there are seconds of time.

2. The difference in the time of Washington and of St. Petersburgh is 7 hr. 9 min. 19$\frac{1}{4}$ sec. What is the difference in their longitudes?

3. When it is 12 o'clock M. at Rochester, N. Y., it is 9 hr. 1 min. 37 sec. A. M. at San Francisco. The long. of Rochester being 77° 51′ W., what is the long. of the latter?

Rule.—*Multiply the difference of time expressed in hours, minutes, and seconds by* 15; *the product will be the difference of longitude in degrees, minutes, and seconds.* Or,

Reduce the difference of time to minutes and seconds, then divide by 4; *the quotient will be the difference of longitude in degrees and minutes.*

4. Noon comes 1 hr. 5 min. 42 sec. sooner at Quebec than at Chicago, whose longitude is 87° 37′ 45″. What is the longitude of Quebec?

5. When the days and nights are of equal length, and it is noon on the first meridian, on what meridian is it then sunrise? Sunset? Midnight?

449. *The following table of the Longitude of places is compiled from the Records of the U. S. Coast Survey.*

Albany	73° 44′ 50″	W.	New York	74° 3′	W.
Ann Arbor	83° 43′	W.	New Orleans	90° 2′ 30″	W.
Astoria, Or.	124°	W.	Paris	2° 20′	E.
Boston	71° 3′ 30″	W.	Rome	12° 27′	E.
Berlin	13° 23′ 45″	E.	Richmond, Va.	77° 25′ 45″	W.
Bombay	72° 54′	E.	San Francisco	122° 26′ 45″	W.
Cincinnati	84° 29′ 31″	W.	St. Paul, Minn.	95° 4′ 55″	W.
Chicago	87° 37′ 45″	W.	St. Louis, Mo.	90° 15′ 15″	W.
Cambridge, Mass.	71° 7′ 40″	W.	Univ. of Virginia	78° 31′ 30″	W.
Jefferson City, Mo.	92° 8′	W.	West Point, N. Y.	73° 57′	W.
Mexico	99° 5′	W.	Washington, D.C.	77° 0′ 15″	W.

450. To find the difference of time between two places, when their longitudes are given.

1. Find the diff. in the time of Cinn. and of St. Paul.

OPERATION.

	95°	4′	55″	Long. of St. P.
	84	29	31	" Cinn.
15)	10°	35′	24″	Diff. of Long.
	42 min.	21 $\frac{3}{5}$ sec.		Diff. of Time.

ANALYSIS.—Since 15° of longitude make a difference of 1 hr. of time and 15′, a difference of 1 min. of time, and 15″, a difference of 1 sec. of time (**446**), there are $\frac{1}{15}$ as many hours, minutes, and seconds of time as there are degrees, minutes, and seconds of longitude.

$$\begin{array}{lrrrll}
 & 10^\circ & 35' & 24'' & & \\
\text{Or,} & & & 4 & \text{min.} & \text{sec.} \\ \cline{2-4}
 & 42\text{ min.} & 21\text{ sec.} & \frac{36}{60} = & 42 & 21\frac{3}{5}
\end{array}$$

Since 1° of long. makes a diff. of 4 min. of time, and 1′ makes a diff. of 4 sec. of time (446), there is a diff. of 4 times as many minutes and seconds of time as there are deg., min., and sec. of long.

2. Find the difference in the time of Ann Arbor, Mich., and of Cambridge, Mass.?*

3. When it is half-past 3 o'clock P.M. at West Point, N. Y., what time is it at Bombay?

RULE.—*Divide the difference of longitude expressed in degrees, minutes, and seconds, by* 15; *the quotient will be the difference of time in hours, minutes, and seconds.* Or,

Multiply the difference of longitude by 4, *and the product will be the difference of time in minutes and seconds, which may be reduced to hours.*

Find the difference in time of

4. Washington, and Rome.
5. Chicago, and Paris.
6. N. Orleans, and N. York.
7. Albany, and Jefferson C'y.
8. Richm'd, and St. Louis.
9. New York, and Mexico.
10. Ann Arbor, and Berlin.
11. Mexico, and San Fran.

12. When it is 6 A.M. at Boston, what time is it at Cincinnati? At Chicago? At St. Louis?

13. When it is 6 P.M. at the University of Va., what time is it at Berlin? At St. Paul? At Astoria, Or.?

14. How much later does the sun rise in New York than in Rome? Than in Paris?

15. In sailing from San Francisco to Bombay, will a chronometer gain or lose time, and how much?

*** Take from the Table the required Longitude of the different places.**

DUODECIMALS.

451. ***Duodecimals*** are fractions of a foot formed by successively dividing by 12; as, $\frac{1}{12}$, $\frac{1}{144}$, $\frac{1}{1728}$, etc.

452. The ***Unit*** of measure is 1 *foot,* which may be a *linear,* a *square,* or a *cubic foot.* The *scale* is uniformly 12.

453. In the duodecimal divisions of a foot, the different orders of units are related as follows:

1′ (inch or prime) = $\frac{1}{12}$ of a foot, or 1 in. Linear Meas.
1″ (second) or $\frac{1}{12}$ of $\frac{1}{12}$ = $\frac{1}{144}$ of a foot, or 1 " Square "
1‴ (third) or $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ = $\frac{1}{1728}$ of a foot, or 1 " Cubic "

Table.

12 Fourths (⁗)	=1 Third . .	1‴	1 *ft.*=12′=144″=1728‴=20736⁗
12 Thirds	=1 Second . .	1″	1′= 12″= 144‴= 1728⁗
12 Seconds	=1 Prime . .	1′	1″= 12‴= 144⁗
12 Primes	=1 Foot . . .	*ft.*	1‴= 12⁗

The marks ′, ″, ‴, ⁗, are called *Indices.*
Duodecimals are used by *artificers* in measuring *surfaces* and *solids.*

ADDITION AND SUBTRACTION.

454. Duodecimals are added and subtracted in the same manner as compound numbers.

WRITTEN EXERCISES.

1. Add 14 ft. 7′ 8″, 16 ft. 3′ 5″, and 21 ft. 9′ 11″.
2. Add 140 ft. 10′ 7″ 9‴, 71 ft. 8″, and 107 ft. 4′ 11″ 3‴.
3. From 54 ft. 9′ 5″ subtract 30 ft. 10′ 8″.

Duodecimals are not much used. The subject is fully treated and applied in "Robinson's Higher Arithmetic."

MULTIPLICATION.

455. In the multiplication of duodecimals, the product of *two* dimensions is *area* or *surface*, and the product of *three* dimensions is *solidity* or *volume*. **(344, 349.)**

WRITTEN EXERCISES.

456. 1. Multiply 9 ft. 8′ by 4 ft. 7′.

OPERATION.

9 ft.	8′	
4 ft.	7′	
5 ft.	7′	8″
38 ft.	8′	
44 ft.	3′	8″

ANALYSIS.—Begin at the right. 8′ × 7′ = 56″ = 4′ 8″. Write the 8″ one place to the right, reserving the 4′ to add to the next product.

Then 9 ft. × 7′ = 63′; 63′ + 4′ = 67′ = 5 ft. 7′, which write in the places of feet and primes.

Next multiply by 4 feet; 8′ × 4 ft. = 32′ = 2 ft. 8′. Write the 8′ in the place of primes, reserving the 2 ft. to add to the next product. Then 9 ft. × 4 ft. = 36 ft.; 36 ft. + 2 ft. = 38 ft., which write in the place of feet. Adding the partial products, the sum equals 44 ft. 3′ 8″, the product required.

2. How many square feet in 4 boards, each 12 ft. 9′ long, and 1 ft. 4′ wide?

RULE.—I. *Write the terms of the multiplier under the corresponding terms of the multiplicand.*

II. *Multiply each term of the multiplicand by each term of the multiplier, beginning with the lowest order of units in each. Reduce each product to higher denominations when possible, and write in their proper places. The sum of the partial products will be the product required.*

3. Multiply 10 ft. 6′ 4″ by 5 ft. 3′ 8″.

4. Find the area of a floor 14 ft. 8′ wide and 16 ft. 5′ long.

5. What are the solid contents of a block of marble 6 ft. 10′ long, 4 ft. 3′ wide, and 1 ft. 9′ thick?

457. SYNOPSIS FOR REVIEW.

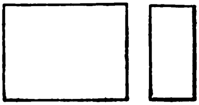
DENOMINATE NUMBERS—CONTINUED.

- 3. REDUCTION.
  - 1. Definition of Reduction.
  - 2. Reduction to Lower Denominations.
    - 1. Principle.
    - 2. Rule, I, II.
  - 3. Reduction to Higher Denominations.
    - 1. Principle.
    - 2. Rule, I, II.
  - 4. Reduction of Denominate Fractions.
    - 430. Rule.
    - 432. Rule, I, II, III.
    - 434. Rule.
    - 436. Rule, I, II, (2.)
- 4. ADDITION OF COMP. NUMBERS.
- 5. SUBTRACTION " "
- 6. MULTIPLICATION " "
- 7. DIVISION. " "
  - 1. How performed.
  - 2. Upon what principles based.
- 8. LONGITUDE AND TIME.
  - 1. Definitions.
    - 1. Longitude.
    - 2. First Meridian.
  - 2. Comparison of Longitude and Time.
  - 3. Rules.
    - 1. To find diff. of long. when diff. of time is given.
    - 2. To find diff. of time when diff. of long. is given.
- 9. DUODECIMALS.
  - 1. Definition.
  - 2. Unit of Measure.
  - 3. Table.
  - 4. Addition and Subtraction.
  - 5. Multiplication.
    - 1. Product of two dimensions.
    - 2. Product of three dimensions.
    - 3. Rule, I, II.

MEASUREMENTS

458. ***Measurements*** involve a practical application of the Weights and Measures to various operations required in the mechanic arts, and to the common business of life.

RECTANGULAR SURFACES.[*]



Rectangle.

459. A ***Rectangle*** is a plane figure bounded by four sides, having all its angles right angles.

It has *two* dimensions—*length and breadth.*

When *all* its sides are equal, it is called a *Square.*

460. The ***Area*** of a rectangle is the surface included within the lines which bound it, and is expressed by the number of times it contains a given *unit of measure.*

461. The ***Unit of Measure*** for surfaces is a *square* each side of which is a unit of some known length.

Thus, the unit of square inches is 1 *square inch;* of square feet, 1 *square foot;* of square yards, 1 *square yard*, etc.

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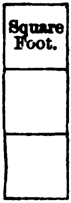
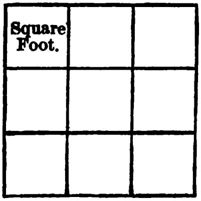
[*] Measurements of plane figures requiring a knowledge of Involution and Evolution are treated at the close of this book under the head of "*Mensuration.*"

The diagram represents a square yard, each side of which is 1 yd. or 3 ft. long, and the whole is divided into square feet, 1 sq. ft. being the *Unit of Measure*. In one row there are 3 sq. ft., in 3 rows there are 3 times 3 sq. ft., or 9 sq. ft. Hence the *area* of 1 sq. yd. is 9 sq. ft.

So the area of a *rectangle* formed by 2 adjacent rows, is expressed by 3 sq. ft. × 2, or 6 sq. ft.





3 sq. ft. × 3 = 9 sq. ft.

462. To find the *area* of a rectangle:

Rule.—*Find the product of the numbers denoting the length and breadth, expressed in the same denomination; the result is the area.*

463. To find either *dimension* of a rectangle:

Rule.—*Divide the area by one dimension; the quotient is the other.*

464. ***Artificers*** compute their work by *linear*, *superficial* or *square*, and *cubic measures*.

1. Glazing and stone-cutting are estimated by the *square foot.*

2. Plastering, paving, ceiling, etc., commonly by the *square foot*, or the *square yard.*

3. Roofing, flooring, partitioning, slating, etc., generally by the *square* of 100 *square feet*; sometimes by the *square foot*, or *square yard.*

4. *One thousand* shingles, averaging 4 in. wide, and laid 5 in. to the weather, are estimated to cover a *square.*

5. Bricklaying is generally estimated by the *thousand bricks.*

WRITTEN EXERCISES.

465. 1. How many square feet in a floor 27 ft. long and 21 ft. wide? How many square yards?

2. How many feet wide is a hall that is 26 ft. long and contains 195 square feet?

3. What is the length of a lawn that contains 305 sq. yd. and is 45 ft. wide?

Find the area of rectangles having the following dimensions:

4. 12⅓ yards square.
5. 18 yd. 2 ft. square.
6. 18⅓ rd. by 20⅓ rd.
7. 5 ch. 14 l. by 6 ch. 25 l.
8. 25 ft. 6 in. by 16 ft. 9 in.
9. 14 yd. 1 ft. 10 in. square.

The area and one dimension given, find the other dimension of the following rectangles:

10. Area 374⅛ square feet, length 20 ft. 6 in.

11. Area 5 A. 41 P., width 7 chains 25 links.

12. Area 180 sq. yd. 4 sq. ft., width 9 yd. 2 ft.

13. How many square yards in the sides of a room 16 ft. long, 12 ft. 6 in. wide, and 9 ft. 3 in. high?

14. How many planks 12 ft. long and 10 in. wide will be required to floor a room which is 24 ft. by 20 ft.?

Find the number of yards in length of carpeting required for rooms of the following dimensions:

15. For a room 24 ft. by 16 ft. 6 in.; carpet 1 yd. wide.

16. For a room 52 ft. by 35 ft.; carpet 2 ft. 4 in. wide.

17. For a room 28 ft. by 23 ft. 9 in.; carpet 30 in. wide.

18. For a room 27 ft. 3 in. by 22 ft. 6 in.; carpet 2 ft. 6 in. wide.

Find the cost of carpeting rooms, their dimensions, and the width and price of carpet being as follows:

19. Floor, 34 ft. by 18 ft. 6 in., carpet 2 ft. wide, at $.94 a yard.

20. Floor, 30 ft. 3 in. by 22 ft., carpet ¾ yd. wide, at $\$1.0\dot{8}$ a yard.

21. Floor, 18⅓ ft. by 16.4 ft., carpet ⅞ yd. wide, at $2⅓ a yard.

22. Floor, 40 ft. by 36 ft., covered with matting 4 ft. wide, at $1.22 a yard.

23. Floor, 26 ft. 6 in. by 18 ft., covered with oil-cloth, at $1.15 a square yard.

24. How many tiles 8 inches square, will lay a floor 48 ft. by 10 ft.?

25. What will be the cost of flagging a side-walk 312 ft. long and 6½ ft. wide, at $2.70 a square yard?

26. What will it cost to cement a cellar bottom 48 ft. 6 in long and 27 ft. wide, at $.45 a square yard?

27. How many squares are there in a partition 104 ft. 9 in. long, and 20 ft. 4 in. high?

28. What is the expense of plastering the sides and ceiling of a room 40 ft. long, 36½ ft. wide, and 22¼ ft. high, at $.36 a square yard, allowing 1375 sq. ft. for doors, windows, and baseboard?

29. Find the cost of glazing 6 windows, each 8 ft. 3′, by 5 ft. 4′, at $.75 a square foot.

30. A room is 24 ft. by 16½ ft., and 18 ft. high. Find the cost of papering its sides with paper 40 in. wide and 8 yd. in a roll, at $1.20 a roll put on, and edging it with gilt moulding next the ceiling, at 9 cents a foot. There are 2 windows, each 2 ft. 4 in. by 5 ft. 8 in., and 2 doors 2 ft. 9 in. by 6 ft. 6 in., and a baseboard 9 in. wide.

31. How many sods, each 16 in. square, will be required to turf a yard 53 ft. 4 in. long and 28 ft. wide?

32. How many yards of silk, ⅝ yd. wide, will be required to line 24 yd. of satin ¾ yd. wide?

33. How many rolls of paper, each 8 yd. long and 18 in. wide, will paper the sides of a room 16 ft. by 14 ft. and 10 ft. high, deducting 124 sq. ft. for doors and windows?

34. Find the cost of lining a tank 5 ft. 8 in. long, 4 ft. wide, and 5 ft. deep, with zinc, weighing 5 lb. to the square foot, at 12 cents a pound, which includes the labor.

35. Find the cost of plastering the walls of a room 12 ft. 11′ square, 9 ft. 3′ high, allowing for 2 windows and 1 door, each 6 ft. 2′ by 2 ft. 4′, at $.28 a square yard.

36. How many shingles 4 in. wide, laid 6 in. to the weather, will cover the roof of a building, the ridge being 46 ft. long, and the girt from eaves to eaves 40 ft., the first course on each of the eaves being double?

37. What will be the cost of wainscoting a room 21 ft. 8 in. by 14 ft. 10 in. and 10 ft. 6 in. high, at $.30 a sq. yd.?

38. Find the cost of slating a roof, 64 ft. 9 in. long and 45 ft. wide, at $15.37½ per square?

LAND.

466. The ***Unit*** of land measure is the *acre*.

Measurements of land are commonly recorded in *square miles*, *acres*, and *hundredths* of an acre. The denomination *rood* is no longer used. See Arts. **341** and **346**.

WRITTEN EXERCISES.

467. 1. How many acres in a farm 120 rods square?

2. A field 80 rd. long contains 16 A.; what is its width?

3. A town 6½ mi. long and 5½ mi. wide is equal to how many farms of 120 A. each?

4. What decimal part of an acre is a piece of land 121 yd. long and 75 feet wide?

5. A rectangular farm containing 435 A. 96 P. is 264 rd. long on one side: what is the length of the other side?

6. What is the value of a farm 189.5 rd. long and 150 rd. wide, at $42¾ an acre?

7. A man having a field 70 rd. square appropriated 5 A. of it to corn, 100 sq. rd. to garden vegetables, and the remainder to meadow. What fractional part of the whole field did the meadow comprise?

8. A rectangular field 50 rd. long contains 10 acres. Another field of the same width contains 5 acres; what is its length?

9. At $2.75 a rod, how much less will it cost to fence a piece of land 80 rd. square than if the same were in the form of a rectangle twice as long and one-half as wide?

10. I bought a piece of land 16 ch. long and 15 ch. wide, at $100 an acre, and dividing it into lots of 6 rd. by 5 rd., sold them at $50 each. What was my gain?

468. ***Government Lands*** are usually surveyed into rectangular tracts, bounded by lines conforming to the cardinal points of the compass.

A *Base-line* on a parallel of latitude, and a *Principal Meridian* intersecting it, are first established. Other lines are then run *six miles* apart, each way, as nearly as possible.

The tracts thus formed are called *Townships,* and contain, as near as may be, 23040 acres.

A line of townships extending north and south is called a *Range.*

The *ranges* are designated by their number east or west of the *principal meridian.*

The *townships* in each range are designated by their number north or south of the *base-line.*

Since the earth's surface is convex, the principal meridians converge as they proceed northward. This tends to throw the townships and sections out of square, and necessitates occasional lines of offset, called "*correction lines.*"

Townships are subdivided into *Sections*, and sections into *Half-Sections, Quarter-Sections, Half-Quarter-Sections, Quarter-Quarter-Sections,* and *Lots.*

Diagram No. 1 shows the sub-divisions of a *Township* into Sections, and how they are *numbered*, commencing at the N. E. corner.

Diagram No. 2 shows the sub-divisions of a *Section*, on an enlarged scale, and how they are *named.*

Diagram No. 1.

A TOWNSHIP

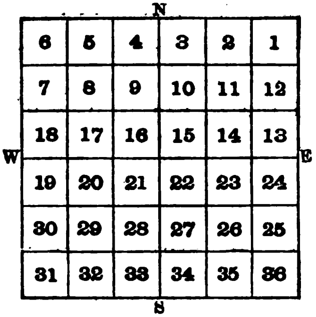


Diagram No. 2.

A SECTION.

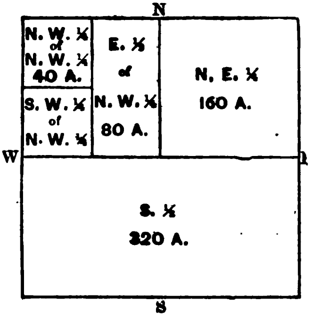


TABLE.

6 mi. × 6 mi. = 36 sq. mi. = 23040 Acres. = 1 Township.
1 " × 1 " = 1 " = 640 " = 1 Section.
1 " × ½ " = ½ " = 320 " = 1 Half-Section.
½ " × ½ " = ¼ " = 160 " = 1 Quarter-Section.
½ " × ¼ " = ⅛ " = 80 " = 1 Half-Quarter-Section.
¼ " × ¼ " = $\frac{1}{16}$ " = 40 " = 1 Quarter-Quarter-Section

A *Lot* is a sub-division of a section, usually of irregular form, on account of bordering upon a navigable river or lake—containing as near as may be the area of a Quarter-Quarter-Section, and described as lot No. 1, 2, 3, etc., of a particular section.

City and village plats are usually sub-divided into *Blocks*, and these into *Lots.*

WRITTEN EXERCISES.

1. If a township of land is equally divided among 288 families, how many acres does each receive? What part of a section?

2. What number of rails will enclose a quarter-section of land with a fence 6 rails high, and 3 lengths for every 2 rods; and what will be the cost of the rails, at $40 per thousand?

3. A man bought the S. ½ of a section of land at $2¼ an acre, and afterward sold the E. ½ of what he bought at $4.37½ an acre. What was his gain?

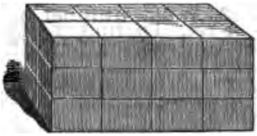
4. If I buy the N.E. ¼ and the E. ½ of N.W. ¼ of a section of land, how many acres do I purchase? What part of a whole section? How are the parts located in respect to each other?

5. A speculator bought of the Ill. Central R. R. Co., the S. ½ of Section 4, township 10 north, range 6 east, at $2 an acre. He afterward sold the E. ½ of S.E. ¼ at $2.75 an acre; the N.W. ¼ of S.E. ¼ at $3½ an acre; and the N. ½ of S.W. ¼ at $3.84 an acre. How many acres has he left? What was his gain on the purchase price of the whole? Draw diagram.

6. A man having purchased a section of land from the U. S. Government at $1.25 an acre, sold the S. ½ of S.W. ¼ at $2.50 an acre; the N.E. ¼ of N.W. ¼ at $1.75 an acre; the W. ½ of S.E. ¼ at $2 an acre; and the W. ½ of S.W. ¼ of N.E. ¼ at $3 an acre. How many acres has he remaining, and what is his gain, provided the remainder is sold at $2¼ an acre? Draw diagram and explain.

RECTANGULAR SOLIDS.

469. A ***Rectangular Solid*** is a body bounded by six rectangular plane *faces*.



The opposite sides are equal and parallel.

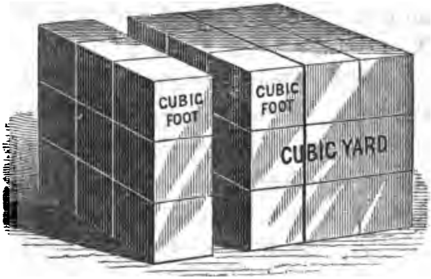
It has *three* dimensions—*length, breadth,* and *thickness.*

When *all* its faces are equal, it is called a *Cube.*

470. The ***Volume*** or ***Solid Contents*** of a body is the space included within the surfaces which bound it, and is expressed by the number of times it contains a given *unit of measure.*

471. The ***Unit of Measure*** for solids is a *cube,* the edge of which is a unit of some known length.

Thus, the unit of cubic inches is a *cube* the edge of which is 1 *inch,* or 1 *cubic inch;* of cubic feet, 1 *cubic foot,* etc.



9 cu. ft. × 3 = 27 cu. ft.

The diagram represents a cubic yard, each face being a square yard, containing 9 sq. ft. If a section 1 ft. thick is cut from one side, it may be divided into 3 times 3 cu. ft., or 9 cu. ft., 1 cu. ft. being the *unit of measure.* And since the cubic yard is 3 ft. thick, it contains 3 such sections, or 3 times 9 cu. ft., which are 27 cu. ft. Hence the *volume* of 1 cu. yd. is 27 cu. ft.

So the volume of a *solid,* formed of two adjacent sections, is expressed by 3 cu. ft. × 3 × 2 = 18 cu. ft.

472. To find the *volume* of a rectangular solid

RULE.—*Find the product of the numbers denoting the three dimensions, expressed in the same denomination; this result is the volume.*

473. To find a *required dimension* of a rectangular solid:

RULE.—*Divide the volume by the product of the numbers denoting the other two dimensions; the quotient will be the required dimension.*

WRITTEN EXERCISES.

474. 1. What are the contents of a rectangular solid 6 ft. long and 4 ft. square?

2. What is the volume of a solid 9 ft. long, 4 ft. wide, and 3 ft. thick?

3. A vat 12 ft. square contains 1224 cu. ft. What is its depth?

4. What is the volume of a bin, the inside dimensions of which are 8 ft. 6 in. by 6 ft. by 4 ft. 4 in.?

5. How many cubic yards of earth must be removed in digging a cellar 36 ft. long, 24 ft. wide, and 6½ ft. deep?

Find the volume of rectangular solids having the following dimensions:

6. Of a cube the edge of which is 1 yd. 1 ft. 9 in.

7. Of a solid 6 yd. 2 ft. 7 in. by 3 ft. 4 in. by 2 ft. 11 in.

8. Of a solid 5 ft. square and the height 6.4 ft.

Find the required dimension of rectangular solids, the volumes and two dimensions being as follows:

9. Volume, 6 cu. ft.; length, 8 ft.; width, 8 ft.

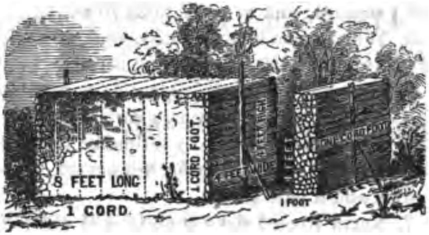
10. Volume, 20 cu. ft.; length, 36 ft.; width, 10 in.

11. Volume, 13 cu. yd. 14 cu. ft. 900 cu. in.; width, 7 ft. 3 in.; height, 5 ft. 6 in.

12. How many cubic feet of air in a room that is 24 ft. 9 in. long, 18 ft. 4 in. wide, and 10 ft. 8 in. high?

A *Cord* of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high.

A *cord-foot* is 1 foot in length of such a pile; that is, 1 ft. long, 4 ft. wide, and 4 ft. high.



13. How many cords in a pile of wood 30 ft. long, 8 ft. wide, and 6 ft. 6 in. high?

14. A pile of wood containing 67½ cords, is 90 ft. long and 12 ft. wide. How high is it?

15. What will be the cost of a pile of wood 12 ft. 6 in. long, 8 ft. wide, and 4 ft. 6 in. high, at $3.75 a cord?

16. What will it cost to dig a cellar 45 ft. long, 28 ft. wide, and 8 ft. 6 in. deep, at $.42 a cubic yard?

17. What must be the length of a load of wood that is 3 ft. high and 5 ft. 4 in. wide, to contain a cord?

18. How many cans, 8 in. by 6 in. by 3 in., can be packed in a box 32 in. by 24 in. by 15 in. in the clear?

19. At $3½ a cord, what is the value of the wood that can be piled under a shed 50 ft. long, 25 ft. wide, and 12 ft. high?

20. In building a house, 200 joists 10 in. by 3 in. were used, which together amounted to 1000 cu. ft. What was the length of each?

475. ***Masonry*** is estimated by the *cubic foot,* and by the *perch;* also by the *square foot* and the *square yard.*

1. *Materials* are usually estimated by cubic measure; the *work* by cubic or square measure.

2. A *Perch* of stone, or of masonry, is 16½ ft. long, 1½ ft. wide, and 1 ft. high, and is equal to 24.75 cu. ft.

3. When stone is built into a wall, 22 cu. ft. make a perch, 2¾ cu. ft. being allowed for mortar and filling.

4. *Embankments* and *Excavations* are estimated by the *cubic yard.*

5. A cubic yard of common earth is called a *load.*

6. *Brickwork* is generally estimated by the *thousand bricks;* sometimes in *cubic feet.* In *walls,* brick-work is estimated at the rate of a brick and a half thick.

7. North River bricks are 8 in. × 3½ × 2¼; Maine bricks are 7½ in. × 3⅜ × 2⅜; Philadelphia and Baltimore bricks are 8¼ in. × 4⅛ × 2⅜; and Milwaukee bricks 8½ in. × 4⅛ × 2⅜.

8. In estimating *material,* allowance is made for doors, windows, and cornices.

9. In estimating the *work,* masons measure each wall on the *outside,* and ordinarily, no allowance is made for doors, windows, and corners; but sometimes an allowance of *one-half* is made, this being, however, a matter of *contract.*

476. To find the number of bricks in a cubic foot of masonry:

RULE.—I. *Add to the face dimensions of the kind of bricks used the thickness of the mortar or cement in which they are laid, and compute the area.*

II. *Multiply this area by the quotient of the thickness of the wall divided by the nnmber of bricks of which it is composed, the product will be the volume of a brick and its mortar in cubic inches.*

III. *Divide* 1728 *by this volume, and the quotient will be the number of bricks in a cubic foot.*

WRITTEN EXERCISES.

477. 1. How many Milwaukee bricks in a cubic foot of wall 12¾ in. wide, laid in courses of mortar ¼ of an inch thick?

OPERATION.

8.5 + .25 = 8.75 in. = *length* of brick and joint.

2.375 + .25 = 2.625 in. = *thickness* of brick and joint.

8.75 × 2.625 = 22.96875 sq. in. = *area* of its face.

12.75 ÷ 3 (number of bricks in width of wall) = 4.25 in. = *width* of brick and mortar.

22.96875 × 4.25 = 97.617 + = *cubic inches* in a brick.

1728 ÷ 97.617 + = 17.7 + = *number of bricks* in a cubic foot.

2. How many bricks, 8 in. × 4 × 2, will be required to build a wall 42 ft. long, 24 ft. high, and 16½ in. thick, laid in courses of mortar ¼ of an inch thick?

3. How many perches of stone, laid dry, will build a wall 60 ft. long, 16½ ft. high, and 18 in. thick?

RULES.—1. *Multiply the number of cubic feet in the wall, or work to be done, by the number of bricks in a cubic foot; the product will be the number of bricks required.*

2. *Divide the number of cubic feet in the work to be done by* 24.75; *the quotient will be the number of perches.*

4. How many perches of masonry in a wall 120 ft. long, 6 ft. 9 in. high, and 18 in. thick?

5. How many bricks in the four walls of a square house 36 ft. long, 24 ft. high, and 12⅜ in. thick, allowing 224 cu. ft. for doors and windows, *one half* for the corners, and ¼ of an inch for each course of mortar?

6. At $.56 a cu. yd., what will it cost to remove an embankment 240 ft. long, 38 ft. wide, and 8.5 ft. high?

7. Find the cost of digging and walling the cellar of a house, whose length is 41 ft. 3 in., and width 33 ft.; the cellar to be 8 ft. deep, and the wall 1½ ft. thick. The excavating will cost $.50 a load, and the stone and mason work $3.75 a perch.

8. How many perches of stone will be required to enclose a lot 16 rd. long and 12 rd. wide, with a wall 6 ft. high and 3 ft. thick, allowing one-half for the corners?

9. A street 650 ft. long and 72 ft. wide, averages 4.5 ft. below grade. Find the cost of filling it in, at $.42 a cu. yd.?

10. What will be the cost of building a wall 60 ft. long, 21⅞ ft. high, and 17 in. thick, of Philadelphia bricks, laid in courses of mortar ¼ of an inch thick, at $12½ per M.?

11. How many cubic feet of masonry in the wall of a cellar 37½ ft. long, 26 ft. wide, and 9 ft. deep, the wall being 2 ft. thick, allowing *one-half* for the corners; and what will be the cost, at $3.85 a perch?

BOARDS AND TIMBER.

478. A ***Board Foot*** is 1 ft. long, 1 ft. wide, and 1 inch thick. Hence 12 board feet make 1 cubic foot.

Board feet are changed to cu. ft. by dividing by 12, and cubic feet are changed to board feet by multiplying by 12.

1. In *Board Measure* all boards are assumed to be 1 in. thick.
2. *Lumber* and *Sawed Timber*, such as plank, scantling, joists, etc., are usually estimated in *board* measure.
3. *Hewn* and *Round Timber* are commonly estimated in *cubic* measure.

479. When lumber is not more than 1 inch thick:

RULE.—*Multiply the length in feet by the width in inches, and divide the product by* 12.

480. When more than 1 inch thick:

RULE.—*Multiply the length in feet by the width and thickness in inches, and divide the product by* 12.

1. If *one* of the dimensions is *inches*, and the other two are *feet*, the product will be *board feet*.

2. If a board tapers regularly, multiply the length by the *mean width*, found by taking half the sum of the two ends.

WRITTEN EXERCISES.

481. 1. Find the contents of a board 15 ft. long and 8 in. wide.

OPERATION.—$\overline{15 \times 8} \div 12 = 10$ board feet.

2. What are the contents in board measure of a joist 16 ft. long, 10 in. wide, and 3 in. thick?

OPERATION.—$\overline{3 \times 10 \times 16} \div 12 = 40$ board feet.

3. How many board feet in 4 boards 16 ft. long, 10 in. wide?

4. How many board feet in 2 joists 17 ft. long, 11 in. wide, and 3 in. thick?

5. Find the contents of a board 18 ft. long, 1 ft. 8 in. wide at one end, and 14 in. at the other.

OPERATION.—$\overline{20\text{ in.}} + \overline{14\text{ in.}} \div 2 = 17$ in; $\overline{18 \times 17} \div 12 = 25\frac{1}{2}$ board ft.

6. Find the cost of 5 boards 12 ft. long, 17 in. wide at one end and 11 in. at the other, at 6 cents a square foot.

7 Find the cost of 10 planks, each 15 ft. long, 16 in. wide, and 3½ in. thick, at $2.25 per hundred feet.

8. What length of board 9 in. wide contains 8 board ft.?

OPERATION.—$\overline{144 \times 8} \div 9 = 128$; $128 \div 12 = 10\frac{2}{3}$ ft., the length.

9. What length may be cut from a board 15 ft. long and 20 in. wide, so as to leave 15 board feet?

10. What must be the width of a board 16 ft. long to contain 12 board feet?

OPERATION.—16 ft.=192 in.; $\overline{144 \times 12} \div 192 = 9$ in., the width.

11. What must be the width of a piece of board 5 ft. 3 in. long, to contain 7 square feet?

12. Find the cost of 3 pieces of timber, each 26 ft. long and 6 in. by 9 in., at $1.75 per hundred board feet.

13. Find the cost of 8 pieces of scantling, 3 in. by 4 in. and 14 ft. long, at $9.50 per thousand board feet.

14. What length of a piece of timber 6 in. by 9 in., will contain 3 cubic feet?

OPERATION.—$\overline{1728 \times 3} \div \overline{9 \times 6} = 96$; $96 \div 12 = 8$ ft., the length.

15. A piece of timber is 10 in. by 16 in.; what length of it will contain 15 cubic feet?

16. What amount of inch lumber will make a box 4 ft. by 3 ft. 6 in. by 2 ft. 6 in. on the outside?

Find the cost of the following:

17. Of 36 boards, 12 ft. long, 11 in. wide, @ $2¼ per C.

18. Of 16 planks, 14½ feet long, 10 in. wide, and 3 in. thick, @ $16½ per M.

19. How many board feet in a stick of timber 36 ft. long, 10 in. thick, 12 in. wide at one end and 9 in. wide at the other end? How many cubic feet?

20. Make a bill for lumber bought by John Osborn of Geo. Mason & Co., of St. Paul, Sept. 20, 1875, as follows:

124 boards,	10 in.	by 16 ft.	@ $15	per M.
120 "	16 "	" 14 "	" $16½	"
40 planks,	2½ × 12	" 15 "	" $18.75	"
96 joists,	3 × 10	" 18 "	" $14	"
60 scantling,	3 × 4	" 12 "	" $12½	"

What is the amount?

21. Find the cost of the flooring for a two-story house at $30 per M., it being 1¼ in. thick, each floor being 48 ft. by 25 ft., no allowance made for waste.

22. A rectangular field, 16 ch. long and 8 ch. wide, is enclosed by a post and board fence; the posts are set 8 ft. apart, the boards are 16 ft. long, and the fence is 5 boards high. The bottom board is 12 in. wide, the top board 6 in., and the other three each 9 in. wide. The posts cost $25 per C., and the boards $14.80 per M. Required the number of posts, the amount of lumber, and the cost of both.

CAPACITY OF BINS, CISTERNS, ETC.

482. The ***Standard Bushel*** of the United States contains 2150.42 cu. in., and is a cylindrical measure 18½ in. in diameter and 8 in. deep.

1. Measures of capacity are all *cubic* measures, solidity and capacity being measured by different units, as seen in the tables.

2. The *Imperial Bushel* of Great Britain contains 2216.192 cu. in.

3. The *English Quarter* contains 8 Imp. bushels, or 8¼ U. S. bu.

4. Grain is shipped from New York by the *Quarter* of 480 lb. (8 U. S. bu.), or by the *ton* of 33⅓ U. S. bushels.

5. A *Register Ton*, used in measuring the entire internal capacity or *tonnage* of vessels, is 100 cubic feet.

6. A *Shipping Ton*, used in measuring *cargoes*, is 40 cubic feet in the U. S., and in England 42 cubic feet.

7. The bushel *heaped measure* is the Winchester bushel heaped in the form of a cone, which cone must be 19½ in. in diameter, and at least 6 in. high.

8. Grain, seeds, and small fruits are sold by *stricken measure.*

9. Corn in the ear, potatoes, coal, large fruits, coarse vegetables, and other bulky articles are sold by *heaped measure.*

10. It is sufficiently accurate in practice to call 5 *stricken* measures equal to 4 *heaped* measures.

483. To find the *exact capacity* of a bin in bushels.

RULE.—*Divide the contents in cubic inches by* 2150.42; *the quotient will represent the number of bushels.*

484. To find the *cubic* contents in a given number of bushels.

RULE.—1. *Multiply the number of bushels by* 2150.42; *the product will be the number of cubic inches, which may be reduced to higher denominations if required.*

Since a standard bushel contains 2150.42 cu. in., and a cubic foot contains 1728 cu. in., a bu. is to a cu. ft. nearly as 5 to 4; or a bu. is equal to 1¼ cu. ft., nearly. Hence for all practical purposes,

2. *Any number of cubic feet diminished by* ⅕ *will represent an equivalent number of bushels.*

Thus, 250 cu. ft. − ⅕ of 250 cu. ft., or 50 cu. ft. = 200, the number of bushels in 250 cu. ft.

3. *Any number of bushels increased by* ¼, *will represent an equivalent number of cubic feet.*

Thus, 200 bu. + ¼ of 200 bu., or 50 bu. = 250, the number of cubic feet in 200 bushels.

WRITTEN EXERCISES.

485. 1. A bin is 6 ft. long, 5 ft. wide, and 4 ft. deep. How many bushels will it hold?

2. A rectangular box will hold 128 bu. What is its volume in cubic feet?

3. How many bushels of wheat can be put in a bin 8 ft. long 6 ft. 6 in. wide, and 3 ft. 4 in. deep?

4. What must be the depth of a bin to contain 240 bu., its length being 10 ft. and its width 5 ft.?

OPERATION.—240 bu. + 60 bu. = 300; $300 \div \overline{10 \times 5} = 6$ ft., the depth.

RULE.—*Divide the contents in cubic feet or inches by the product of the two dimensions, in the same denomination.*

5. What must be the length of a bin that is 6 ft. wide and 4½ feet deep, to contain 324 bushels?

6. What must be the width of a bin 12 ft. long and 10 ft. deep, to contain 900 bushels of shelled corn?

7. A bin that holds 100.8 bu. is 7 ft. long and 6 ft deep. How wide is it?

8. How many bushels will fill a bin that is 8.5 ft. long, 4.25 ft. wide, and 3¾ ft. deep?

9. A bin 10 ft. long, 6 ft. wide, and 4 ft. deep, will hold how many bushels of oats? Of potatoes?

10. How many bushels of apples will a wagon-box hold, that is 12 ft. long, 3 ft. wide, and 2 ft. 6 in. deep? How many bushels of barley?

11. A bin 20 ft. long, 12 ft. wide, and 5 ft. deep, is full of wheat. What is its value at $2 a bushel?

12. A bin 7 ft. long, 6 ft. wide, and 5 ft. deep is ¾ full of rye. What is its value at $1.37½ a bushel?

13. A farmer's entire crop of barley just filled a bin 10 ft. long, 6 ft. wide, and 5 ft. deep. What was its value, at $1.78 per cental?

14. A crib, the inside dimensions of which are 15 ft. long, 7 ft. 4 in. wide, and 8 ft. high, is full of corn in the ear. If 2 bu. of ears make 1 bu. of shelled corn, what is the value of the whole, when shelled, at $.92 a bushel?

15. If one bushel or 60 lb. of wheat make 48 lb. of flour, how many barrels of flour can be made from the contents of a bin 10 ft. long, 5 ft. wide, and 4 ft. deep, filled with wheat?

16. How many tons of ice can be packed in a building 40 ft. long, 30 ft. wide, and 20 ft. high, a cubic foot of ice weighing 58½ pounds?

17. John Wheatley & Co. bought 12400 bu. of wheat, delivered in New York, at $1.50 a bushel. They shipped the same to Liverpool, paying 6s. sterling per quarter freight, and sold the entire cargo at 12s. per cental. Making no allowance for exchange or for waste, what was the gross gain in U. S. money.

1. *Coal.* Ordinary anthracite coal measures from 36 to 40 cu. ft. to the ton; bituminous coal, from 36 to 45 cu. ft. to the ton.

2. Lehigh, white ash, egg size, measures about 34½ cu. ft. to the ton (2000 lb.); Schuylkill, white ash, 35 cu. ft., and of gray or red ash, 36 cu. ft. to the ton.

3. Coal is bought and sold in large quantities by the ton; in small quantities by the bushel, the conventional rate being 28 bu. (5 pecks) to a ton, or about 43.5 cu. ft.

18. How many tons of red ash coal, egg size, will a bin 17 ft. long, 6 ft. wide, and 3 ft. deep, contain?

19. A bin 6 ft. long, 4 ft. deep, and 5 ft. 9 in. wide is full of Lehigh white ash coal. Find its value at $6.75 a ton.

20. A large crib 10 yd. long, 6 yd. wide, and 6 yd. deep is filled with Schuylkill red ash coal. Find the number of tons it contains, and its value at $5¼ a ton?

21. A bin 7 ft. long, 5 ft. wide, and 5 ft. deep is half-full of Schuylkill white ash coal. Find its value at $5.90 a ton.

486. The ***Standard Liquid Gallon*** of the United States contains 231 cu. in., and is equal to about 8⅓ lb. Avoir. of pure water.

1. The half-peck, or *dry gallon*, contains 268.8 cubic inches.

2. *Six dry* gallons are equal to nearly *seven liquid* gallons.

3. The *Imperial Gallon* of Great Britain contains 277.274 cu. in., and is equal to about 1.2 U. S. Liquid Gallons.

487. COMPARATIVE TABLE OF MEASURES OF CAPACITY.

	Cubic in. in one gallon.	Cubic in. in one quart.	Cubic in. in one pint.	Cubic in. in one gill.
Liquid Measure	231	$57\frac{3}{4}$	$28\frac{7}{8}$	$7\frac{7}{32}$
Dry Measure ($\frac{1}{8}$ pk.) . .	$268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$	$8\frac{2}{5}$

A *cubic foot* of pure water weighs 1000 oz., equals $62\frac{1}{2}$ lb. Avoir.

488. To find the *exact* capacity of a vessel or space in gallons:

RULE.—*Divide the contents in cubic inches by* 231 *for liquid gallons, or by* 268.8 *for dry gallons.*

489. To reduce gallons to cubic inches:

RULE.—*Multiply the given number of liquid gallons by* 231; *then reduce to higher denominations if required.*

WRITTEN EXERCISES.

490. 1. How many gallons of water will a cistern hold, that is 4 ft. square and 6 ft. deep?

OPERATION.—$(4 \times 4 \times 6 \times 1728) \div 231 = 718\frac{10}{77}$ gal.

2. How many gallons will a tank 4 ft. long, 3 ft. wide, and 1 ft. 8 in. deep contain?

3. How many barrels of water will a vat hold that contains 43659 cubic inches?

4. How many cubic feet in a space that holds 48 hhd.?

5. How many hogsheads will a cistern 11 ft. long, 6 ft wide, and 7 ft. deep contain?

6. Find the weight of water in a bath-tub 6 ft. long, 3 ft. wide, and 1 ft. 9 in. deep.

7. How many gallons will a space contain that is 22.5 ft. long, 3.25 ft. wide, and 6.4 ft. deep?

8. A man constructed a cistern to hold 32 hhd., the bottom being 6 ft. by 8 ft. What was its depth?

9. How many more cubic inches in 189.5 gallons dry measure than in 189.5 gallons liquid measure?

10. Find the number of gallons in a cubic foot, correct to 4 decimal places.

11. A tank in the attic of a house is 6 ft. 6 in. long, 4 ft. wide, and 3 ft. 6 in. deep. How many gallons of water will it hold, and what will be its weight?

12. If 64 quarts of water are put into a vessel that will exactly hold 64 quarts of wheat, how much will the vessel lack of being full?

13. If a man buy 10 bu. of chestnuts at $5 a bushel, dry measure, and sell the same at 25 cents a quart, liquid measure, how much does he gain?

14. A cistern 5 ft. by 4 ft. by 3 ft. is full of water. If it is emptied by a pipe in 1 hr. 30 min., how many gallons are discharged through the pipe in a minute?

15. A vat that will hold 5000 gallons of water will hold how many bushels of corn?

16. A tank is 7 ft. deep, 4 yd. long, and 3 yd. broad. What weight of water is in it when just *half-full?*

17. A cellar 40 ft. long, 20 ft. wide, and 8 ft. deep is half-full of water. What will be the cost of pumping it out, at 6 cents a hogshead?

18. A reservoir 24 ft. 8 in. long by 12 ft. 9 in. wide is full of water. How many cubic feet must be drawn off to sink the surface 1 foot? How many gallons?

19. How many imperial gallons will a cistern contain that is 7 ft. 3 in. long, 3 ft. 8 in. deep, and 2 ft. 10 in. wide?

491. The ***Avoirdupois Pound*** contains 7000 Troy grains; hence the Troy pound is $\frac{5760}{7000} = \frac{144}{175}$ of an Avoir. pound; but the Troy ounce contains $\frac{5760}{12} = 480$ grains, and the Avoir. ounce, $\frac{7000}{16} = 437.5$ grains.

The pound, ounce, and grain of Apothecaries' Weight are the same as those of Troy Weight, the *ounce* being *differently divided.*

492. COMPARATIVE TABLE OF WEIGHTS.

		Troy.		Avoirdupois.		Apothecaries'.
1 pound	=	5760 grains	=	7000 grains	=	5760 grains.
1 ounce	=	480 "	=	437.5 "	=	480 "
		175 pounds	=	144 pounds	=	175 pounds.

WRITTEN EXERCISES.

493. 1. Change 10 lb. 8 oz. Avoir. weight to Troy.

OPERATION.—10 lb. 8 oz. = 168 oz.; 168 oz. × 437.5 = 73500 gr., and 73500 gr. ÷ 480 = $153\frac{1}{8}$ oz. = 12 lb. $9\frac{1}{8}$ oz. Troy.

2. Change 15 lb. 10 oz. 12 pwt. Troy to Avoirdupois.

OPERATION.—15 lb. 10 oz. 12 pwt. = 190.6 oz.; 190.6 oz. × 480 = 91488 gr.; and 91488 gr. ÷ 437.5 = $209\frac{101}{875}$ oz = 13 lb. $1\frac{101}{875}$ oz.

3. Find the value in Troy weight, of 9 lb. 10 oz. Avoir.

4. What is the value in Avoirdupois weight, of 16 lb. 8 oz. 10 pwt. 12 gr. Troy?

5. What is the value of a coffee urn, weighing 2 lb. 14 oz. Avoir., at $1.80 per ounce Troy?

6. How many ounces of gold are equal in weight to 6 lb. of lead?

7. If 8 lb. Avoir. of drugs are bought for $$12\frac{1}{2}$ a pound, and retailed at the rate of $16.25 a pound, Apothecaries' weight, what is the gain on the whole?

8. What is the difference in the weight of $42\frac{3}{8}$ lb. of iron and 42.375 lb. of gold?

494. SYNOPSIS FOR REVIEW.

DENOMINATE NUMBERS—CONTINUED. MEASUREMENTS.

- 1. RECTANGULAR SURFACES.
  - 1. Rectangle.
  - 2. Area of Rectangle.
  - 3. Unit of Measure.
  - 4. Rules.
    - 1. *To find area of rect.*
    - 2. *To find either dimen.*
- 2. ARTIFICERS' WORK. How computed.
- 3. GOVERNMENT LANDS.
  - Range.
  - Township.
  - Sub-divisions.
  - *Table.*
- 4. RECTANGULAR SOLIDS.
  - 1. Definitions.
    - 1. *Rect. Solid.*
    - 2. *Volume.*
  - 2. Unit of Measure.
  - 3. Rules.
    - 1. *To find volume of a rectangular solid.*
    - 2. *To find a required dimension.*
- 5. MASONRY.
  - 1. How estimated.
  - 2. Rules.
    - 1. *To find number of bricks in a cu. ft. of masonry.*
    - 2. *To find number of Pch. of stone in a given work.*
- 6. BOARDS AND TIMBER.
  - 1. Definition of board foot.
  - 2. Rules, 1, 2.
- 7. CAPACITY OF BINS, ETC.
  - 1. Standard Bushel of U. S.
  - 2. Rules.
    - 1. *To find capacity of bins.*
    - 2. *To find cu. contents of a given number of gal.*
    - 3. *To find either dimen.*
  - 3. Standard Liquid Gallon of U. S.
  - 4. Comp. Table of Meas. of Capacity.
  - 5. Rules.
    - 1. *To find capacity of a vessel in gallons.*
    - 2. *To reduce gal. to cu. in.*
    - 3. *To reduce cu. ft. to bu.*
    - 4. *To reduce bu. to cu. ft.*
    - 5. *To find 1 dimen. of a bin.*

PERCENTAGE

ORAL EXERCISES.

495. 1. What is $\frac{1}{100}$ of \$100? $\frac{5}{100}$? $\frac{25}{100}$? $\frac{18}{100}$?

2. What is $\frac{3}{100}$ of \$500? Of \$700? Of \$1000?

3. What is $\frac{5}{100}$ of \$600? $\frac{7}{100}$? $\frac{21}{100}$? $\frac{30}{100}$?

4. How many *hundredths* of \$100 are \$5? \$7? \$18?

5. How many hundredths of \$500 are \$25? \$35? \$50?

6. How many hundredths of any number is $\frac{1}{2}$ of it? $\frac{1}{5}$?

496. ***Percentage*** is a term applied to computations in which 100 is employed as a *fixed measure*, or standard.

The term *percentage* is also used to denote the result found by applying that standard to any given number.

497. ***Per Cent.*** is an abbreviation of the Latin phrase *per centum*, which signifies *by the hundred.*

Thus, 5 *per cent.* means 5 of every 100, or $\frac{5}{100}$, the five standing for the numerator, and the words "*per cent.*" for the denominator 100.

The phrase "*per cent.*," wherever it occurs, should invariably suggest to the mind a *decimal* to the *hundredths* place. Thus, 25 *per cent.* = $\frac{25}{100}$ or .25.

498. The ***Sign of Per Cent.*** is %. It is read *per cent.* Thus 6% is read 6 per cent.

6 per cent., 6%, $\frac{6}{100}$, and .06 are equivalent expressions; the first two are used in the statement of questions, the other two in performing the operations.

7. How many *hundredths* of a number is 7% of it? 9%? $3\frac{1}{3}$%? 15%? $8\frac{2}{3}$%? $6\frac{3}{4}$%? 25%? $18\frac{4}{5}$%? 45%?

8. What *per cent.* of a number is $\frac{4}{100}$ of it? $\frac{7}{100}$? .08? .12$\frac{1}{2}$? $\frac{26}{100}$? $\frac{35}{100}$? .025? .00$\frac{1}{2}$? .04$\frac{3}{4}$? .375? .0325?

499. What *per cent.* of a number is $\frac{1}{3}$ of it?

Analysis.—Since the whole of any number is $\frac{100}{100}$, $\frac{1}{3}$ of the same is $\frac{1}{3}$ of $\frac{100}{100}$, or $\frac{33\frac{1}{3}}{100}$, equal to 33$\frac{1}{3}$%. Hence, etc.

What % of a number is $\frac{1}{2}$ of it? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{8}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{2}{3}$? $\frac{5}{6}$? $\frac{9}{20}$? $\frac{12}{25}$? $\frac{7}{8}$? $\frac{9}{40}$? $\frac{27}{50}$?

500. What *fractional* part of a number is 12$\frac{1}{2}$% of it?

Analysis.—12$\frac{1}{2}$% is $\frac{12\frac{1}{2}}{100}$, or $\frac{25}{200}$, equal to $\frac{1}{8}$. Hence, etc.

What part of a number is 8$\frac{1}{3}$% of it? 16$\frac{2}{3}$%? 15%? 20%? 37$\frac{1}{2}$%? 7$\frac{1}{2}$%? 6$\frac{1}{4}$%? 25%? 66$\frac{2}{3}$%? 75%?

501. What *part* of a number is $\frac{1}{5}$% of it?

Analysis.—$\frac{1}{5}$% is $\frac{\frac{1}{5}}{100}$, equal to $\frac{1}{500}$. Hence, etc.

What is $\frac{1}{2}$% of a number? $\frac{1}{3}$%? $\frac{5}{8}$%? $\frac{7}{16}$%? $\frac{3}{4}$%? $\frac{4}{5}$%? $\frac{7}{10}$%? $\frac{5}{6}$%? $\frac{11}{12}$%? $\frac{9}{20}$%?

502. Any *per cent.* may be expressed either as a *decimal* or as a fraction, as shown in the following

Table.

Per cent.	Decimal.	Fraction.	Per cent.	Decimal.	Fraction.
1%	.01	$\frac{1}{100}$	75%	.75	$\frac{3}{4}$
2%	.02	$\frac{1}{50}$	100%	1.00	
4%	.04	$\frac{1}{25}$	125%	1.25	1$\frac{1}{4}$
6%	.06	$\frac{3}{50}$	$\frac{1}{2}$%	.005	$\frac{1}{200}$
10%	.10, or .1	$\frac{1}{10}$	$\frac{3}{4}$%	.0075	$\frac{3}{400}$
20%	.20, or .2	$\frac{1}{5}$	8$\frac{1}{3}$%	.08$\frac{1}{3}$	$\frac{1}{12}$
25%	.25	$\frac{1}{4}$	12$\frac{1}{2}$%	.125	$\frac{1}{8}$
50%	.50	$\frac{1}{2}$	16$\frac{1}{4}$%	.1625	$\frac{13}{80}$

WRITTEN EXERCISES.

503. Change to expressions having the *per cent.* sign.

1. $.15$; $.085$; $.33\frac{1}{3}$; $.375$; $.00\frac{7}{8}$; $1\frac{1}{3}$; $1\frac{1}{8}$; $.75\frac{3}{4}$.

2. $2\frac{1}{4}$; $\frac{7}{16}$; $.00\frac{4}{5}$; $\frac{16}{40}$; $\frac{5}{8}$; $\frac{7}{12}$; $.00125$; $\frac{3}{4}$; $2\frac{3}{4}$.

Change to the form of *decimals*,

3. $5\frac{1}{2}\%$; $9\frac{1}{4}\%$; 20%; $3\frac{1}{8}\%$; $\frac{4}{5}\%$; $\frac{7}{16}\%$; $1\frac{3}{4}\%$; $112\frac{1}{2}\%$.

Change to the form of *fractions*,

4. 24%; $\frac{3}{4}\%$; $6\frac{1}{4}\%$; $37\frac{1}{2}\%$; $\frac{5}{8}\%$; $3\frac{1}{4}\%$; 120%; 75%.

504. In the applications of percentage, at least three elements are considered, viz.: the *Rate*, the *Base*, and the *Percentage*. Any two being given, the other can be found.

505. The ***Rate*** is the number per cent. or the number of *hundredths*. Thus, in 5%, .05 is the *rate*. Hence,

Rate per cent. is the decimal which denotes how many *hundredths* of a number are to be taken or expressed.

506. The ***Base*** is the number of which the per cent. is taken.

Thus, in the expression, 5% of \$15, the *base* is \$15.

507. The ***Percentage*** is the result obtained by taking a certain per cent. of the base.

Thus, in the statement, 6% of \$50 is \$3, the rate is .06, the base \$50, and the *percentage* is \$3.

508. The ***Amount*** is the sum of the base and the percentage.

Thus, if the base is \$80, and the percentage \$5, the *amount* is \$80 + \$5 = \$85.

509. The ***Difference*** is the remainder found by subtracting the percentage from the base.

Thus, if the base is \$80, and the percentage \$5, the *difference* is \$80 − \$5 = \$75.

510. The base and rate being given to find the percentage.

ORAL EXERCISES.

1. What is 10% of 140?

Analysis.—10% is $\frac{10}{100}=\frac{1}{10}$, and $\frac{1}{10}$ of 140 is 14. Hence 10% of 140 is 14.

What is
2. 5% of $80?
3. 7% of 200 lb.?
4. 6% of 150 men?
5. 25% of 120 mi.?

How much is
6. 12½% of 72 gal.?
7. 40% of 60 sheep?
8. 8% of 50 bu.?
9. 50% of $240?

Find the amount
10. Of 100 A. + 27%.
11. Of $75 + 5%.
12. Of 32 doz. + 12½%.

Find the difference
13. Of 90 hhd. — 10%.
14. Of 63 Cd. — 33⅓%.
15. Of $200 — 2½%.

16. A farmer had 150 sheep, and sold 20% of them. How many had he left?

17. A mechanic who received $20 a week had his salary increased 8%. What were his daily wages then?

18. From a hhd. of molasses 33⅓% was drawn. How many gallons remained?

19. A grocer bought 150 dozen eggs, and found 16⅔% of them bad or broken. How many were salable?

20. A train of cars running 25 miles an hour increases its speed 12½%. How far does it then run in an hour?

511. **Principle.**—*The percentage of any number is the same part of that number as the given rate is of* 100%.

WRITTEN EXERCISES.

512. 1. What is 17% of $4957?

OPERATION.

$$\begin{array}{r} \$4957 \\ .17 \\ \hline \$842.69 \end{array}$$

ANALYSIS.—Since 17% is .17, the required percentage is .17 of $4957, or $4957 × .17, which is $842.69.

What is

2. 35% of 695 lb.?
3. 75% of $8428?
4. $12\frac{1}{2}\%$ of £2105?

Find

5. $33\frac{1}{3}\%$ of 8736 bu.
6. $\frac{1}{8}\%$ of $35000.
7. 120% of $171.24.

RULE.—*Multiply the base by the rate. Or, take such a part of the base as the rate is of* 100%.

This rule may be briefly expressed by the following

FORMULA.—*Percentage* = *Base* × *Rate.*

What

8. Is $4\frac{1}{2}\%$ of 312.8 rd.?
9. Is 105% of $5728?
10. Is $3140.75 + $1\frac{1}{4}\%$?
11. Is $2\frac{2}{3}$ mi. + $7\frac{1}{2}\%$?
12. Is 400 ft. — $3\frac{1}{3}\%$?

Find

13. 84% of $25\frac{5}{7}$ bu.
14. 25% of $\frac{7}{8}$ of a ton.
15. $\frac{3}{4}\%$ of 16400 men.
16. $\frac{2}{3}\%$ of $\frac{5}{6}$ of a year.
17. $\frac{5}{8}\%$ of $\frac{16}{25}$ of a hhd.

18. The bread made from a barrel of flour weighs 35% more than the flour. What is the weight of the bread?

19. A man having a yearly income of $4550 spends 20% of it the first year, 25% of it the second year, and $37\frac{1}{2}\%$ of it the third year. How much does he save in 3 years?

20. A man receives a salary of $1600 a year. He pays 18% of it for board, $8\frac{1}{3}\%$ for clothing, and 16% for incidentals. What are his yearly expenses, and what does he save?

21. A man owning ⅘ of a cotton-mill, sold 35% of his share for $24640. What part of the whole mill did he still own, and what was its value?

22. Smith had $5420 in bank. He drew out 15% of it, then 20% of the remainder, and afterward deposited 12½% of what he had drawn. How much had he then in bank?

513. The base and percentage being given to find the rate.

ORAL EXERCISES.

1. What per cent. of 25 is 3?

Analysis.—Since 3 is 3/25 of 25, it is 3/25 of 100%, or 12%. Hence, 3 is 12% of 25.

What per cent.

2. Of 24 is 18?
3. Of $16 are $4?
4. Of 200 figs are 20 figs?
5. Of 40 lb. are 15 lb.?
6. Of 12½ bu. are 2½ bu.?
7. Of 2 A. are 80 sq. rd.?
8. Of 1 da. are 16 hr.?

What per cent.

9. Are 6¼ mi. of 12½ mi.?
10. Are 18 qt. of 30 qt.?
11. Are 16⅔ cents of $1?
12. Is $¼ of $25?
13. Is ⅔ of 8/9?
14. Is 6/7 of 2⅓?
15. Is ¾ of 3¾?

16. ⅔ of an acre is what per cent. of it?

17. ⅜ of a cargo is what per cent. of it?

18. 2¼ times a number is what per cent. of it?

19. If $6 are paid for the use of $30 for a year, what is the rate per cent.?

20. If a milkman adds 1 pint of water to every gallon of milk he sells, what per cent. does he add?

514. Principle.—*The rate is the number of hundredths which the percentage is of the base.*

WRITTEN EXERCISES.

515. 1. What per cent. of 72 is 48 ?

OPERATION.

$48 \div 72 = .66\frac{2}{3} = 66\frac{2}{3}\%$

Or, $\frac{48}{72} = \frac{2}{3}$; $100\% \times \frac{2}{3} = 66\frac{2}{3}\%$

ANALYSIS.—Since the percentage is the *product* of the base and rate, the rate is the quotient found by dividing the percentage by the base; and 48 divided by 72 is $\frac{48}{72} = \frac{2}{3} = .66\frac{2}{3}$; hence the rate is $66\frac{2}{3}\%$. Or,

Since 48, the percentage, is $\frac{2}{3}$ of the base, the rate is $\frac{2}{3}$ of 100%, or $66\frac{2}{3}\%$.

What per cent.

2. Of 300 is 75 ?

3. Of 66 is $16\frac{1}{2}$?

4. Of \$20 are \$21.60 ?

What per cent.

5. Of \$18 are 90 cents ?

6. Of 560 lb. are 80 lb. ?

7. Of 980 mi. are 49 mi. ?

RULE.—*Divide the percentage by the base. Or, take such a part of* 100% *as the percentage is of the base.*

FORMULA.—*Rate = Percentage ÷ Base.*

What per cent.

8. Of \$480 are \$26.40.

9. Of 192 A. are 120 A. ?

10. Of 15 mi. are 10.99 mi. ?

11. Of 46 gal. are 5 gal. 3 qt. ?

12. Of \$4 are 30 cents ?

13. Of 6 bu. 1 pk. are 4 bu. 2 pk. 6 qt. ?

What per cent.

14. Are 448 da. of 5600 da. ?

15. Are 5 lb. 10 oz. of 15 lb. Avoir. ?

16. Is 13.5 of 225 ?

17. Is $\frac{21}{35}$ of $\frac{56}{105}$?

18. Is $3\frac{5}{8}$ of $18\frac{1}{8}$?

19. Is $22\frac{4}{5}$ of 182.4 ?

20. A grocer sold from a hogshead containing 600 lb. of sugar, $\frac{1}{4}$ of it at one time, and $\frac{1}{3}$ of the remainder at another time. What per cent. of the whole remained ?

21. A merchant owes \$15120, and his assets are \$9828. What per cent. of his debts can he pay ?

516. The rate and percentage being given to find the base.

ORAL EXERCISES.

1. 18 is 3% of what number?

Analysis.—Since 3%, or $\frac{3}{100}$, of a certain number is 18, $\frac{1}{100}$ is $\frac{1}{3}$ of 18, or 6, and $\frac{100}{100}$ is 600. Hence 18 is 3% of 600.

Of what number

2. Is 15 25%?

3. Is 24 75%?

4. Is 48 8%?

5. Is 1.2 6%?

Of what are

6. 30 lb. 20%? 25%?

7. $84 12%? 21%?

8. 15 bu. 30%? 50%?

9. 16 doz. 12½%? 8⅓%?

10. 12½% of 96 is 33⅓% of what number?

517. Principle.—*The base is as many times the percentage as* 100% *is times the rate.*

WRITTEN EXERCISES.

518. 1. 144 is 75% of what number?

OPERATION.

$$144 \div .75 = 192$$

$$\text{Or, } 100 \div 75 = \tfrac{100}{75} = \tfrac{4}{3}$$

$$144 \times \tfrac{4}{3} = 192$$

Analysis.—Since the percentage is the *product* of the base by the rate, the base is equal to the percentage divided by the rate; and 144÷.75 is 192. Or,

Since the rate is .75, the percentage is $\frac{75}{100}$, or $\frac{3}{4}$ of the base; hence the base is $\frac{4}{3}$ of the percentage, and $\frac{4}{3}$ of 144 is 192.

2. $54 are 15% of what?

3. $18.75 are 2½% of what?

4. 4.56 A. are 5% of what?

5. 39.6 lb. are 7½% of what?

Rule.—*Divide the percentage by the rate. Or, take as many times the percentage as* 100% *is times the rate.*

Formula.—*Base = Percentage ÷ Rate.*

Of what number			Of what		
6.	Is 828	120% ?	10.	Are $281.25	37½%?
7.	Is 6199	105½%?	11.	Are $4578	84%?
8.	Is .43	71⅔% ?	12.	Are 37½ bu.	6¼%?
9.	Is 31¼	31¼% ?	13.	Are 1260 bbl.	12½%?

14. 25% of 800 bu. is 2½% of how many bushels?

15. A farmer sold 3150 bushels of grain and had 30% of his entire crop left. What was his entire crop?

16. A man drew 25% of his bank deposits, and expended 33⅓% of the money thus drawn in the purchase of a horse worth $250. How much money had he in bank at first?

17. If a man owning 45% of a steamboat sells 16⅔% of his share for $5860, what is the value of the whole boat?

18. If $295.12 are 13⅓% of A's money, and 4⅔% of A's money is 8% of B's, how much more money has A than B?

519. The amount, or the difference, and the rate being given to find the base.

ORAL EXERCISES.

1. What number increased by 25% of itself amounts to 60?

ANALYSIS.—Since 60 is the number increased by 25% of itself, it is $\frac{125}{100}$, or $\frac{5}{4}$ of the number; and if $\frac{5}{4}$ of the number is 60, the number itself is 4 times $\frac{1}{5}$ of 60, or 48.

2. What number increased by 8⅓% of itself is 130?

3. $70 are 40% more than what sum?

4. A man sold a saddle for $18, which was 12½% more than it cost him. What did it cost him?

5. A grocer sold flour for $8.40 a barrel, which was 16⅔% more than he paid for it. What did he pay for it?

6. What number diminished by 20% of itself is 40?

ANALYSIS.—Since 40 is the number diminished by 20% of itself, it is $\frac{80}{100}$, or $\frac{4}{5}$ of the number; and if $\frac{4}{5}$ of the number is 40, the number itself is 5 times $\frac{1}{4}$ of 40, or 50.

7. What number diminished by 5% of itself is 38?

8. What sum diminished by 50% of itself is $20.50?

9. 68 yd. are 15% less than what number?

10. A tailor, after using 75% of a piece of cloth, had 9¾ yards left. How many yards in the whole piece?

11. A sells tea at $.90 a pound, which is 10% less than he paid for it. What did he pay for it?

WRITTEN EXERCISES.

520. 1. What sum increased by 37% of itself is $2055?

OPERATION.

$$1+.37=1.37$$

$$\$2055\div1.37=\$1500$$

Or,

$$\tfrac{100}{137}\text{ of }\$2055=\$2055\div137\times100=\$1500$$

ANALYSIS.—Since the number is increased 37%, or by .37 of itself, $2055 is 137%, or 1.37 the number. Hence $2055 divided by 1.37, is the base or required number. Or,

Since $2055, the amount, is $\frac{137}{100}$ of the base, 100 times $\frac{1}{137}$ of $2055, or $1500, is the base.

2. What number increased by 18% of itself equals 2950?

3. What sum increased by 15% of itself is $6900?

4. What number diminished by 12% of itself is 2640?

OPERATION.

$$1-.12=.88$$

$$2640\div.88=3000$$

$$\text{Or, }\overline{2640\div22}\times25=3000$$

ANALYSIS.—Since the number is diminished 12%, or by .12 of itself, 2640 is 88%, or .88 of the number. Hence 2640 divided by .88 is the base or required number. Or,

Since 2640, the difference, is $\frac{88}{100}$ or $\frac{22}{25}$ of the base, 25 times $\frac{1}{22}$ of 2640, or 3000, is the base.

5. If the difference is $1000 and the rate 20%, what is the base?

6. What sum diminished by 35% of itself equals $4810?

Rule.—*Divide the amount by* 1 *plus the rate; or, divide the difference by* 1 *minus the rate.*

Formula.—*Base* $= \begin{cases} Amount \div (1 + Rate). \\ Difference \div (1 - Rate). \end{cases}$

What number increased	What number diminished
7. By 12% of itself is 3800?	11. By 7½% of itself is 740?
8. By 10% is 39600?	12. By 4% is 312 acres?
9. By 15% is $2616.25?	13. By 8% is $2281.60?
10. By 22% is 1098 bu.?	14. By 37½% is $234.625?

15. A man sold 160 acres of land for $4563.20, which was 8% less than it cost. What did it cost an acre?

16. A speculator bought 48 bales of cotton, and afterward sold the whole for $2008.80, losing 7%. What was the cost of each bale?

17. A dealer bought a quantity of grain by measure and sold it by weight, thereby gaining 1½% in the number of bushels. He sold at 10% above the purchase price, and received $4910.976 for the grain. Required the cost.

18. A merchant, after paying 60% of his debts, found that $3500 would discharge the remainder. What was his whole indebtedness?

19. The net profits of a mill in two years were $6970, and the profits the second year were 5% greater than the profits the first year. What were the profits each year?

20. A man sold two houses at $2500 each; for one he received 20% more than its value and for the other 20% less. Required his loss.

APPLICATIONS OF PERCENTAGE.

521. The applications of percentage are those which are independent of *time*, as, Profit and Loss, Commission, Stocks, etc.; and those in which time is considered, as, Interest, Discount, Exchange, etc.

Since some one of the *four formulas* of percentage already considered will apply to any of these applications, the following will serve as a *general*

RULE.—*Note what elements of Percentage are given in the problem, and what element is required, and then apply the special rule or formula for the corresponding case.*

PROFIT AND LOSS.

522. ***Profit and Loss*** are terms used to express the gain or loss in business transactions.

523. Gains and losses are usually estimated at a *rate per cent.* on the cost, or the money or capital invested.

524. The operations involve the *same principles* as those of Percentage.

525. The corresponding terms are the following:

1. The ***Base*** is the *Cost,* or capital invested.
2. The ***Rate*** is the *per cent.* of profit or loss.
3. The ***Percentage*** is *profit* or *loss.*
4. The ***Amount*** is the cost *plus* the profit, or the *Selling Price.*
5. The ***Difference,*** is the cost *minus* the loss, or the *Selling Price.*

ORAL EXERCISES.

526. 1. A horse that cost $200 was sold at a gain of 12%. What was the gain, and the selling price?

ANALYSIS.—Since the gain was 12%, it was $\frac{12}{100}$ of $200, which is $24; and the selling price was $200 + $24 = $224. Hence, etc. (**510.**)

2. A saddle that cost $25 sold at a loss of 10%. What was the loss, and the selling price?

3. A tailor bought cloth at $6 a yard, and wished to sell it at a gain of 25%. At what price must he sell it?

4. For how much must a grocer sell tea that cost $.60 a pound, to gain 30%?

5. A merchant buys gloves at $.75 a pair, and sells them at a profit of 33⅓%. For how much does he sell them?

6. Bought a carriage for $160, and, after paying 10% for repairs, sold it at 12½% profit. What was the gain, and the selling price?

7. If butter bought at 36 cents a pound is sold at a loss of 16⅔%, what is the selling price?

8. What must be the selling price of coffee that cost 25 cents a pound, in order to gain 20%?

9. At what price must an article that cost $5 be sold, to gain 100%? 120%? 150%? 200%?

527. 1. A merchant bought cloth at $5 a yard, and sold it at $6 a yard. What was the gain per cent.?

ANALYSIS.—The whole gain is the difference between $6 and $5, which is $1. Since $5 gain $1, or ⅕ of itself, the gain per cent. is ⅕ of 100% or 20%. Hence, etc. (**513.**)

2. What is gained per cent. by selling coal at $7 a ton, that cost $6 a ton?

3. Sold a piano for $300, which was ¾ of what it cost. What was the loss per cent.?

4. Sold melons for \$.75 that cost \$.50. What was the gain per cent.?

5. What is gained per cent. by selling pine-apples at 30 cents each, that cost \$15 a hundred?

6. Sold a sewing machine at a loss of ⅛ of what it cost. What was the loss per cent.?

7. What % is gained on goods sold at *double* the cost?

8. What % is lost on goods sold at *one-half* the cost?

9. What per cent. profit does a grocer make who buys sugar at 10 cents and sells it at 12 cents?

10. What per cent. is gained on an article bought at \$3 and sold at \$5?

528. 1. A dealer sold flour at a profit of \$2 a barrel, and gained 25%. What was the cost?

ANALYSIS.—Since the gain was 25% = $\frac{25}{100}$, or ¼, \$2 is ¼ of the cost; \$2 is ¼ of 4 times \$2, or \$8. Hence, etc. (**516.**)

2. Sold hats for \$1 less than cost, and lost 16⅔%. What did they cost?

3. A merchant sells silk at a profit of \$1½ a yard, which is 40% gain. What did it cost, and what is the selling price?

4. If corn selling for 21 cents a bushel more than cost gives a profit of 30%, what did it cost?

5. Sold sheep at \$2½ more than cost, which was a profit of 50%. What did they cost, and what is the selling price?

6. Shoes sold at \$.50 above cost give a profit of 12½%. What did they cost?

7. A farmer, by selling a cow for \$12 less than she cost, lost 33⅓%. What did she cost?

8. A grocer sells a certain kind of tea for 6 cents a pound more than cost and gains 5%. What did it cost?

529. 1. A watch was sold for $120, at a gain of 20%. What was the cost?

ANALYSIS.—Since the gain was 20%, or $\frac{1}{5}$, of the cost, $120, the selling price, is $\frac{6}{5}$ of the cost. $\frac{1}{6}$ of $120, or $20, is $\frac{1}{5}$ of the cost, and $\frac{5}{5}$, or the cost itself, is 5 times $20, or $100. Hence, etc. (**518.**)

2. Sold tea at $.90 a pound, and gained 25%. What did it cost?

3. A newsboy, by selling his papers at 4 cents each, gains 33⅓%. What do they cost him?

4. A man sold a horse and harness for $330, which was 10% more than they cost. What was their cost?

5. If 20% is lost by selling wheat at $1.60 a bushel, what would be gained if sold at 20% above cost?

6. John Rice lost 40% on a reaper, by selling it for $60. For what should he have sold it to gain 40%?

7. If, by selling books at $2 a volume, there is a gain of 25%, at what price must they be sold to lose 15%?

8. Two pictures were sold for $99 each; on one there was a gain of 10%, on the other a loss of 10%. Was there a gain or loss on the sale of both, and how much?

WRITTEN EXERCISES.

530. 1. A hogshead of sugar bought for $108.80 was sold at a profit of 12½%. What was the gain?

OPERATION.—$108.80 × .12½ = $13.60. (**512.**)

FORMULA.—*Profit* or *Loss* = *Cost* × *Rate %*.

Find the *Profit* or *Loss*,

2. On land that cost $1745, and was sold at a gain of 20%.
3. On goods that cost $3120, and were sold at 27% gain.
4. On a boat bought for $2545½, and sold at 25% loss.

5. On goods bought for $2560.75, and sold at 8% loss.

6. On 25 tons of iron rails bought at $58 a ton, and sold at an advance of 17½%.

7. A merchant pays $6840 for a stock of spring goods, and sells them at an advance of 26½% on the purchase price. After deducting $375 for expenses, what is his gain?

8. A miller bought 1000 bushels of wheat at $1.84 a bushel, and sold the flour at 16⅔% advance on the cost of the wheat. What was his profit?

9. Bought 128 tons of coal at $5.12½ a ton, and sold it at a gain of 22%. What was the entire profit?

10. A ship, loaded with 3840 bbl. of flour, being overtaken by a storm, found it necessary to throw 37½% of her cargo overboard. What was the loss at $7.65 a bbl.?

11. A man bought a pair of horses for $450, which was 25% less than their real value, and sold them for 25% more than their real value; what was his gain?

531. 1. Bought a house for $4380. For what must it be sold to gain 14½%?

OPERATION.—$4380 × (1 + .14½) or 1.145 = $5015.10. (**512.**)

2. At what price must pork, bought at $18.40 a barrel, be sold, to lose 15%?

OPERATION.—$18.40 × (1 − .15), or .85 = $15.64. (**512.**)

FORMULA.—Selling Price = $\begin{cases} \text{Cost} \times (1 + \text{Rate \% of Gain}). \\ \text{Cost} \times (1 - \text{Rate \% of Loss}). \end{cases}$

Find the *Selling Price*,

3. Of goods bought at $187.50, and sold at 11% gain.

4. Of beef bought at $12½ a barrel, and sold at 9½% loss.

5. Of cotton bought at $.14, and sold at a gain of 21⅜%.

6. Of cloth that cost $5¼ a yard, and was sold at a profit of 18⅓%?

7. At what price must goods that cost $3½ a yard be marked, to gain 25%? To lose 20%?

8. Sold a lot of damaged goods at a loss of 15%. What was the selling price of those that cost $.62½? $1.25?

9. Bought a hogshead of sugar containing 9 cwt. 56 lb. for $86.04, and paid $4.78 freight and cartage. At what price per pound must it be sold to gain 20%.

532. 1. Bought wool at $.48 a pound, and sold it at $.60 a pound. What per cent. was gained?

OPERATION.—$.60 − $.48 = $.12; and $.12 ÷ $.48 = .25 = 25%. (**515.**)

2. Sold for $10.02 an article that cost $12. What was the loss per cent.?

OPERATION.—$12 − $10.02 = $1.98; and $1.98 ÷ $12 = .16½ = 16½%.

FORMULA.—*Rate % = Profit or Loss ÷ Cost.*

Find the *rate per cent.* of profit or loss,

3. On sugar bought at 8 cents and sold at 9½ cents.

4. On tea bought at $1, and sold at $.87½.

5. On goods that cost $275, and were sold for $330.

6. On grain bought for $1.25 a bushel, and sold for $1.60 a bushel.

7. On a sewing-machine sold for $72.96, at a gain of $9.12.

8. On goods sold for $425.98, at a loss of $134.52.

9. Bought paper at $3 a ream, and sold it at 25 cents a quire. What was the gain per cent.?

10. A dealer bought 108 bbl. of apples at $4.62½, and sold them so as to gain $114.88½. What was his gain %?

11. If ½ of an acre of land is sold for ¾ the cost of an acre, what is the gain per cent.?

12. If $\frac{4}{5}$ of an acre of land is sold for the cost of $\frac{1}{2}$ of an acre, what is the loss per cent.?

13. If $\frac{5}{8}$ of a chest of tea is sold for what the whole chest cost, what is the gain per cent. on the part sold?

533. 1. A speculator sold grain at a profit of $33\frac{1}{3}$%, by which he made 25 cents on a bushel. What did it cost?

OPERATION.—\$.25 ÷ .33$\frac{1}{3}$ = \$.75. Or, \$.25 ÷ $\frac{1}{3}$ = \$.75. (**518.**)

2. Lost \$45.75 on the sale of a horse, which was 20% of the cost. What was the cost?

OPERATION.—\$45.75 ÷ .20 = \$228.75. Or \$45.75 ÷ $\frac{1}{5}$ = \$228.75.

FORMULA.—*Cost* = *Profit* or *Loss* ÷ *Rate %*.

Find the *Cost*,

3. Of goods sold at \$1500 profit, or a gain of 16%.

4. Of flour sold at a loss of \$.88, or 10%, on a barrel.

5. Of wheat sold at a loss of 6 cents, or 4%, on a bu.

6. Of lumber sold at an advance of \$4.95 per M., or 35% gain.

7. If a grocer sells his stock at a profit of 15%, what amount must he sell to clear \$2500?

8. A and B engage in speculation. A gains \$2000, which is $12\frac{1}{2}$% of his capital, and B loses \$500, which is 5% of his capital. What sum did each invest?

534. 1. A furniture dealer sold two parlor sets for \$450 each; on one he made 15%, on the other he lost 15%. What did each cost him?

$$\text{OPERATION.—}\begin{cases}\$450 \div (1+.15) = \$391.30+\text{, cost of one.}\\ \$450 \div (1-.15) = \$529.41+\text{, cost of the other. } (\mathbf{520.})\end{cases}$$

$$\text{FORMULA.—}\textit{Cost} = \textit{Selling Price} \div \begin{cases}(1 + \textit{Rate \% of gain.})\\ (1 - \textit{Rate \% of loss.})\end{cases}$$

Find the *Cost*,

2. Of coal sold at $6, being at a loss of 12½%.

3. Of grain sold at $.96 a bushel, at a gain of 28%.

4. Of silk sold for $5.40 a yard, at a profit of 10%.

5. Of hops sold at 16 cents a pound, at a loss of 20%.

6. Of fruit sold for $207.48, at a loss of 15%.

7. Having used a carriage 1 year, I sold it for $125, which was 25% below cost. What should I have received had I sold it for 10% above cost?

8. B sold a span of horses to C and gained 12½%; C sold them to D for $550, and lost 16⅔%. What did the horses cost B?

9. If a piece of property increases in value each year at the rate of 25% on the value of the previous year, for 4 years, and then is worth $16000, what did it cost?

535. 1. Bought cloth at $3.60 a yard. At what price must it be marked that 12½% may be abated from the asking price, and still a profit made of 16⅔%?

OPERATION.— $\begin{cases} \textit{Selling Price} = \$3.60 \times (1 + .16\frac{2}{3}) = \$4.20. \\ \textit{Marking Price} = \$4.20 \div (1 - .12\frac{1}{2}) = \$4.80. \end{cases}$ **(519.)**

2. At what price must shovels that cost $1.12 each be marked in order to abate 5%, and yet make 25% profit?

3. How must a watch be marked, that cost $120, so that 4% may be deducted and a profit of 20% be made?

4. A merchant, on opening a case of goods that cost $.80 a yard, finds them slightly damaged. How must he mark them, to fall 25% in his asking price, and sell at cost?

5. Bought land at $60 an acre; how much must I ask an acre, that I may deduct 25% from my asking price, and still make 20% on the purchase price?

COMMISSION.

536. An ***Agent*** or ***Commission Merchant*** is a person who buys or sells merchandise, or transacts other business for another, called the *Principal.*

537. ***Commission*** is the fee, or compensation, allowed an agent or commission merchant for transacting business, and is usually computed at a certain rate per cent. of the money involved in the transaction.

538. A ***Consignment*** is a quantity of goods sent to a commission merchant to be sold.

539. The ***Consignor*** is the person who sends the goods for sale. A consignor is sometimes called a *Shipper.*

540. The ***Consignee*** is the person to whom the goods are sent. He is sometimes called a *Correspondent.*

541. The ***Net Proceeds*** of a sale or other transaction is the sum of money that remains after all expenses of commission, etc., are paid.

542. A ***Guaranty*** is security given by a commission merchant to his principal for the payment of goods sold by him on credit.

543. An ***Account Sales*** is a written statement made by a commission merchant to his principal, containing an account of goods sold, their price, the expenses, and the net proceeds.

544. A ***Broker*** is a person who buys or sells stocks, bills of exchange, real estate, etc., for a commission, which is called *Brokerage.*

545. The *principles* and *operations* of Percentage involved in Commission and Brokerage are the same as those already treated.

546. The following are the corresponding terms:

1. The ***Base*** is the amount of sales, money invested, or collected.
2. The ***Rate*** is the per cent. allowed for services.
3. The ***Percentage*** is the Commission or Brokerage.
4. The ***Amount*** or ***Difference*** is the amount of sales, *plus* or *minus* the commission.

WRITTEN EXERCISES.

547. Find the *Commission* or *Brokerage*,

1. On a sale of flour for $2575, at 2½%.

OPERATION.—$2575 × .025 = $64.87½. (**512.**)

FORMULA.—*Amount of Sales* × *Rate %* = *Commission.*

2. On the purchase of a farm for $13750, at 2¾%.
3. On the sale of a mill for $9384, at ⅞%.
4. On the sale of $21680 worth of wool, at 1¾%.
5. On the sale of 250 bales of cotton, averaging 520 lb., at 14⅝ cents a pound; commission 1½%.
6. On the sale of 175 shares of stock, at $92¼ a share; brokerage, ⅛%.
7. On the sale at auction of a house and the furniture for $9346.80, at 6¼%.
8. A commission merchant sells 225 bbl. of potatoes at $3.25 per bbl., and 316 bbl. of apples at $4½ per bbl. What is his commission at 4½%?

548. Find the *rate* of commission or brokerage,

1. When $89 commission is paid for selling goods for $3560.

OPERATION.—89 ÷ 3560 = .02½ = 2½%. (**515.**)

FORMULA.—*Commission ÷ Amount of Sales = Rate %.*

2. When $165 com. is paid for selling goods for $4950.

3. When $63 is paid for collecting a debt of $1260.

4. When $117.75 is paid for selling a house for $7850.

5. When $235.40 is paid for buying 26750 lb. of wool at 32 cents a pound.

6. When $125 is paid for the guaranty and sale of goods for $2500.

7. Paid my N. O. agent $74.25 for buying 26400 lb. of rice, at 4½ ct. a lb. What was the rate of his commission?

549. Find the *Amount of Sales,*

1. When a commission of $147 is charged at 3½%.

OPERATION.—$147 ÷ .035 = $4200. (**517.**)

FORMULA.—*Commission ÷ Rate % = Amount of Sales.*

2. When $92.80 commission is paid at 3⅕%.

3. When $210 commission is charged at 6%.

4. When $24 brokerage is paid at ¼%.

5. When $135 commission is charged at 1⅛%.

6. Paid an attorney $72.03 for collecting a note, which was a commission of 7½%. What was the face of the note?

550. Find the *Amount of Sales,*

1. When the net proceeds are $4875, commission 2½%.

OPERATION.—$4875 ÷ .975 = $5000. (**519.**)

FORMULA.—*Net Proceeds ÷ (1 — Rate %) = Amt. of Sales.*

When the net proceeds are $3281.25, commission 12½%

3. When the net proceeds are $560, and the com. 4%.

4. After deducting 6¼% commission and $132 for storage, my correspondent sends me $23654.25 as the net proceeds of a consignment of pork and flour. What was the gross amount of the sale?

551. Find the *amount to be invested,*

1. If $9500 is remitted to a correspondent to be invested in woolen goods, after deducting 5% commission.

OPERATION.—$9500 ÷ 1.05 = $9047.62. (**519.**)

FORMULA.—*Amount Remitted* ÷ (1 + *Rate %*) = *Sum Invested.*

2. If $4908 be remitted, deducting 4½% commission.

3. If $3246.20 be remitted, deducting 2% commission.

4. If $1511.25 be remitted, deducting ¾% commission.

5. If $10701.24 be remitted, deducting ½% brokerage.

6. A dealer sends his agent in Havana $6720.80, with which to purchase oranges and other fruits, after deducting his commission of 5%. What sum did the agent invest, and what was the amount of his commission?

7. What amount of sugar can be bought at 8 cents a pound, for $2523.40, after deducting a commission of 1¾%.

8. Remitted to a stockbroker $10650, to be invested in stocks, after deducting ¼% brokerage. What amount of stock did he purchase?

9. A broker received $45337.50 to invest in bond and mortgage, after deducting a commission of 2¼%. What amount did he invest, and what was his commission?

10. Sent $250.92 to my agent in Boston, to be invested in prints at 15 cents a yard, after taking out his commission of 2%. How many yards ought I to receive?

REVIEW.

ORAL EXERCISES.

552. 1. If stoves bought at $36 each are sold at a profit of 8⅓%, what is the gain?

2. What will be the expense of collecting a tax of $1000, allowing 5%?

3. What will a broker receive for selling $600 worth of stock, at ⅜% brokerage?

4. A man having $250 spent $80. What per cent. of his money had he left?

5. If a man sells a building lot that cost $300, at an advance of 166⅔%, what is his gain?

6. ⅗ of 30% is what per cent. of 72%? Of 144%? Of 180%? 240%?

7. Bought a horse for 20% less than $200, and sold him for 10% more than $200. What per cent. was gained?

8. How many bushels of wheat at $2 a bushel can an agent buy for $2040, and retain 2% on what he expends as his commission?

9. If by selling land at $150 an acre I lose 25%, how must I sell it to gain 40%?

10. A boy bought bananas for $3 a hundred, and sold them for 5 cents each. What per cent. did he gain?

11. Bought cannel coal at $19 a ton, which was 5% less than the market price. What was the market price?

12. Paid an agent $150, or a commission of 1½%, for selling my house. For what sum was the house sold?

13. If an article is sold so as to gain ⅖ as much as it cost, what per cent. is gained?

14. A merchant tailor sold some linen coats at $1.80 each, which was 33⅓% below the marked price. What was the marked price?

15. A grocer bought 40 gal. of maple syrup at the rate of 4 gal. for $6, and sold it at the rate of 5 gal. for $8. What was his whole gain, and his gain per cent.?

16. How much wheat must a farmer take to mill that he may bring away the flour of 4½ bushels, after the miller takes his toll of 10%?

WRITTEN EXERCISES.

553. 1. After taking out 15% of the grain in a bin, there remained 40 bu. 3⅕ pk. How many bushels were there at first?

2. The net profits of a farm in 2 years were $3485, and the profits the second year were 5% greater than the profits the first year. What were the profits each year?

3. A has 32% more money than B; what per cent. less is B's money than A's?

4. Bought 450 bushels of wheat at $1.25 per bushel, and sold it at $1.40 per bushel. What was the whole gain, and the gain per cent.?

5. A man drew out of the bank ⅜ of his money, and expended 30% of 50% of this for 728 bu. of wheat, at $1.12½ a bushel. What sum had he left in bank?

6. Sold goods to the amount of $47649, at a profit of 16⅔%. Required the cost and the total gain.

7. A broker received $37.50 for selling some uncurrent money, charging ⅛% brokerage. How much did he sell?

8. If ⅘ of a farm is sold for what ⅚ of it cost, what is the gain per cent.?

9. An architect charged ¼% for plans and specifications, and 1⅝% for superintending a building that cost $25000. What was the amount of his fee?

10. If a stationer marks his goods 50% above cost, and then deducts 50%, what per cent. does he make or lose?

11. Sold a farm for $14700, and lost 12%. What per cent. should I have gained by selling it for $21000?

12. If an article bought at 20% below the asking price is sold at 16% below that price, what is the rate of gain?

13. A commission merchant sold a consignment of goods for $5250, and charged 3¼% commission, and 2½% for a guaranty. Find the net proceeds.

14. Smith & Jones bought a stock of groceries for $13680. They sold ¼ of the entire stock at 15% profit, ⅓ at 18¾%, ⅙ at 20%, and the remainder at 33⅓% profit. What was the whole gain, and the average gain per cent.?

15. Give the marking prices at 25% advance, of the following bill of goods, and the amount when sold at a reduction of 10% from those prices:

1	Case of Prints,	450 yd.,	@ $.12
3	Pieces Cassimeres,	65 "	@ 3.25
1	Bale Ticking,	244 "	@ .20
25	Dress Shawls,		@ 7.36
1	Gr. gross Clark's Thread,	144 doz.,	@ .70
50	Gross Buttons,		@ 1.00

16. How much would the above bill of goods amount to if sold at 5⅓% below a marking price of 15% above cost?

17. What would be the net proceeds of a sale of 18 cwt. 75 lb. of sugar, at $9⅜ per cwt., allowing 2½% commission, and $16¾ for other charges?

18. A broker receives $7125 to invest in cotton, at 11¼ cents a pound. If his commission is 2½%, how many pounds of cotton can he buy?

19. If the sale of potatoes at $.75 a barrel above cost gives a profit of 18¾%, how much must be added to this price to realize a profit of 31¼%?

20. An agent in Chicago purchases 1000 bbl. of flour at $6.80, and pays 5 cents a barrel storage for 30 days; also, 3000 bu. of wheat at $1.20. He charges a commission of 1½% on the flour, and 1 cent a bushel on the wheat. What sum of money will balance the account, and what is the amount of his commission?

21. An agent in Boston received 28000 lb. of Texas cotton, which he sold at $.12½ a pound. He paid $45.86 freight and cartage, and after retaining his commission, he remits his principal $3252.89 as the net proceeds of the sale. What was the rate of his commission?

22. The following bill of goods was sold at auction:

1½ bbl. A Sugar,	312 lb.,	@ $.12½	that	cost	$.11¼
½ “ Pulv.“	96 “	@ .14⅛	“	“	.14
1 Chest Y. H. Tea,	84 “	@ 1.10	“	“	1.12½
1 Box Soap,	60 “	@ .13	“	“	.10¾
1½ Sacks Java Coffee,	110 “	@ .22½	“	“	.24¼
184 lb. Codfish,		@ .07½	“	“	.08⅜

Allowing a commission of 4½% for selling, find the entire profit or loss, and the gain or loss per cent. on the whole.

23. A merchant in New York imported 2400 yd. of English cloth, for which he paid in London 10s. sterling a yard, and the total expenses were $255. He sold the cloth for $3.81 a yard, U. S. money. What was his whole gain, and his gain per cent.?

554. SYNOPSIS FOR REVIEW.

PERCENTAGE.

- 1. DEFINITIONS.
  - 1. Percentage. 2. Per Cent. 3. Sign of Per Cent. 4. Rate, or Rate %. 5. Base. 6. Percentage. 7. Amount. 8. Difference.
- 2. ELEMENTS.
  - 1. How many considered.
  - 2. How many must be given.
- 3. **510.** 1. Principle. 2. Rule. 3. Formula.
- 4. **513.** 1. Principle. 2. Rule. 3. Formula.
- 5. **516.** 1. Principle. 2. Rule. 3. Formula.
- 6. **519.** 1. Principle. 2. Rule. 3. Formula.
- 7. APPLICATIONS OF PERCENTAGE.
  - 1. Diff't kinds.
    - 1. *Without Time.*
    - 2. *With Time.*
  - 2. General Rule.
- 8. PROFIT AND LOSS.
  - 1. Definition.
  - 2. To estimate gains and losses.
  - 3. Corresponding terms.
    - 1. *Base.*
    - 2. *Rate.*
    - 3. *Percentage.*
    - 4. *Am't and Diff.*
- 9. COMMISSION.
  - 1. Definitions.
    - 1. *Agent, or Commission Merchant.*
    - 2. *Commission.*
    - 3. *Consignment.*
    - 4. *Consignor.*
    - 5. *Consignee.*
    - 6. *Net Proceeds.*
    - 7. *Guaranty.*
    - 8. *Account Sales*
    - 9. *Broker.*
  - 2. Prin. and Operations Involved.
  - 3. Corresponding terms.
    - 1. *Base.*
    - 2. *Rate.*
    - 3. *Percentage.*
    - 4. *Am't and Diff.*

INTEREST

ORAL EXERCISES.

555. 1. When 5% is charged for the use of money, how many dollars should be paid for the use of $100? For the use of $200? Of $500? Of $50?

2. At 7% a year, what should be paid for the use of $100 for 2 years? Of $200 for 3 years?

3. If $500 is loaned for 3 years, what should be paid for its use, at 5% a year? At 6% a year?

4. If I borrow $250, and agree to pay 4% a year for its use, how much will be due the lender in 5 years?

5. If $7 is paid for the use of $100 for 1 year, what is the per cent.?

6. If $50 is paid for the use of $100 for 5 years, what is the per cent.?

7. If $14 is paid for the use of $200 for 1 year, what is the per cent.?

8. At 6%, what decimal part of the money borrowed is equal to the money paid for its use? At 7%? 8%? 9%?

DEFINITIONS.

556. ***Interest*** is a sum paid for the use of money.

557. The ***Principal*** is the sum for the use of which interest is paid.

558. The ***Rate of Interest*** is the per cent., or number of hundredths, of the principal, paid for its use for *one year*.

559. The ***Amount*** is the sum of the principal and the interest.

560. ***Legal Interest*** is the interest according to the rate per cent. *fixed by law.*

561. ***Usury*** is a higher rate of interest than is allowed by law.

562. The *legal* rates of interest in the different States are as follows:

Name of State.	Rate.		Name of State.	Rate.	
Alabama	8%	......	Minnesota	7%	12%
Arkansas*	6%	Any.	Mississippi	6%	10%
Arizona	10%	Any.	Missouri	6%	10%
California*	10%	Any.	Montana	10%	....
Canada and Ireland	6%	......	New Hampshire	6%	......
Connecticut	7%	......	New Jersey	6%	......
Colorado*	10%	Any.	New York	6%	......
Dakota	7%	Any.	North Carolina	6%	8%
Delaware	6%	......	Nebraska	10%	15%
Dist. Columbia	6%	10%	Nevada*	10%	Any.
England and France	5%	......	Ohio	6%	8%
Florida*	8%	Any.	Oregon	10%	12%
Georgia	7%	10%	Pennsylvania	6%	......
Idaho	10%	8%	Rhode Island*	6%	Any.
Illinois	6%	......	South Carolina*	7%	Any.
Indiana	6%	10%	Tennessee	6%	10%
Iowa	6%	10%	Texas	8%	12%
Kansas	7%	12%	Utah*	10%	Any.
Kentucky	6%	10%	Vermont	6%	......
Louisiana	5%	8%	Virginia	6%	12%
Maine*	6%	Any.	West Virginia	6%	......
Maryland	6%	......	Washington T.*	10%	Any.
Massachusetts*	6%	Any.	Wisconsin	7%	10%
Michigan	7%	10%	Wyoming	12%	.....

1. When the rate per cent. is not specified in accounts, notes, mortgages, contracts, etc., the *legal* rate is always understood.

2. Where *two* rates are specified, any rate above the lower, and not exceeding the higher, is allowed, *if stipulated in writing.*

3. In the States marked thus (*) the rate per cent. is unlimited if agreed upon by the parties in writing.

563. In the operations of interest there are *five* parts, or elements, namely :

The *Principal ;* the *Rate per Cent. per Annum* (for one year) ; the *Interest ;* the *Time* for which the principal is lent ; and the *Amount,* or sum of the Prin. and Int.

564. These terms correspond respectively to *Base, Rate, Percentage,* and *Amount* in Percentage, excluding *Time,* which is an additional element in Interest.

ORAL EXERCISES.

565. 1. At 3% for 1 yr., what decimal part of the principal equals the interest ? At 5% ? At 8% ? At 12½% ?

2. What is the interest of $20 for 1 year at 5% ?

ANALYSIS.—Since the interest of any sum at 5% for 1 yr. is .05 of the principal, the interest of $20 for 1 yr. at 5% is .05 of $20, or $1.

3. What is the interest of $50 for 1 yr. at 5% ? 6% ? 7% ?

4. What is the interest of $80 for 1 yr. at 7%? 8%? 10%?

5. At 7% for 5 yr., what decimal part of the principal equals the interest ?

ANALYSIS.—Since the interest at 7% for 1 yr. is .07 of the principal, the interest for 5 yr. is 5 times .07, or .35 of the principal. Or, it is 5 times the interest for 1 year.

6. At 6% for 3 yr., what decimal or fractional part of the principal equals the interest ? At 7% for 6 yr.? At 5% for 5 yr. ? At 6¼% for 2 yr.? At 10% for 4 yr. ?

7. Find the interest of $30 for 3 yr. at 5%.

ANALYSIS.—Since the interest of any sum at 5% for 1 yr. is .05 of the principal, for 3 yr. it is .15, and .15 of $30 is $4.50. Or, the interest for 1 yr. is .05 of $30, or $1.50, and for 3 yr. it is 3 times as much, or $4.50.

8. Find the int. at 6% of $20 for 2 yr. Of $40 for 3 yr.

9. Find the int. at 8% of $5 for 5 yr. Of $10 for 10 yr.

10. At 8% for 2 yr. 6 mo., what decimal part of the principal equals the interest?

ANALYSIS.—Since the interest of any sum for 1 yr. at 8% is .08 of the principal, the interest on the same for 2 yr. 6 mo. is 2½ times .08, or .20 of the principal. Or, it is 2½ times the interest for 1 yr.

11. At 6% for 3 yr. 3 mo., what decimal part of the principal equals the interest? At 9% for 3 yr. 3 mo.?

12. Find the int. of $9 for 2 yr. 4 mo. at 7%. At 8%.

13. What is the int. of $1000 for 2 yr. 3 mo. at 10%? For 4 yr. 6 mo.? For 5 yr. 3 mo.? For 8 mo.?

566. PRINCIPLE.—*The interest is the product of three factors; namely, the principal, rate per annum, and time (expressed in years or parts of a year).*

WRITTEN EXERCISES

567. To find the interest or amount of any sum at any rate per cent., for years and months.

1. Find the amount of $97.50, at 7%, for 2 yr. 6 mo.

OPERATION.	
$97.50	
.07	
$6.8250	Int. for 1 yr.
2½	
17.0625	Int. for 2 yr. 6 mo.
97.50	Principal.
$114.5625	Amount.

ANALYSIS.—Since the interest of any sum at 7% for 1 yr. is .07 of the principal, the interest of $97.50 at 7% for 1 yr. is .07 of $97.50, or $6.825; and the interest for 2 yr. 6 mo. is 2½ times the interest for 1 yr., or $17.06¼, and $17.06¼ + $97.50 = $114.56¼, the *Amount.*

Find the interest and the amount,

2. Of $450 for 3 yr. 9 mo. at 6%. For 8 mo. at 7%.

3. Of $247 for 5 yr. 3 mo. at 5½%. For 10 mo. at 8%.

4. Of $500 for 4 yr. 2 mo. at 10%. For 11 mo. at 5%.

RULE.—I. *Multiply the principal by the rate, and the product is the interest for* 1 *year.*

II. *Multiply the interest for* 1 *year by the time in years, and the fraction of a year; the product is the required interest.*

III. *Add the principal to the interest for the amount.*

FORMULA.—*Interest = Principal × Rate × Time.*

Find the interest,

5. Of $36.40 for 1 yr. 7 mo. at 6%. At 7%. At 7½%

6. Of $750.50 for 3 yr. 1 mo. at 5%. At 8%. At 9%

7. Of $1346.84 for 2 yr. 4 mo. at 6¼%. At 7½%.

8. Of $138.75 for 4 yr. 3 mo. at 10%. At 12½%.

9. Find the amount of $640 for 5 yr. 6 mo. at 7%.

10. Find the amount of $56.64 at 8% for 3 yr. 3 mo.

11. Made a loan of $1040 for 1 yr. 9 mo. at 7½%. How much is due at the end of the time?

12. If a note for $375, on interest at 8%, dated June 10, 1874, be paid Sept. 10, 1876, what amount will be due?

568. To find the interest on any sum of money, for any time, at any rate per cent.

OBVIOUS RELATIONS BETWEEN TIME AND INTEREST.

I. The interest on any sum for 1 year at 1% is .01 of the principal.

It is therefore equal to the principal with the decimal point removed *two places* to the left.

II. The interest for 1 mo. is $\frac{1}{12}$ of the interest for 1 yr.

III. The interest for 3 days is $\frac{3}{30}$, or $\frac{1}{10}$, of the interest for 1 month; hence any number of days may readily be reduced to *tenths* of a month by dividing by 3.

IV. The interest on any sum for 1 month, multiplied by the number of *months* and *tenths* of a month in the given time, and the product by the number expressing the *rate*, will be the required interest.

569. 1. Find the int. of $361.20 for 1 yr. 3 mo. 24 da. at 7%.

OPERATION.

$3.612	(.01 of the Prin.) Int. for 1 yr. at 1% (**568,** I).
.301	Int. for 1 mo. at 1% (**568,** II).
15.8	Number of months and tenths (**568,** III).
$4.7558	Int. for 1 yr. 3 mo. 24 da. at 1%.
7	
$33.2906	Int. for 1 yr. 3 mo. 24 da. at 7% (**568,** IV).

What is the interest,

2. Of $137.25 for 1 yr. 6 mo. 10 da. at 6%? At 4%?
3. $510.50 for 3 yr. 7 mo. 15 da. at 5%? At 8%?
4. Of $1297.60 for 2 yr. 11 mo. 18 da. at 7%? At 7½%?

RULE.—I. To find the interest for 1 yr. at 1%.

Remove the decimal point in the given principal two places to the left.

II. To find the interest for 1 mo. at 1%.

Divide the interest for 1 *year by* 12.

III. To find the interest for *any time* at 1%.

Multiply the interest for 1 *month by the number of months and tenths of a month in the given time.*

IV. To find the interest at any rate %.

Multiply the interest at 1% *for the given time by the number expressing the given rate.*

5. Find the int. of $781.90 for 1 yr. 1 mo. 12 da. at 7%.
6. Find the int. of $3000 for 11 mo. 21 da. at 10%.

7. What is the amt. of $1049 for 2 yr. 3 mo. 9 da. at 6½% ?

8. What is the amt. of $216.75 for 3 yr. 5 mo. 11 da. at 8%?

9. Required the int. of $250 from Jan. 1, 1873, to May 10, 1875, at 7%.

10. Required the amount of $408.60 from Aug. 20 to Dec. 18, 1876, at 10%.

11. What is the interest on a note for $515.62, dated March 1, 1873, and payable July 16, 1875, at 7%?

12. A man sold his house and lot for $12500; the terms were, $4000 in cash on delivery, $3500 in 9 mo., $2600 in 1 yr. 6 mo., and the balance in 2 yr. 4 mo., with 6% interest. What was the whole amount paid?

570. SIX PER CENT. METHOD.

At 6% per annum, the interest of $1

For 12 mo.	is 6 cents,	or .06	of the	principal.
" 2 " or $\frac{1}{6}$ of 12 mo.,	" 1 cent,	" .01	"	"
" 1 " " $\frac{1}{12}$ " 12 "	" $\frac{1}{2}$ "	" .005	"	"
" 6 da. " $\frac{1}{5}$ " 1 "	" $\frac{1}{10}$ "	" .001	"	"
" 1 " " $\frac{1}{6}$ " 6 da. "		.000$\frac{1}{6}$	"	"

571. PRINCIPLES.—1. *The interest of any sum at 6% is* ONE-HALF *as many hundredths of the principal as there are* MONTHS *in the given time.*

2. *The interest of any sum at 6% is* ONE-SIXTH *as many thousandths of the principal as there are* DAYS *in the given time.*

Thus, the interest on any sum at 6% for 1 yr. 3 mo., or 15 mo., is ½ of .15, or .075, of the principal; and for 18 da. it is ⅙ of .018, or .003, of the principal. Hence, for 1 yr. 3 mo. 18 da., it is .075 + .003 = .078 of the principal.

It is evident that an odd month is ½ of .01, or .005; and that any number of days less than 6 is such a fractional part of .001 as the days are of 6 days.

ORAL EXERCISES.

572. What is the interest,

1. Of $1 at 6% for 1 year? 2 years? 3 years? 5 years? 8 years? 12 years?

2. Of $1 at 6% for 1 month? 2 mo.? 3 mo.? 4 mo.? 5 mo.? 7 mo.? 9 mo.? 10 mo.? 15 mo.? 18 mo?

At 6%, what is the interest,

3. Of $1 for 1 yr. 4 mo.? 1 yr. 7 mo.? 2 yr. 2 mo.?

4. Of $1 for 1 day? 6 da.? 12 da.? 19 da.? 24 da.? 33 da.? 36 da.? 45 da.? 63 da.?

5. Of $1 for 1 mo. 12 da.? For 3 mo. 15 da.? For 6 mo. 25 da.? For 7 mo. 11 da.? For 11 mo. 18 da.?

Find the interest,

6. Of $1, at 6%, for 1 yr. 3 mo. 6 da. For 1 yr. 9 mo. 18 da. For 1 yr. 5 mo. 19 da.

7. Of $1 at 6% for 2 yr. 1 mo. 9 da. For 3 yr. 24 da.

8. Of $1 at 6% for 5 yr. 5 mo. 5 da. For 4 yr. 7 mo. 10 da.

At 6%, find the interest,

9. Of $1 for 2 yr. 6 mo. Of $2. Of $3. Of $5.

10. Of $1 for 4 yr. 2 mo. Of $10. Of $20. Of $30.

11. Of $5 for 1 yr. 4 mo. For 2 yr. For 2 yr. 8 mo.

12. Of $1 for 33 da. For 63 da. For 93 da. For 123 da.

13. Of $6 for 33 da. Of $4 for 63 da. Of $2 for 93 da.

14. If the interest of a certain principal at 6% is $18, what would the interest be at 5%? 7%? 8%? 9%?

5% is ⅙ less than 6%; 7% is ⅙ more than 6%; 8% is ⅓ more, etc.

15. If the interest of a certain principal is $16, what would the int. be at 3%? 4½%? 5%? 7½%? 8%? 12%?

16. If the interest of a certain principal is $30, what would the int. be at 2%? 4%? 7%? 8%? 10%? 14%?

WRITTEN EXERCISES.

573. 1. What is the int. of $427.20 at 6% for 2 yr. 5 mo. 27 da.?

OPERATION.

2 yr. 5 mo.	= 29 mo.	$427.20
½ of .29	= .145	.149½
⅙ of .027	= .004½	$63.8664
Int.	= .149½ of the Prin.	

ANALYSIS.—Since the interest of $1 for 2 yr. 5 mo. 27 da. is $.149½, or of any sum is .149½ of the principal (**571**), $427.20 × .149½ = $63.866 + is the required interest.

Find the interest at 6% of

2. $597.25 for 7 mo. 18 da.
3. $418.75 for 1 mo. 25 da.
4. $309.18 for 2 yr. 24 da.
5. $1298 for 3 yr. 1 mo. 13 da.
6. $2000 for 2 yr. 7 mo. 24 da.
7. $4010 for 1 yr. 1 mo. 13 da.

RULE.—*Multiply the given principal by the decimal expressing the interest of* $1; *or by the decimal expressing one-half as many hundredths as there are months, and one-sixth as many thousandths as there are days, in the given time, and the product will be the required interest.*

To find the interest at any other per cent. by this method, increase or diminish the interest at 6% by such part of itself as the given rate is greater or less than 6%.

574. To compute ***Accurate Interest,*** that is, reckoning 365 da. to the year, use the following

RULE.—*Find the interest for years and aliquot parts of a year by the common method, and for days take such part of* 1 *year's interest as the number of days is of* 365. Or,

When the time is in days and less than 1 *year, find the interest by the common method, and then subtract* $\frac{1}{73}$ *part of itself for the common year, or* $\frac{1}{61}$, *if it be a leap year.*

1. Find the accurate interest of $1560 for 45 da. at 7%.

The *exact int.* of $1560 for 45 da. at 7% $= \frac{\$109.20 \times 45}{365} = 13.46+.$

Or it is $\$13.65 - \frac{\$13.65 \times 1}{73} = \$13.46+.$

2. Find the exact int. of $1600 for 1 yr. 3 mo. at 6%.

3. What is the difference between the *exact* interest of $645.40 at 8% for 1 yr. 3 mo. 20 da. and the interest reckoned by the 6% method?

4. Find the exact interest of $875.60 at 7% for 63 da.

5. Required the exact interest on three U. S. Bonds of $1000 each, at 6%, from May 1 to Oct. 15.

6. What is the exact interest on a $500 U. S. Bond, at 5%, from Nov. 1 to April 10 following?

575. Find the interest, by any of the ordinary methods,

1. Of $721.56 for 1 yr. 4 mo. 10 da. at 6%.
2. Of $54.75 for 3 yr. 24 da. at 5%.
3. Of $1000 for 11 mo. 18 da. at 7%.
4. Of $3046 for 7 mo. 26 da. at 8%.
5. Of $1821.50 from April 1 to Nov. 12 at 6%.
6. Of $700 from Jan. 15 to Aug. 1 at 10%.
7. Of $316.84 from Oct. 20 to March 10 at 7%.

What is the amount

8. Of $3146 for 2 yr. 3 mo. 10 da. at 7%?
9. Of $96.85 for 3 yr. 1 mo. 27 da. at 6%?
10. Of $1008.80 for 10 mo. 16 da. at 6½%?
11. Of $2000 for 15 da. at 12½%?
12. Of $137.60 for 127 da. at 10%?
13. If $1671.64 be placed at interest June 1, 1874, what amount will be due April 1, 1876, at 7%?

14. How much is the interest on a note for $600, dated Feb. 1, 1872, and payable Sept. 25, 1875, at 6%?

15. If a man borrow $9700 in New York, and loan it in Colorado, what will it gain at legal int. in a year?

16. Required the interest of $127.36 from Dec. 12, 1873, to July 3, 1875, at 4½%.

17. A note of $250, dated June 5, 1874, was paid Feb. 14, 1875, with interest at 8%. What was the amount?

18. A note for $710.50, with interest after 3 mo., at 7%, was given Jan. 1, 1874, and paid Aug. 12, 1876. What was the amount due?

19. A man engaged in business was making 12½% annually on his capital of $16840. He quit his business and loaned his money at 7½%. What did he lose in 2 yr. 3 mo. 18 da. by the change?

20. A man borrows $2876.75, which belongs to a minor who is 16 yr. 5 mo. 10 da. old, and he is to retain it until the owner is 21 years old. What will then be due at 8% simple interest?

21. A speculator borrowed $9675, at 6%, April 15, 1874, with which he purchased flour at $6.25. May 10, 1875, he sold the flour at $7⅜ a barrel, cash. What did he gain by the transaction?

22. A man borrows $10000 in Boston at 6%, reckoning 360 da. to the year, and lends it in Ohio at 8%, reckoning 365 da. to the year. What will be his gain in 146 days?

23. A tract of land containing 450 acres was bought at $36 an acre, the money paid for it being loaned at 5½%. At the end of 3 yr. 8 mo. 24 da., ⅜ of the land was sold at $40 an acre, and the remainder at $38½ an acre. What was gained or lost by the transaction?

PROBLEMS IN INTEREST.

576. Interest, time, and rate given, to find the principal.

ORAL EXERCISES.

1. What sum of money will gain $10 in 1 yr. at 5%?

ANALYSIS.—The interest of $1 for 1 yr. at 5% is .05 of the principal, and therefore $10 ÷ .05, or $200, is the required sum. Or,

Since $.05 is the interest of $1, $10 is the interest of as many dollars as $.05 is contained times in $10, or 200 times. Hence, etc.

What sum of money will gain,

2. $20 int. in 2 yr. at 5%?
3. $25 int. in 5 yr. at 5%?
4. $60 int. in 2 yr. at 6%?
5. $84 int. in 2 yr. at 7%?
6. $50 int. in 6 mo. at 10%?
7. $30 int. in 3 mo. at 8%?

WRITTEN EXERCISES.

577. 1. What sum of money, put at interest $3\frac{1}{2}$ yr. at 6%, will gain $346.50?

OPERATION.

Int. of $1 for $3\frac{1}{2}$ yr. at 6% = $.21.
$346.50 ÷ $.21 = 1650 times;
$1 × 1650 = $1650.

ANALYSIS.—Same as in oral exercises. (**576.**)

What *principal*

2. Will gain $49.50 in 1 yr. 3 mo. at 6%? At 5%?
3. Will gain $153.75 in 3 mo. 24 da. at 7%? At 8%?

RULE.—*Divide the given interest by the interest of $1 for the given time, at the given rate.*

FORMULA.—*Principal* = *Interest* ÷ (*Rate* × *Time*).

What sum of money

4. Will gain $213 in 5 yr. 10 mo. 20 da. at 7%?
5. Will gain $173.97 in 4 yr. 4 mo. at 6%? At 12%?

6. A man receives semi-annually $350 int. on a mortgage at 7%. What is the amount of the mortgage?

578. Amount, rate, and time given, to find the principal.

ORAL EXERCISES.

1. What sum of money will amount to $107 in 1 yr. at 7%?

ANALYSIS.—Since the interest is .07 of the principal, the amount is 1.07, or $\frac{107}{100}$, of it. If $107 is $\frac{107}{100}$ of the principal, $\frac{1}{100}$ of the principal is $\frac{1}{107}$ of $107, or $1; and $\frac{100}{100}$, or the principal itself, is $100. Or,

Since $1.07 is the amount of $1, $107 is the amount of as many dollars as $1.07 is contained times in $107, or $100.

What sum of money will amount to

2. $130 in 5 yr. at 6%?
3. $228 in 2 yr. at 7%?
4. $412 in 6 mo. at 6%?
5. $250 in 10 yr. at 10%?
6. $350 in 15 yr. at 5%?
7. $260 in 3 yr. 9 mo. at 8%?

WRITTEN EXERCISES.

579. 1. What sum will amount to $337.50 in 5 yr. at 7%?

OPERATION.

Am't of $1 for 5 yr. at 7% = $1.35.
$337.50 ÷ $1.35 = 250 times;
$1 × 250 = $250.

ANALYSIS.—Same as in oral exercises. (578.)

What *principal*

2. Will amount to $1028 in 4 mo. 24 da. at 7%?
3. Will amount to $1596 in 2 yr. 6 mo. at 5½%?
4. Will amount to $1531.50 in 3 mo. 18 da. at 7%?

RULE.—*Divide the given amount by the amount of* $1 *for the given time, at the given rate.*

FORMULA.—*Prin.* = *Amt.* ÷ (1 + *Rate* × *Time*).

5. What is the principal which in 217 days, at 5½%, amounts to $918.73?

6. What principal in 3 yr. 4 mo. 24 da. will amount to $761.44 at 5%?

580. Principal, interest, and time given, to find the rate.

ORAL EXERCISES.

1. At what rate will $100 gain $14 in 2 years?

ANALYSIS.—Since the interest of $100 is $14 for 2 yr., it is $7 for 1 yr., and $7 is .07 of $100, the principal. Hence the rate is 7%. Or, Since the interest of $100 for 2 yr. at 1% is $2, $14 is as many per cent. as $2 is contained times in $14, or 7%.

At what rate will

2. $300 gain $60 in 4 yr.?
3. $500 gain $100 in 5 yr.?
4. $400 gain $84 in 3 yr.?
5. $5 gain $1 in 3 yr.?
6. $120 gain $60 in 10 yr.?
7. $150 double itself in 10 yr.?

WRITTEN EXERCISES.

581. 1. At what rate per cent. will $1600 gain $280 interest in 2½ years?

OPERATION.

Int. of $1600 at 1% for 2½ yr. = $40.
$280 ÷ $40 = 7 times; 1% × 7 = 7%.

ANALYSIS. — Same as in oral exercises. (**580.**)

At what *rate per cent.*

2. Will $2085 gain $68.11 in 5 mo. 18 da.?

3. Will $1500 gain $252 in 2 yr. 4 mo. 24 da.?

RULE.—*Divide the given interest by the interest of the given principal, for the given time, at 1%.*

FORMULA.—*Rate = Int. ÷ (Prin. × 1% × Time).*

4. A house that cost $14500 rents for $1189. What per cent. does it pay on the investment?

5. At what rate will $1500 amount to $1684.50 in 2 yr. 18 da.?

6. At what rate per *month* will $2000 gain $120 in 90 da.?

7. A man invests $15600, which gives him an annual income of $1620. What rate of interest does he receive?

8. At what rate per annum will any sum double itself in 4, 6, 8, and 10 years, respectively?

At 1%, any sum will double itself in 100 yr.; hence, to double itself in 4 yr., the rate will be as many times 1% as 4 yr. are contained times in 100 yr,, or 25%, etc.

9. At what rate per annum will any sum triple itself in 2, 5, 7, 12, and 20 years, respectively?

10. I invest $49500 in a business that pays me $297 a month. What annual rate of interest do I receive?

11. Which is the better investment, and how much, one of $4200, yielding $168 semi-annually, or one of $7500, producing 712\frac{1}{2}$ annually?

582. Principal, interest, and rate given, to find the time.

ORAL EXERCISES.

1. In what time will $200 gain $56 at 7%?

ANALYSIS.—The given interest, $56, is $\frac{56}{200}$, or .28, of the principal, $200; therefore, the time is as many years as .07, the given rate, is contained times in .28, or 4 times. Hence, etc.

Or, the interest of $200 at 7% for 1 yr. is $14; therefore, the time is as many years as $14 are contained times in the given interest, $56, or 4 years. Hence, etc.

In what time will

2. $40 gain $10 at 5%?
3. $500 gain $100 at 4%?
4. $25 gain $20 at 6%?
5. $1000 gain $250 at 5%?
6. $5 gain 90 cents at 6%?
7. $50 gain $12½ at 10%?

WRITTEN EXERCISES.

583. 1. In what time will $840 gain $78.12 at 6%?

OPERATION.

$840 × .06 = $50.40 Int. for 1 yr.
$78.12 ÷ $50.40 = 1.55.
1 yr. × 1.55 = 1 yr. 6 mo. 18 da.

ANALYSIS.—Same as in the oral exercises. (**582.**)

In what time

2. Will $175.12 gain $6.43 at 6%?

3. Will $1000 amount to $1500 at 7½%?

RULE.—*Divide the given interest by the interest of the given principal, at the given rate for* 1 *year.*

FORMULA.—*Time* = *Interest* ÷ (*Prin.* × *Rate*).

4. In what time will $8750 gain $1260 at 2% a month?

5. How long must $1301.64 be on interest to amount to $1522.92 at 5%?

6. How long will it take any sum of money to double itself at 3%, 5%, 6%, 7½%, and 10%, respectively?

At 100%, any sum of money will double itself in 1 year; hence to double itself at 10%, it will require as many years as 10% is contained times in 100%, or 10 yr.

7. How long will it take any sum to triple itself at 4%, 5%, 7%, 8%, and 12½%, respectively?

8. In what time will the interest of $120, at 8%, equal the principal? Equal *half* the principal? Equal *twice* the principal?

COMPOUND INTEREST.

584. ***Compound Interest*** is interest not only on the principal, but on the interest added to the principal when it becomes due.

ORAL EXERCISES.

585. 1. What is the comp. int. of $500 in 2 yr. at 6%?

ANALYSIS.—The simple interest of $500 for 2 yr. is $60; the interest of the first year's interest, $30, for the second year is $1.80, which, added to $60, gives $61.80, the compound interest. Or,

The interest of $500 for 1 yr. at 6% is $30, and the amount is $530, which is the principal for the second year; the interest of $530 for 1 yr. at 6% is $31.80, which added to $530 gives $561.80, the final amount; and deducting $500, the original principal, gives $61.80, the compound interest.

What is the *compound interest*

2. Of $600 for 2 yr. at 5%?
3. Of $100 for 2 yr. at 7%?
4. Of $300 for 2 yr. at 10%?
5. Of $1000 for 2 yr. at 5%?

What is the *amount* at compound interest,

6. Of $800 for 2 yr. at 5%?
7. Of $2000 for 2 yr. at 10%?
8. Of $400 for 2 yr. at 4%?
9. Of $500 for 2 yr. at 8%?

WRITTEN EXAMPLES.

586. 1. What is the comp. int. of $750 for 2 yr. at 6%?

OPERATION.

$750	Prin. for 1st yr.
1.06	
$795	Prin. for 2d yr.
1.06	
$842.70	Total amount.
750.	
$92.70	Compound int.

ANALYSIS.—Since the amount is 1.06 of the principal, the amount at the end of the first year is $795, which is the principal for the 2d year, and the amount at the end of the 2d year is $842.70. Hence, by subtracting the given principal, $750, the result is the compound interest, $92.70.

2. What will $350 amt. to in 3 yr. at 7%, comp. int.?

3. What is the compound int. of $1200 for 3 yr. at 5%?

RULE.—I. *Find the amount of the given principal for the first period of time at the end of which interest is due, and make it the principal for the second period.*

II. *Find the amount of this principal for the next period; and so continue till the end of the given time.*

III. *Subtract the given principal from the last amount, and the remainder will be the compound interest.*

When the time contains months and days, less than a single period, find the amount up to the end of the last period, and compute the simple interest upon that amount for the remaining months and days, which add to find the total amount.

4. What will $864.50 amount to in 4 yr. at 8%, compound interest?

5. What is the compound interest of $680 for 2 yr. at 7%, interest being payable semi-annually?

6. What is the compound interest of $460 for 1 yr. 5 mo. 18 da. at 6%, interest payable quarterly?

7. What will be the amount of $1250 in 3 yr. 7 mo. 18 da. at 5%, interest being semi-annual?

8. Find the compound interest of $790 for 9 mo. 27 da. at 8%, payable quarterly.

The computation of compound interest may be abridged by using the following table.

To use the table, multiply the given principal by the number in the table corresponding to the given number of years and the given rate. If the interest is not annual, reduce the time to periods, and the rate proportionally. Thus, 2 yr. 6 mo., by semi-annual payments, at 7%, is the same as 5 yr. at 3½%; and 1 yr. 9 mo., quarterly payments, at 8%, the same as 7 yr. at 2%.

587. TABLE *showing the amt. of* $1, *at* 2½, 3, 3½, 4, 5, 6, 7, 8, 9, 10, 11, *and* 12%, *compound int., from* 1 *to* 20 *years.*

Yrs.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1 123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1 795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2 396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2 526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Yrs.	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.070000	1.080000	1.090000	1.100000	1.110000	1.120000
2	1.144900	1.166400	1.188100	1.210000	1.232100	1.254400
3	1.225043	1.259712	1.295029	1.331000	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.464100	1.518070	1.573519
5	1.402552	1.469328	1.538624	1.610510	1.685058	1.762342
6	1.500730	1.586874	1.677100	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.076160	2.210681
8	1.718186	1.850930	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.839420	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.518170	2.812665	3.138428	3.498450	3 895976
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363493
14	2.578534	2.937194	3.341727	3.797498	4.310440	4 887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130394
17	3.158815	3.700018	4.327633	5.054470	5.895091	6.866040
18	3.379932	3.996019	4.717120	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.612760
20	3.869684	4.660957	5.604411	6 727500	8.062309	9.646291

9. Find by the table the compound interest of $950 for 1 yr. 5 mo. 24 da., at 10%, interest payable quarterly.

OPERATION.

1 yr. 5 mo. 24 da. = 5 quarters of a year + 2 mo. 24 da.
10% per annum = 2½% per quarter.
Amount for 5 yr. at 2½% = 1.131408 of principal.
$950 × 1.131408 = $1074.837, amount for 1 yr. 3 mo.
Interest of $1074.837 for 2 mo. 24 da. at 10% = $25.079.
$1074.837 + $25.079 = $1099.916, total amount.
$1099.916 − $950 = $149.916, compound interest.

10. Find the amount, at compound interest, of $749.25 for 10 yr. 4 mo., at 7%, interest payable semi-annually.

11. What sum placed at simple interest for 3 yr. 10 mo. 18 da., at 7%, will amount to the same as $1500 placed at compound interest for the same time, and at the same rate, payable semi-annually?

12. At 8%, interest compounded quarterly, how much will $850 amount to in 1 yr. 10 mo. 20 da.?

13. What will $500 amount to in 20 yr. at 7%, comp. int.?

14. A father at his death left $12500 for the benefit of his only son, 14 yr. 8 mo. 12 da. old, the money to be paid him when he should be 21 years of age, with 6% interest compounded semi-annually. What did he receive?

ANNUAL INTEREST.

588. ***Annual Interest*** is interest on the principal and on each year's interest remaining unpaid, but so computed as not to increase the original principal.

It is allowed in the case of promissory notes and other contracts which contain the words, "with interest payable annually," or with "compound interest." In such cases, the interest is not compounded beyond the second year.

WRITTEN EXERCISES.

589. 1. Find the annual interest and amount of $8000 for 5 yr., at 6% per annum.

OPERATION.

Int. of $8000 for 5 yr. at 6%=$2400.

" " $480 for 10 yr. at 6% = $288.

$2400+$288=$2688, Annual int.

$8000+$2688=$10688, Amount.

ANALYSIS.—The interest on $8000 for 1 yr. at 6% is $480, and for 5 yr. is $2400.

The interest for the first year, remaining unpaid, draws interest for 4 yr.; that for the second year, for 3 yr.; that for the third year, for 2 yr.; and that for the fourth year, for 1 yr., the sum of which is equal to the interest of $480 for 4 yr.+3 yr.+2 yr.+1 yr.=10 yr.; and the interest of $480 at 6% for 10 yr. is $288. Hence the total amount of interest is $2400+$288, or $2688, and the amt. is $10688.

2. What is the annual interest of $1500 for 4 yr. at 7%?

RULE.—*Compute the interest on the principal for the given time and rate, to which add the interest on each year's interest for the time it has remained unpaid.*

To obtain the latter, when the interest has remained unpaid for a number of years, multiply the interest for one year by the product of the number of years and half that number diminished by one.

Thus, if the time is 9 yr., the interest for 1 yr. should be multiplied by 9 × (9 − 1) ÷ 2, or 9 × 4 = 36. Since the interest for the first year draws 8 years' interest, that for the second year 7 years' interest, etc., and the sum of the series 8+7+6+5+4+3+2+1 is 36.

3. What will $3500 amt. to in 10 yr., annual int., at 8%?

4. What is the difference between the annual interest and the compound interest of $2500 for 6 yr. at 6%?

5. Find the amt. of $575, at 8% annual int., for 9½ yr.

6. $800. MACON, June 15, 1872.

Four years after date, for value received, I promise to pay Robert E. Park, or order, eight hundred dollars, with interest at seven per cent., payable annually.

J. W. BURKE.

What amount is due on this note at maturity, no interest having been paid?

PARTIAL PAYMENTS.

590. ***Partial Payments*** are payments in part of the amount of a note, bond, or other obligation.

591. ***Indorsements*** are the acknowledgment of such payments, written on the back of the note, bond, etc., stating the time and amount of the same.

592. A ***Promissory Note*** is a written promise to pay a certain sum of money, on demand or at a specified time.

593. The ***Maker or Drawer*** of the note is the person who signs it.

594. The ***Payee*** is the person to whom, or to whose order, the money is paid.

595. An ***Indorser*** is a person who, by signing his name on the back of the note, makes himself responsible for its payment.

596. The ***Face*** of a note is the sum of money made payable by the note.

597. A ***Negotiable Note*** is one made payable to bearer, or to any person's order. When so made, it can be sold or transferred.

WRITTEN EXERCISES.

1. $800. **New York, Jan. 1st, 1874.**

One year after date, I promise to pay Caleb Barlow, or order, eight hundred dollars, for value received, with interest. JAMES DUNLAP.

Indorsed as follows: April 1, 1874, $10; July 1, 1874, $35; Nov. 1, 1874, $100. What was there due Jan. 1, 1875?

Analysis.—The interest of $800 for 3 mo., from Jan. 1 to April 1, at 7%,* is $14; am't, $814. Since the payment is less than the interest, it cannot be deducted for a new principal without compounding the int., which is illegal; hence, find the int. of $800 to the time of the next payment, 3 mo., which is $14, and the amt. to that time, $828, from which deduct the sum of the two payments, or $45, leaving $783, a new principal. The int. of $783 for 4 mo., to Nov. 1, is $18.27; amt., $801.27; from which deduct the third payment, $100, leaving $701.27, the next principal, the amt. of which for 2 mo., to Jan. 1, 1874, is $709.45, sum due at that time.

Principle.—*The principal must not be increased by the addition of interest due at the time of any payment, so as to compound the interest.*

Upon this principle is based the rule prescribed by the Supreme Court of the United States:

U. S. Rule.—I. *Find the amount of the given principal to the time of the first payment, and if this payment equals or exceeds the interest then due, subtract it from the amt. obtained, and treat the remainder as a new principal.*

II. *If the interest exceed the payment, find the amount of the same principal to a time when the sum of the payments equals or exceeds the interest then due, and subtract the sum of the payments from that amount.*

III. *Proceed in the same manner with the remaining payments.*

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* At the date of these payments the legal rate of interest in N. Y. was 7%.

$500. PHILADELPHIA, Feb. 1, 1875.

2. *Three months after date, I promise to pay to J. B Lippincott & Co., or order, five hundred dollars, with interest, without defalcation. Value received.*

JAMES MONROE.

Indorsed as follows: May 1, 1875, $40; Nov. 14, 1875, $8; April 1, 1876, $18; May 1, 1876, $30. What was due Sept. 16, 1876?

OPERATION.

Face of note, or principal		$500.00
Interest to May 1, 1875, 3 mo., at 6%		7.50
Amount		507.50
Payment, to be subtracted		40.00
2d principal		467.50
Int. of $467.50 to Nov. 14, 1875, 6 mo. 13 da.	$15.04	
Int. of $467.50 to April 1, 1876, 4 mo. 17 da.	10.67	25.71
Amount		493.21
Sum of payments, to be subtracted		26.00
3d principal		467.21
Int. to May 1, 1876, 1 mo.		2.34
Amount		469.55
Payment, to be subtracted		30.00
4th principal		439.55
Int. to Sept. 16, 1876, 4 mo. 15 da.		9.89
Amount due		$449.44

3. What was the amount due October 25, 1873, upon a note for $1500, dated New Orleans, April 1, 1872, and on which the following payments were endorsed: June 5, 1872, $300; Oct. 15, 1872, $37.75; May 1, 1873, $97.25; Aug. 6, 1873, $495?

$700. DETROIT, Nov. 1, 1873.

4. *On demand, I promise to pay Charles Smith, or order, seven hundred dollars, with interest. Value received.*

ABRAHAM ISAACS.

Indorsed as follows: Dec. 5, 1873, $75; Jan. 10, 1874, $350; April 11, 1874, $11.25; May 15, 1874, $250. What was due Sept. 1, 1874?

497\frac{39}{100}$. CHICAGO, Mar. 15, 1874.

5. *Three months after date, we promise to pay James Kelly, or order, four hundred and ninety seven* $\frac{39}{100}$ *dollars, with interest at* 6%. *Value received.*

BROWN, NICHOLS & CO.

Indorsed as follows: Nov. 3, 1874, $57.50; June 15, 1875, $22.25; Aug. 1, 1875, $125; Sept. 15, 1875, $175. What was due Jan. 1, 1876?

598. The following method of computation is often used by merchants in the settlement of notes and of interest accounts running a year or less; hence called the MERCANTILE RULE:

I. *Find the amount of the note or debt from its date to the time of settlement.*

II. *Find the amount of each payment from its date to the time of settlement.*

III. *Subtract the sum of the amounts of payments from the amount of the note or debt:*

An accurate application of this rule requires that the time should be reduced to *days*, and that the interest should be computed by the rule for days (574).

For the Vermont State method of computation, and also of assessing taxes, see pages 491–495.

1. On a note for $600 at 7%, dated Feb. 15, 1874, were the following indorsements: March 25, 1874, $150; June 1, 1874, $75; Oct. 10, 1874, $100. What was due Dec. 31, 1874?

OPERATION.

Am't of $600 from Feb. 15 to Dec. 31, 319 da.,		$636.71
" " $150 " Mar. 25 " " 281 da.,	$158.08	
" " $75 " June 1 " " 213 da.,	78.06	
" " $100 " Oct. 10 " " 82 da.,	101.57	337.71
Balance due Dec. 31, 1874,		$299.00

2. A note for $950, dated Jan. 25, 1876, payable in 9 mo., at 7% interest, had the following indorsements: March 2, 1876, $225; May 5, 1876, $174.19; June 29, 1876, $187.50; Aug. 1, 1876, $79.15. What was the balance due at the time of its maturity?

3. Payments were made on a debt of $1750, due April 5, 1875, as follows: May 10, 1875, $190; July 1, 1875, $230; Aug. 5, 1875, $645; Oct. 1, 1875, $372. What was due Dec. 31, 1875, interest at 6%?

DISCOUNT.

599. ***Discount*** is a certain per cent. deducted from the price-list of goods, or an allowance made for the payment of a debt or other obligation before it is due.

600. The ***Present Worth*** of a debt payable at a future time without interest, is such a sum as, being put at legal interest, will amount to the debt when it becomes due.

601. The ***True Discount*** is the difference between the whole debt and the present worth.

ORAL EXERCISES.

602. 1. What is the present worth of a debt of $224, to be paid in 2 yr., at 6%?

ANALYSIS.—Since in 2 yr., at 6%, the int. is .12 of the principal, the amt. is 1.12 of it; therefore, $224, the debt, is 1.12, or $\frac{112}{100}$ of the present worth, and $\frac{100}{100}$, or the present worth itself, is $200 Or, since $1.12 is the amt. of $1, $224 is the amt. of as many dollars as $1.12 is contained times in $224, or $200. (**578.**)

What is the present worth
2. Of $315, due in 10 mo., at 6%?
3. Of $570, due in 2 yr., at 7%?
4. Of $408, due in 3 mo., at 8%?
5. Of $51, due in 4 mo., at 6%?
6. Of $440, due in 2 yr., at 5%?
Find the true discount at 6%,
7. Of $1019, due in 3 mo. 24 da.
8. Of $102.20, due in 4 mo. 12 da.
9. Of $5035, due in 1 mo. 12 da.

WRITTEN EXERCISES.

603. 1. What are the present worth and the true discount, of $362.95, payable in 7 mo. 12 da., at 6%?

OPERATION.

Amt. of $1, for 7 mo. 12 da., at 6% = $1.037.
$362.95 ÷ $1.037 = 350 times.
$1 × 350 = $350, Present Worth.
$362.95 — $350 = $12.95, True Discount.

ANALYSIS.—Since the amount of $1 for 7 mo. 12 da. at 6% is $1.037 (**579**), $362.95 is the amount of as many dollars as $1.037 is contained times in $362.95, or 350 times. Hence the *present worth* is $350; and the *true discount* is $362.95—$350, or $12.95.

2. What is the present worth of a debt of $287.75 to be paid in 3 mo. 18 da. at 7%?

3. What is the true discount on a debt of $2202.90 due in 8 mo. 12 da. at 7%?

RULE.—I. *Divide the debt by the amount of $1 for the given rate and time, and the quotient is the present worth.*

II. *Subtract the present worth from the debt, and the remainder is the true discount.*

FORMULA.—*Present Worth = Debt ÷ Amt. of $1.*

Hence the present worth is the *principal* of which the true discount is the *interest*, and the whole debt the *amount*.

4. Bought a house and lot for $19500 cash, and sold them for $22000, payable one-fourth in cash and the remainder in 1 yr. 6 mo. How much ready money did I gain, computing discount at 6%?

5. A merchant buys goods for $4200 on 4 mo. credit, but is offered a discount of 3% for cash. If money is worth ½% a month, what is the difference?

6. Bought a bill of lumber amounting to $3500, on 6 mo. credit; 2 months afterward paid on account $1500, and 1 month later, $1000. Find the present worth of the balance, at the time of the second payment, int. at 7%.

7. A merchant holds two notes, one for $356.25 due Dec. 1, 1875, and the other for $497.50, due Feb. 1, 1876. What would be due him in cash on both notes Sept. 15, 1875, at 6%?

8. A bookseller bought books, worth $300, at retail prices, getting a discount of 33⅓%; he sold them at the retail prices, on 6 mo. time: money being worth 6%, what per cent. profit did he make?

9. A speculator bought 230 bales of cotton, each bale containing 470 lb., at 11¾ cents a pound, on a credit of 9 mo. He at once sold the cotton for $13000 cash, and paid the pres. worth of the debt at 7%. What was his gain?

10. Which is the more profitable, to buy flour at $8.75 a barrel on 6 mo. credit, or at $8.60 on 2 mo., money being worth 7%?

11. A person sold goods to the amount of $3750, 15% payable in cash, 25% in 3 mo., 20% in 4 mo., and the remainder in 6 mo. What ready money would discharge the whole debt, money being worth 6%?

BANK DISCOUNT.

604. A ***Bank*** is a corporation chartered by law for the safe-keeping and loaning of money, or the issuing of bills for circulation as money.

605. ***Bank Bills*** or ***Notes*** are promissory notes issued by banks, and payable on demand.

A bank which issues notes to circulate as money is called a *Bank of Issue;* one which lends money by discounting notes, a *Bank of Discount;* and one which takes charge of money belonging to other parties, called *depositors*, a *Savings Bank*, or *Bank of Deposit.* Some banks perform two and others all of these duties.

606. ***Bank Discount*** is a deduction made for interest in advancing money upon a note not due, or payment by a borrower, in advance, of interest upon money loaned to him. It is equal to the interest at the given rate for the given time (including the days of grace) on the whole sum specified to be paid.

607. ***Days of Grace*** are the three days allowed by law for the payment of a note after the expiration of the time specified in the note. They are counted in by bankers in discounting notes.

608. The ***Maturity*** of a note is the expiration of the whole time, including the days of grace.

609. The ***Term of Discount*** is the time from the discount of a note to its maturity.

610. A ***Bank Check*** is a written order for money by a depositor, upon a bank.

611. The ***Proceeds,*** or ***Avails*** of a note is the sum received for it when discounted, that is, the face of the note less the discount.

612. A ***Protest*** is a formal declaration in writing, made by a Notary-Public, at the request of the holder of a note, to give legal notice to the maker and the indorsers of its non-payment.

1. The failure to protest a note on the third day of grace releases the indorsers from all obligation to pay it.

2. If the third day of grace or the maturity of a note occurs on Sunday, or a legal holiday, it must be paid on the day previous.

3. The transaction of borrowing money at a bank is conducted as follows: The borrower presents a note, either made or indorsed by himself, payable at a specified time, and receives for it a sum equal to the face less the interest for the time it has to run, including the days of grace. A note for discount at a bank must be made payable to the order of some person, by whom it must be indorsed. When the note bears interest, the discount is computed on its face plus the interest for the time it has to run.

613. Bank discount being simple interest, the following are corresponding terms:

The ***Face of the Note*** is the *principal.*
The ***Term of Discount*** is the *time.*
The ***Bank Discount*** is the *interest.*
The ***Proceeds*** is the *principal less the interest.*

614. To find the bank discount and proceeds of a note.

ORAL EXERCISES.

1. What is the bank discount on a note for $2000 due in 2 mo. 15 da. at 6%, and the proceeds?

Analysis.—After adding 3 da., the time is 2 mo. 18 da.; the interest for which at 6% is .013 of the principal; .018 of $2000 is $26, the *bank discount*, and $2000 – $26 equals $1974, or the *proceeds.*

What are the *bank discount* and the *proceeds* of a note

2. Of $80 for 5 mo. 27 da., at 7%?
3. Of $100 for 2 mo. 21 da., at 6%?
4. Of $200 for 8 mo. 9 da., at 7%?
5. Of $150 for 4 mo. 21 da., at 5%?
6. Of $100 for 30 da., at 6%?

WRITTEN EXERCISES.

615. 1. Required the bank discount and proceeds of a note for $1250 due in 90 days, at 7%.

OPERATION.

$$\frac{\$1250 \times .07}{365} \times 93 = \$22.32, \text{ Bank Discount.}$$

$$\$1250 - \$22.29 = \$1227.71, \text{ Proceeds.}$$

Analysis.—The interest of $1250 for 93 da., at 7%, reckoning 365 da. to the year, is $22.29, which is the *bank discount.* If 360 da. are reckoned to the year, the bank disc't is $22.604. Deducting the bank disc't from the face of the note, the remainder is the *proceeds.*

Rule.—I. *Compute the interest on the face of the note (or if it bears interest, on its amount at maturity), for three days more than the specified time, and the result is the bank discount.*

II. *Subtract the discount from the face of the note, or its amount at maturity, and the remainder is the proceeds.*

2. What is the bank discount, and what is the proceeds of a note for $597.50, due in 60 da., at 6% ?

3. What will be the proceeds of a note for $1615, due in 90 da. with interest at 7%, discounted at the Nassau Bank in New York?

4. Sold a farm, containing 173 A. 95 P., for $62¼ an acre, and received payment as follows: $2000 cash, and the balance in a note payable in 5 mo. 18 da. at 7% interest, which was discounted at a bank. How much ready money did the farm bring?

Find the *date of maturity*, the *term of discount*, and the *proceeds* of the following:

957\frac{37}{100}$. CHICAGO, July 27, 1875.

5. Three months after date, I promise to pay to the order of D. L. Moody, nine hundred fifty-seven and $\frac{37}{100}$ dollars, for value received.

*Discounted Aug. 10, at 8%. WILLIAM THOMSON.

916\frac{25}{100}$. SAN FRANCISCO, Feb. 5, 1874.

6. Two months after date, we jointly and severally agree to pay C. H. Thomas, or order, nine hundred sixteen and $\frac{25}{100}$ dollars *with interest* at 8%, value received.

Discounted at Marine Bank, JAMES BARNES.
Feb. 21, at 10%. GEORGE CHILDS.

1315\frac{75}{100}$. NEW YORK, May 1, 1875.

7. Ninety days after date, I promise to pay to the order of Ivison, Blakeman, Taylor & Co., one thousand three hundred fifteen and $\frac{75}{100}$ dollars, for value received.

Discounted May 15, at 7%. WILLIAM HEWSON.

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* Banks usually count the actual number of *days* in the given time, and 365 days to the year.

$1250. **Boston, June 12, 1876.**

8. Six months after date, I promise to pay Knight, Adams & Co., or order, twelve hundred fifty dollars, *with interest* at 6 per cent., value received.

Discounted at a broker's, S. R. Brown.
Nov. 15, at 6%.

616. The proceeds and time of a note given, to find the face.

ORAL EXERCISES.

1. For what sum must a note be drawn, at 2 mo. 15 da., at 6%, so that the proceeds when discounted may be $987 ?

Analysis.—The bank discount for 2 mo. 18 da. at 6% is .013 of the face of the note, and the proceeds must therefore be 1 — .013, or .987 of the face; and if .987 of the face is $987, the whole face of the note is $1000.

Required the face of a note, so that the proceeds may be

2. $972, for 4 mo. 21 da. at 7%.
3. $194, for 5 mo. 27 da. at 6%.
4. $97.60, for 3 mo. 15 da. at 8%.
5. $980, for 4 mo. 21 da. at 5%.
6. $184, for 9 mo. 15 da. at 10%.

WRITTEN EXERCISES.

617. 1. What must be the face of a note at 9 mo. 27 da., interest 8%, so that the proceeds may be $448 ?

OPERATION.

The bank discount of $1 for 10 mo. at 8% is $.066⅔.
The proceeds of $1 = $1 — $.066⅔ or $.933⅓.
Hence $448 ÷ .933⅓ = $480, the face of the note.

2. What is the face of a note at 30 da., the proceeds of which, when discounted at bank, at 7%, are $1425 ?

RULE.—*Divide the given proceeds by the proceeds of $1 for the time and rate given; the quotient is the face of the note.*

FORMULA.—$Face = Proceeds \div (1 - \overline{Rate \times Time})$.

3. Find the face of a 3 mo. note the proceeds of which, discounted at 2% a month, is $675.

4. The proceeds of a note are $1915.75, the time 3 mo., and the rate of interest 7%; what is the face of the note?

5. Bought merchandise for $2250, cash; for what sum must I draw my note at 3 mo., so as to obtain that sum at the bank, interest at 7%?

6. The avails of a 3 months note, when discounted at 7½%, were $315.23; what was the face of the note?

7. For what sum must a note dated April 5, for 90 da., be drawn, so that when discounted at 7%, on April 21, the proceeds may be $650?

8. For how much must I draw my note at 90 da., in order that when discounted at a bank, at 7%, its avails will pay for 137½ yd. of cloth at $2⅝ a yard?

SAVINGS-BANK ACCOUNTS.

618. A ***Savings-Bank*** is designed chiefly to accommodate depositors of small sums of money.

Interest is allowed semi-annually on all sums that have been on deposit for a certain time, if not drawn out before the regular day of paying interest—generally on the 1st of January and of July.

Savings-banks generally allow interest only from the commencement of each quarter; but in some banks money deposited previous to the 1st day of any month draws interest from that date to the day of declaring interest dividends, provided it has not been previously withdrawn.

WRITTEN EXERCISES.

619. 1. A person had on deposit Jan. 1, 1874, $150. His subsequent deposits were, Feb. 3, $35; March 29, $20; April 10, $43; May 15, $26. His drafts during the same time were, Jan. 15, $50; Feb. 27, $15; April 19, $45. What interest was due July 1st, at 6%?

OPERATION.

Date of Int. paym'ts.	Balance 1st of month.	Smallest Bal. during mo.	Interest for 1 month.	Smallest Bal. dur'g Q'rter.	Interest for 1 Quarter.
JAN. 1	$150				
Feb. 1	100	$100	$.50		
Mar. 1	120	100	.50		
APR. 1	140	120	.60	$100	$1.50
May 1	138	138	.69		
June 1	164	138	.69		
JULY 1	164	164	.82	138	2.07
			$3.80		$3.57

Balance due, with int. by *monthly* periods, $167.80.
" " " " " *quarterly* " $167.57.

ANALYSIS.—At the end of January, the balance due is $100, which having been on deposit for the month, draws interest for 1 mo.; at the end of February, the balance is $120; but the *smallest balance* during the month is $100; hence interest is allowed only on that sum. The same principle applies to the other balances. If only *quarterly periods* of interest are allowed, the interest is calculated at the end of each quarter on the smallest balance during the quarter, or, in this case, on $100, April 1, and $138, July 1.

2. Find the balance, due July 1, on the following account: *Deposits*, Jan. 15, $175; April 10, $60; May 31, $110. *Drafts*, March 5, $75; May 1, $35; June 10, $50. Interest at 6%, from the 1st day of each month.

3. A person deposits in a savings-bank the following sums: Jan. 1, $350; Feb. 5, $150; March 15, $75; May 10, $30; June 15, $100. During the same time he draws, Jan. 15, $150; Feb. 10, $200; March 31, $50; June 1, $75. What interest at 6%, payable from the 1st of each month, must be added to the account July 1?

4. Balance the following, Jan. 1, 1875: Balance due to Margaret Brown, July 1, 1874, $275. Deposits received as follows: Aug. 1, $125; Sept. 15, $57; Oct. 10, $350. Drafts paid: July 15, $100; Sept. 1, $150; Nov. 15, $68; Dec. 15, $125. Interest at 6%, from the 1st of each quarter, July 1 and Oct. 1.

RULE.—*At the end of each term compute the interest for the term on the smallest balance on deposit at any time during the quarter; and at the end of each period of six months add to the balance of principal the whole amount of interest due, and the sum will be the principal at the commencement of the next six months.*

5. How much was due Jan. 1, 1876, on the following account, allowing interest, computed from the 1st of each quarter, Jan. 1 to July 1, at 6% per annum?

Dr. GREENWICH SAVINGS BANK, *in acct. with* MARY WILLIAMS. *Cr.*

1874.				1874.			
Jan. 1	To Cash	$136	00	Sept. 15	By Check	$75	00
Mar. 17	" "	25	00	1875.			
Aug. 1	" "	87	50	Jan. 20	" "	87	50
1875.				Mar. 3	" "	50	00
June 11	" "	150	00				
Nov. 17	" "	72	00				

620. SYNOPSIS FOR REVIEW.

PERCENTAGE—CONTINUED

- 10. INTEREST.
  - 1. Defs.
    - 1. *Interest.* 2. *Principal.* 3. *Rate*
    - 4. *Amt.* 5. *Legal Int.* 6. *Usury*
  - 2. Corresponding Elements.
  - 3. 1. Principle. 2. Rule, I, II, III.
  - 4. Relations between Time and Interest. I, II, III, IV.
  - 5. **569.** Rule, I, II, III, IV.
  - 6. 6% Method.
    - 1. Principles, 1, 2.
    - 2. Rule.
  - 7. Accurate Interest. Rule.
  - 8. Problems.
    - **576.** 1. *Rule.* 2. *Formula*
    - **578.** 1. *Rule.* 2. *Formula*
    - **580.** 1. *Rule.* 2. *Formula*
    - **582.** 1. *Rule.* 2. *Formula*
- 11. COMPOUND INTEREST.
  - 1. Definitions—Compound Interest.
  - 2. Rule, I, II, III.
- 12. ANNUAL INTEREST.
  - 1. Definitions.
  - 2. Rule, I, II.
- 13. PARTIAL PAYMENTS.
  - 1. Defs.
    - 1. *Part. Pay'ts.* 2. *Indorsem'ts.* 3. *Promissory Note.* 4. *Maker or Drawer.* 5. *Payee.* 6. *Indorser.* 7. *Face of a Note.* 8. *Negotiable Note.*
  - 2. Principle. U. S. Rule, I, II. Merc. Rule.
- 14. DISCOUNT.
  - 1. Defs.
    - 1. *Discount.* 2. *Present Worth.*
    - 3. *True Discount..*
  - 2. Rule, I, II.
- 15. BANK DISCOUNT.
  - 1. Defs.
    - 1. *Bank.* 2. *Bank Bills or Notes.* 3. *Bank Discount.* 4. *Days of Grace.* 5. *Maturity of Note.* 6. *Term of Discount.* 7. *Bank Check.* 8. *Proceeds or Avails* 9. *Protest.*
  - 2. Corresponding Terms.
  - 3. **614.** Rule, I, II.
  - 4. **616.** Rule.
- 16. SAVINGS-BANK ACCOUNTS—Rule.

STOCKS

621. A ***Corporation*** is an association of individuals authorized by law to transact business as a single person.

622. A ***Charter*** is the legal act of incorporation defining the powers and obligations of the body incorporated.

623. The ***Capital Stock*** of a corporation is the capital or money contributed, or subscribed to carry on the business of the company.

624. ***Certificates of Stock*** or ***Scrip*** are the papers or documents issued by a corporation, specifying the number of shares of the joint capital which the holders own.

625. A ***Share*** is one of the equal parts into which capital stock is divided.

The value of a share in the original contribution of capital varies in different companies. In bank, insurance, and railroad companies, it is usually $100.

626. ***Stocks*** is a general term applied to shares of stock of various kinds, Government and State bonds, etc.

Stockholders are the owners of stock, either by original title or by subsequent purchase. The stockholders constitute the company.

627. The ***Par Value*** of stock is the sum for which the scrip or certificate was issued.

628. The ***Market Value*** of stock is the sum for which it can be sold.

Stock is *at par* when it can be sold for its original or face value, or 100%; it is *above par*, or *at a premium*, when it will bring more than its face value; and it is *below par*, or *at a discount*, when it sells for less than its face value. Thus, when stock is at par, it is quoted at 100; when it is 5% above par, at 105; and when it is 5% below par, at 95.

629. ***Premium, Discount,*** and ***Brokerage*** are each a *percentage* computed upon the par value of the stock as the base.

630. A ***Stock Broker*** is a person who buys and sells stocks, either for himself, or as the agent of another.

631. ***Stock-jobbing*** is the buying and selling of stocks with the view to realize gain from their rise and fall in the market.

632. An ***Installment*** is a portion of the capital stock required of the stockholders as a payment on their subscription.

633. An ***Assessment*** is a sum required of stockholders, to meet the losses, or to pay the business expenses of the company.

634. A ***Dividend*** is a sum paid to the stockholders from the profits of the business.

Dividends and assessments are a *percentage* computed upon the par value of the stock as the *base.*

635. ***Net Earnings*** are the moneys left from the profits of a business after paying expenses, losses, and the interest upon the bonds.

636. A ***Bond*** is a written instrumeut securing the payment of a sum of money at or before a specified time.

The principal bonds dealt in by brokers are Government, State, City, and Railroad bonds.

637. ***U. S. Bonds*** are of two kinds; viz., those which are payable at a fixed date, and those which, while payable at a fixed date, may be paid at an earlier specified time, as the Government may elect.

1. The former are quoted in commercial transactions by the rate of interest which they bear; thus, United States bonds bearing 6% interest are quoted *U. S. 6's*. The latter are quoted in commercial transactions by a combination of the two dates; thus, *U. S. 5-20's*, or *U. S. 6's 5-20*, means bonds of U. S. bearing 6% interest, and payable at any time from 5 to 20 years, as the Government may choose.

2. When it is necessary to distinguish different issues bearing the same rate of interest, the year at which they become due is also mentioned; thus, *U. S. 5's of '71; U. S. 5's of '74; U. S. 6's, 5-20, of 84; U. S. 4's of 1907.*

3. The 5-20's were issued in 1862, '64, '65, '67, and '70. They bore interest at 6%, paid semi-annually in gold, but have nearly all been refunded at a lower rate of interest.

4. Bonds issued by States, cities, etc., are quoted in a similar manner. Thus, *S. C. 6's* are bonds bearing 6% interest, issued by the State of South Carolina.

638. A ***Coupon*** is a certificate of interest attached to a bond, to be cut off and presented for payment when the interest is due.

639. ***Currency*** is a term used to denote the circulating medium employed as a substitute for gold and silver. It consists, at present, in the United States, of U. S. Legal-tender Notes, or "Greenbacks," and the Bills issued by the Nat. Banks, and secured by U. S. Bonds.

If from any cause the paper medium depreciates in value, gold becomes an object of investment, the same as stocks. Gold being of fixed standard value, its fluctuations in price indicate changes in the value of the currency. Hence, when gold is said to be at a *premium*, currency is virtually below par, or at a *discount*.

ORAL EXERCISES.

640. 1. Find the cost of 100 shares of Chicago and Rock Island Railroad stock at 90; brokerage ⅛%.

ANALYSIS.—Since the cost of one share is 90% of $100, or $90, the cost of 100 shares is 100 times $90, or $9000, to which add the brokerage, ⅛% of $10000, or $12½, and the sum $9012½, is the entire cost of the stock.

2. What cost 50 shares of N. Y. Central R. R. Stock, at par; brokerage, ¼%?

3. Find the cost of 10 shares of Bank Stock at 104; brokerage ⅛%.

4. What is the cost of $2000 U. S. 4's, at 112; brokerage ⅛%?

641. 1. A broker has $5010 to invest in bank stock at 25% premium; how many shares can he buy, charging ¼% for brokerage?

ANALYSIS.—Since the stock sells at 25% premium, each share with brokerage will cost $125¼; hence he can buy as many shares as $125¼ are contained times in $5010, or 40 shares.

2. A speculator invested $52000 in Ohio and Mississippi R. R. stock at 25¾, allowing ¼% brokerage; how many shares did he buy?

3. If I invest $2350 in U. S. 4's, at 117⅜, brokerage ⅛%, how many $1000 bonds do I receive?

642. 1. A man bought a number of shares of mining stock at 60, and sold the same at 68, and gained $800 by the transaction. How many shares did he buy?

ANALYSIS.—Since he bought at 60% and sold at 68%, he gained 8% of the par value; hence $800 is 8% of $10000, the par value, and the number of shares at $100 each is 100.

2. Bought R. R. stock at 90, and sold at par, gaining $1000. Required the number of shares.

3. I purchased stock at 110 and sold at 98, losing $1200. How many shares did I buy?

4. A broker bought some stock at par, and sold it at 95, losing $2000. How many shares did he buy?

643. 1. What sum must be invested in California 7's, at 110, to obtain therefrom an annual income of $1400?

Analysis.—Since the annual income is $7 on each share, the number of shares must be equal to $1400 ÷ $7, or 200 shares; and 200 shares at $110 amount to $22000, the required investment.

2. What sum must I invest in stock at 115, paying 10% yearly dividends, to realize an income of $2000?

3. What sum must be invested in N. Y. 7's at 103⅛, in order to receive therefrom an annual income of $2100?

644. 1. What per cent. does money yield which is invested in 8% stock at 120?

Analysis.—Since each share costs $120, and pays $8 income, the per cent. will be $\frac{8}{120}$, or $\frac{1}{15}$ of 100%, equal to 6⅔%.

2. What per cent. does stock yield when bought at 90, paying 6% dividends? When bought at 75? At 120?

3. What per cent. of interest does stock yield, which pays 5% semi-annual dividends, if bought at 150? At 140? At 120?

645. 1. What should be paid for stock yielding 6% dividends, in order to realize an annual interest on the investment of 8%?

Analysis.—Since the annual dividend on each share is $6, this must be 8% of the sum required; and if 8% is $6, 1% is $¾, and 100% is $75. Hence the stock must be bought for 75.

2. For what must stock that pays 7% dividends be bought to realize 10% interest? 9%? 8%?

3. For what should Missouri 6's be bought to pay 5% interest? 5½%? 6½%? 8%?

646. 1. How much currency can be bought for $500 in gold, when the latter is at a premium of 10%?

Analysis.—Since $1 in gold is worth $1.10 in currency, $500 in gold is worth 500 times $1.10, or $550. Hence, etc.

2. How much currency can be bought for $200 in gold, when the latter is at a premium of 9%?

3. What is $1000 in gold worth in currency, when the former is at a premium of 12%? Of 9½%? Of 10½%?

647. 1. How much gold can be bought for $440 in currency, when the former is at a premium of 10%?

Analysis.—Since $1 in gold is worth $1.10 in currency, $440 will buy as many dollars in gold as $1.10 is contained times in $440, or $400 in gold. Hence, etc.

2. How much gold selling at 9% premium will $1090 in currency buy? $218? $654?

3. How much gold at 11% premium will $444 buy?

WRITTEN EXERCISES.

648. Find the *cost*

1. Of 220 shares of bank stock, the market value of which is 103¾, brokerage ¼%.

Operation.—(103¾% + ¼%) of $100 = $104, cost of 1 share.
$104 × 220 = $22880, cost of 220 shares. (**640.**)

Formula.—*Entire Cost* = (*Market Value of* 1 *Share* + *Brokerage*) × *No. of Shares.*

2. Find the cost of 350 shares of Western Union Telegraph stock, market value 97¾, brokerage ⅛%.

3. A broker bought for me 15 one-thousand-dollar U. S. bonds at 112¼, brokerage ⅛%. What was their cost?

4. My broker sells for me 125 shares of stock at 127%. What should I receive, the brokerage being ¼%?

649. Find the *number of shares*

1. Of bank stock at 105, that can be bought for $25260, including brokerage at ¼%?

OPERATION.—(105% + ¼%) of $100 = $105¼, cost of 1 share.
$25260 ÷ $105¼ = 240, No. of shares. (**641.**)

FORMULA.—*No. of Shares = Investment ÷ Cost of* 1 *Share.*

2. How many shares of N. J. Central R. R. stock at 107¾, brokerage ¼%, can be bought for $27000?

3. How many shares of Mo. 6's at 97¾, brokerage ¼%, will $21560 purchase?

4. Bought Pacific Mail at 29½, and sold at 31¼, paying ⅛% brokerage each way. How many shares will gain $330?

OPERATION.—(31¼% − 29½%) − ¼% = 1½%, gain.
$330 ÷ $1.50 = 220, No. of shares. (**642.**)

FORMULA.—*No. of Shares = Whole Gain or Loss ÷ Gain or Loss per Share.*

5. How many shares of stock bought at 97½ and sold at 102¼, brokerage ⅛% each way, will gain $990?

6. Lost $1680 by selling N. Y. Central at 101 that cost 104. Brokerage being ¼% each way, how many shares did I sell?

7. How many shares of the Bank of Commerce bought at 110½ and sold at 116¾, brokerage ⅛% on the purchase and the sale, will gain $1200?

650. Find the *amount of investment*

1. In 5 per cent. bonds, at 111, so as to realize therefrom an annual income of $2500.

OPERATION.—$2500 ÷ $5, income on 1 share = 500, No. of shares.
$111, price of 1 share × 500 = $55500, investment. (**643.**)

FORMULA.—*Investment = Price of* 1 *Share* × *No. of Shares.*

2. What sum must be invested in Tennessee 6's at 85, to yield an annual income of $1800?

3. How much money must be invested in any stock at 105½, which pays 5% semi-annual dividends, to realize an annual income of $2000?

4. What sum invested in stock at $63 per share, will yield an income of $550, the par value of each share being $50, and the stock paying 10% annual dividends?

651. Find the *rate per cent. of income,* realized

1. From bonds paying 8% interest, bought at 110.

OPERATION.—$8, interest per share ÷ $110, cost per share = $.07\frac{3}{11}$, or $7\frac{3}{11}$%. (**644.**)

FORMULA.—*Rate % of Income = Interest per Share ÷ Cost per Share.*

2. If stock paying 10% dividends is at a premium of 12½%, what per cent. of income will be realized on an investment in it?

3. Which will yield the better income, 8% bonds at 110, or 5's at 75?

4. Which is the more profitable, and how much, to buy New York 7's at 105, or 6 per cent. bonds at 84?

5. What per cent. of income does stock paying 10% dividends yield, if bought at 106?

6. What per cent. will stock which pays 5% dividends yield, if bought at a discount of 15%?

7. What rate per cent. of income shall I receive, if I buy U. S. 5's at a premium of 10%, and receive payment at par in 15 years?

652. Find *at what price stock must be bought*

1. That pays 6% dividends, so as to realize an income of 7½% on the investment.

OPERATION.—.06 ÷ .075 = .80 or 80%, price of stock. (**645.**)

FORMULA.—*Price of Stock = Dividend ÷ Rate of Income.*

2. What must be paid for 5% bonds, that the investment may yield 8%?

3. How much premium may be paid on stock that pays 10% dividends, so as to realize 7½% on the investment?

4. What must I pay for 5 per cent. bonds, that my investment may yield 7%?

5. At what price must stock, of the par value of $50 a share, and that pays 6% dividends, be bought, to yield an income of 7½%?

6. At what price must 6% stock be bought, to pay as good an income as 8% stock bought at par? As 9% stock?

653. Find the *value in currency,*

1. Of $3750 in gold, quoted at 110½.

OPERATION.—$1.10½ × 3750=$4143.75, value in currency. (**646.**)

FORMULA.—*Total Value in Currency = Value of $1 in Currency × No. of Dollars in Gold.*

2. Find the value of $4975 in gold, at a premium of 9⅞%.

3. What is the semi-annual interest of $8000 6% gold-bearing bonds worth in currency, when gold is at 111¾?

4. A merchant bought a bill of goods, for which he was to pay $7000 in currency, or $6625 in gold. Gold being at 109⅝, which is the better proposition, and how much in currency?

654. Find the *value in gold,*

1. Of $2150 in currency, when gold is at a premium of 10½%.

OPERATION.—$2150 ÷ 1.105 = $1945.70, value in gold. (**647.**)

FORMULA.—*Total Value in Gold* = *Amt. of Currency* ÷ (1 + *Premium*).

2. What is $4500 in currency worth in gold, when the latter is at a premium of 12⅛%? At 11½%? At 9⅛%?

3. How much money must be invested in U. S. 6's at 111, when gold is quoted at 110⅛, in order to obtain a semi-annual income of $2210 in currency?

4. The Mechanics Bank of New York having $109737.50 to distribute to the stockholders, declares a dividend of 6¼%; what is the amount of its capital?

5. A man owns a house which rents for $1450, and the tax on which is 2¾% on a valuation of $8500. He sells for $15300, and invests in stock at 90, that pays 7% dividends. Is his yearly income increased or diminished, and how much?

6. If I have $36500 to invest, and can buy N. Y. Central 6's at 85, or N. Y. Central 7's at 95, how much more profitable will the latter be than the former?

7. Which is the better investment, a mortgage for 3 yr. of $5000, paying 7% interest, and purchased at a discount of 5%, and paid in full, without cost, at maturity, or 50 shares of stock at 95, paying 8% dividends, and sold at the expiration of three years at 98?

8. Henry Ivison, through his broker, invested a certain sum of money in New York State 6's at 107½, and twice as much in U. S. 5's, at 98½, brokerage in each case ½%. The annual income from both investments was $3348. How much did he invest in each kind of stock?

9. A gentleman invested $12480 current funds in U. S. 6 per cent. bonds, at 104. What was his annual income in currency when gold was 110?

INSURANCE.

655. ***Insurance*** is a contract of indemnity against loss or damage. It is of two kinds: insurance on *property*, and insurance on *life*.

656. The ***Insurer*** or ***Underwriter*** is the party who takes the risk or makes the contract.

657. The ***Policy*** is the written contract between the parties.

658. The ***Premium*** is the sum paid for insurance, and is a certain per cent. of the sum insured.

659. Insurance business is generally conducted by Companies, which are either *Joint-stock Companies*, or *Mutual Companies*.

A *Stock Insurance Company* is one in which the capital is owned by individuals called *stockholders*. They alone share the profits, and are liable for the losses.

A *Mutual Insurance Company* is one in which the profits and losses are divided among those who are insured.

Some companies are conducted upon the *Stock* and *Mutual* plans combined, and are called *Mixed Companies*.

Insurance on property is principally of two kinds: *Fire Insurance*, and *Marine and Inland Insurance*.

660. ***Fire Insurance*** is indemnity for loss of property by fire.

661. ***Marine and Inland Insurance*** is indemnity for loss of vessel or cargo, by casualties of navigation on the ocean, or on inland waters.

Transit Insurance refers to risks of transportation by land only, or partly by land and partly by water. The same policy may cover both Marine and Transit Insurance.

Stock Insurance is indemnity for the loss of cattle, horses, etc. Most insurance companies will not take risks to exceed two-thirds or three-fourths the appraised value of the property insured.

When only a part of the property insured is destroyed or damaged, the insurers are required to pay only the estimated loss; and sometimes the claim is adjusted by repairing or replacing the property, instead of paying the amount claimed.

662. The operations are based on the principles of Percentage, the corresponding terms being as follows:

1. The ***Base*** is the *amount* of *insurance.*
2. The ***Rate*** is the *per cent.* of premium.
3. The ***Percentage*** is the *premium.*

ORAL EXERCISES.

663. 1. How much must be paid for insuring a house and furniture for $4000, at $1\frac{1}{4}$% premium?

ANALYSIS.—Since the premium is $1\frac{1}{4}$%, or $\frac{5}{400}$, equal to $\frac{1}{80}$ of the sum insured, the premium on $4000 will be $\frac{1}{80}$ of $4000, or $50. Hence, etc. (**510.**)

2. What will be the annual premium of insurance, at $\frac{3}{4}$%, on a building valued at $8000?

3. What will be the cost of insuring a quantity of flour, valued at $1500, at ⅘%?

4. What must be paid for insuring a case of merchandise, worth $640, at 2½%?

5. A man owns ¾ of a boat-load of corn valued at $1800, and insures his interest at 1⅜%. What premium does he pay?

6. Paid $6 for insuring $300; what was the rate?

Analysis.—Since the premium on $300 is $6, the premium on $1 is $\frac{1}{300}$ of $6, or $.02, equal to 2%. Hence, etc. (**513.**)

7. Paid $12 for an insurance of $800; find the rate.

8. Paid $24 for an insurance of $1000; find the rate.

9. At 2%, what amount of insurance can be obtained for $30 premium?

Analysis.—Since 2% is $\frac{2}{100}$ or $\frac{1}{50}$ of the amount insured, $30, the given premium, is $\frac{1}{50}$ of the amount insured; and $30 is $\frac{1}{50}$ of 50 times $30, or $1500. Hence, etc. (**516.**)

What amount of insurance can be obtained,

10. On a house, for $75, at 3% premium?

11. On a boat load of flour, for $150, at ¾%?

12. On a car load of horses, for $90, at 4½%?

13. On a store and its contents, for $105, at 1¾%?

WRITTEN EXERCISES.

664. Find the *Premium*

1. For insuring a building for $14500, at 1½%.

Operation.—$14500 × .015 = $217.50. (**512.**)

Formula.—*Premium = Amount Insured × Rate*

Find the premium for insuring

2. A house valued at $5700, at ¾%.

3. Merchandise for $2750, at ⅞%.

4. A fishing craft, for $15000, at 1¼%.

5. If I take a risk of $25000, at 1¾%, and re-insure ⅓ of it at 2¼%, what is my balance of the premium?

665. Find the *Rate of Insurance,*

1. If $36 is paid for an insurance of $2400.

OPERATION.—$36 ÷ $2400 = .015, or 1½%. (**515.**)

FORMULA.—*Rate of Insurance = Premium ÷ Sum Insured.*

What is the rate of insurance,

2. If $280 is paid for an insurance of $16000?

3. If $4.30 is paid for an insurance of $860?

4. A tea merchant gets his vessel insured for $20000 in the Royal Company, at ¾%, and for $30000 in the Globe Company, at ½%. What rate of premium does he pay on the whole insurance?

666. To find the *Amount of Insurance.*

1. A speculator paid $262.50 for the insurance of a cargo of corn, at 1½%. For what amount was the corn insured?

OPERATION.—$262.50 ÷ .015 = $17500, the sum insured. (**518.**)

FORMULA.—*Sum Insured = Premium ÷ Rate.*

2. If it cost $93.50 to insure a store for one-half of its value, at 1⅜%, what is the store worth?

3. Paid $245 insurance at 4⅜% on a shipment of pork, to cover ⅝ of its value. What was its total value?

4. A merchant shipped a cargo of flour worth $3597, from New York to Liverpool. For what must he insure it at 3¼%, to cover the value of the flour and premium?

OPERATION.—$3597 ÷ (1 − .03¼) or .9675 = $3717.829. (**520.**)

5. An underwriter agrees to insure some property for enough more than its value to cover the premium, at the rate of 26 cents per $100. If the property is worth $22163, what should be the amount of the policy?

6. For what sum must a policy be issued to insure a dwelling-house, valued at $35000, at ¼%, a carriage-house worth $9500, at ⅘%, and furniture worth $4500, at ⅜%, 10% being deducted from the premium, which is to be covered by the policy?

7. A person insured his house for ¾ of its value at 40 cents per $100, paying a premium of $73.50. What was the value of the house?

8. A dealer shipped a cargo of lumber from Portland to New York; the amount of insurance, including the value of the lumber and the premium paid, at 1¾%, was $25200. What was the value of the lumber?

9. A merchant had 500 bbl. of flour insured for 80% of their cost, at 3¼%, paying $107.25 premium. At what price per barrel must he sell the flour to gain 20%.

LIFE INSURANCE.*

667. ***Life Insurance*** is a contract by which a company agrees to pay a certain sum, in case of the death of the insured during the continuance of the policy.

668. A ***Term Life Policy*** is an assurance for one or more years specified.

669. A ***Whole Life Policy*** continues during the life of the insured.

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* See note at bottom of page 346.

Premiums may be paid annually for life, or in 5, 10, or more installments (called 5-payment, 10-payment policies, etc.), or the entire premium may be paid in one sum in advance.

The premium is computed at a certain sum or rate per $1000 insured, the rate varying with the age of the insured at the time the policy is issued.

A policy of endowment is not in all respects *an insurance policy*, but is rather a covenant to pay a stipulated sum at the end of a certain period to the person named, if living.

Most companies issue a form of policy that combines the principles of Term Life Assurance and Simple Endowment, called for brevity Endowment Policy. Hence,

670. An ***Endowment Policy*** is one in which the assurance is payable to the person insured at the end of a certain number of years named, or to his heirs if he die sooner.

An endowment policy is really two policies in one, and the assured pays the premiums of both.

671. A ***Dividend*** is a share of the premiums or profits returned to a policy-holder in a mutual life insurance company.

672. A ***Table of Mortality*** shows how many persons per 1000 at each age are expected to die per annum.

673. A ***Table of Rates*** shows the premium to be charged for $1000 assurance at the different ages.

Such a table is based upon the table of mortality, and the probable rates of interest for money invested, with a margin or loading for expenses.

674. The following condensed table gives data from the American Experience Table of mortality, and the annual premium on the kinds of policies most in use.

AMERICAN EXPERIENCE TABLE—MORTALITY AND PREMIUMS.

AGE.	MORTALITY TABLE.	ANNUAL PREMIUM PER $1000.					
		LIFE TABLE.					ENDOWMENT (AND TERM LIFE). 10 years.
		One Year Term (*Net*).	*Whole Life.*				
			Payments during life.	Payment for 10 yr. only.	Payment for 5 yr. only.	Single Payment.	
25	8.1	7.75	$19.89	$42.56	$73.87	$326.58	$103.91
26	8.1	7.82	20.40	43.37	75.25	332.58	104.03
27	8.2	7.88	20.93	44.22	76.69	338.83	104.16
28	8.3	7.95	21 48	45.10	78.18	34 .31	104.29
29	8.3	8.02	22.07	46.02	79.74	352.05	104.43
30	8.4	8.10	22.70	46.97	81.36	359.05	104.58
31	8.5	8.18	23.35	47.98	83.05	366.33	104.75
32	8.6	8.28	24.05	49.02	84.80	373 89	104.92
33	8.7	8.38	24.78	50.10	86.62	381.73	105.11
34	8.8	8.49	25.56	51 22	88 52	389.88	105.31
35	8.9	8.60	26.38	52.40	90.49	398.34	105,53
40	9.8	9.42	31.30	59.09	101 58	445.55	106.90
45	11.2	10.73	37.97	67.37	115.02	501.69	109.07
50	13.8	13.25	47.18	77.77	131.21	567 13	112.68

The actual net cost of insurance for a single year at each age given in the table, on the mortality assumed, is as many dollars and tenths of a dollar as there are deaths, but discounted for 1 year. Thus, at age 25, deaths 8.1 per 1000, net cost, which is $8.10, discounted at 4½% by the insurance law, $7.75. If this sum, $7.75, is loaded for expenses at, say 25%, the total premium for 1 year is $9.69, if at 40%, then it would be $10.85.

In a Term Life Policy the premium may vary, increasing slightly each year of the term, according to the assumed increasing liability to decease, or it may be *averaged* for the term so as to be the same each year.

NOTE.—As there is no uniformity in the Tables and Methods used by different Life Insurance Companies, the pupil may very properly omit this subject.

WRITTEN EXERCISES.

675. To find the amount of premium

1. For a life policy of $5000 issued to a person 30 years old.

OPERATION.—$22.70 × 5 = $113.50.

2. For a life policy of $7500, age being 45.

RULE.—*Multiply the premium for* $1000 *assurance by the number of thousands.*

FORMULA.—*Premium = Rate per* $1000 × *No. of thousands.*

3. Find the annual premium for an endowment policy of $10000, payable in 10 years, age 35.

4. What premium must a man aged 30 pay annually for life, for a life policy of $5000?

What premium annually for 10 years?

What premium annually for 5 years?

What premium in a single payment?

OPERATION.

$22.70 × 5.000 = $113.50
$46.97 × 5.000 = $234.85
$81.36 × 5.000 = $406.80
$359.05 × 5.000 = $1795.25

ANALYSIS.—Multiply the rate per thousand dollars, found in the Life Table, opposite age 30, by the number of thousands, expressing the hundreds, tens, and units decimally.

5. What annual premium will a man aged 35 years pay to secure an endowment policy for $5000, payable to himself in 10 years, or to his heirs, if death occurs before?

6. If he dies at the beginning of the ninth year, how much will the assurance cost, reckoning simple interest at 6%?

7. How much less would he have paid in the whole life (annual payment) plan, interest included?

8. A man aged 45 insures his life for $7500 on the single-payment plan, and dies 3 yr. 5 mo. afterward. How much less would his insurance have cost him had he insured on the annual payment plan, reckoning int. at 6%?

9. A person aged 27 takes out a 10-year endowment policy for $5000; the dividends reduce his annual premiums 15% on the average. Computing annual interest at 7% on his premiums, does he gain or lose, and how much?

10. A man aged 35 years took out a life policy for $12000, on the 5-payment plan, and died 3 yr. 6 mo. afterward. What was gained to his estate by insuring, computing compound interest on his payments at 7%, also adding two dividends of $95 each?

TAXES.

676. A ***Tax*** is a sum of money assessed on the person, property, or income of an individual, for any public purpose.

677. A ***Poll Tax*** or ***Capitation Tax*** is a certain sum assessed on every male citizen liable to taxation. Each person so taxed is called a *poll.*

678. A ***Property Tax*** is a tax assessed on property, according to its estimated, or assessed, value.

Property is of two kinds: *Real Property,* or *Real Estate,* and *Personal Property.*

679. ***Real Estate*** is fixed property; such as houses and lands.

680. ***Personal Property*** is of a movable nature; such as furniture, merchandise, ships, cash, notes, mortgages, stock, etc.

681. An ***Assessor*** is an officer appointed to determine the taxable value of property, prepare the assessment rolls, and apportion the taxes.

682. A ***Collector*** is an officer appointed to receive the taxes.

683. An ***Assessment Roll*** is a schedule, or list, containing the names of all the persons liable to taxation in the district or company to be assessed, and the valuation of each person's taxable property.

684. The ***Rate of Property Tax*** is the rate per cent. on the valuation of the property of a city, town, or district, required to raise a specific tax.

WRITTEN EXERCISES.

'**685.** 1. What sum must be assessed to raise $836000 net, after deducting the cost of collection at 5%? / 0

OPERATION.—$836000 ÷ .95 = $880000. (**519.**)

FORMULA.—*Sum to be raised* ÷ (1 — *Rate of Collection*) = *Sum to be Assessed.*

| 2. What sum must be assessed to raise a net amount of $11123, and pay the cost of collecting at 2%?

\3. In a certain district, a school-house is to be built at a cost of $18500. What amount must be assessed to cover this and the collector's fees at 3%?

4. The expense of building a public bridge was $1260.52, which was defrayed by a tax upon the property of the town. The rate of taxation was 3¼ mills on a dollar, and the collector's commission was 3⅛%. What was the valuation of the property?

5. In a certain town a tax of $5000 is to be assessed. There are 500 polls, each assessed 75 cents, and the valuation of the taxable property is $370000. What will be the rate of property tax, and how much will be A's tax, whose property is valued at $7500, and who pays for 2 polls?

OPERATION.—$.75 × 500 = $375, amt. on polls.
$5000 − $375 = “ “ property.
$4625 ÷ $370000 = .0125, rate of taxation.
$7500 × .0125 = $93.75, A's property tax.
$93.75 + $1.50 = $95.25, A's whole tax.

RULE.—I. *Find the amount of poll tax, if any, and subtract it from the whole amount to be assessed; the remainder is the property tax.*

II. *Divide the property tax by the whole amount of taxable property; the quotient is the rate of taxation.*

III. *Multiply each man's taxable property by the rate of taxation, and to the product add his poll tax, if any; the result is the whole amount of his tax.*

A table such as the following is a great aid in calculating the amount of each person's tax, according to the ascertained rate.

ASSESSOR'S TABLE. (*Rate* .0087.)

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	$.0087	$ 9	$.0783	$ 80	$ .696	$ 700	$ 6.09
2	.0174	10	.087	90	.783	800	6.96
3	.0261	20	.174	100	.87	900	7.83
4	.0348	30	.261	200	1.74	1000	8.70
5	.0435	40	.348	300	2.61	2000	17.40
6	.0522	50	.435	400	3.48	3000	26.10
7	.0609	60	.522	500	4.35	4000	34.80
8	.0696	70	.609	600	5.22	5000	43.50

6. Find by the table the tax of a person whose property is valued at $3475, the rate being .0087.

Operation.—Tax on $3000 = $26.10
" " 400 = 3.48
" " 70 = .609
" " 5 = .0435
" " $3475 = $30.2325, or $30.23.

Find by the table the tax of a person whose property

7. Is $2596 and who pays for 5 polls at $.50.

8. Is $9785, polls 3 at $.75.

9. Is $12356, polls 4 at $1.25.

10. Is $25489, polls 5 at $.95.

11. A tax of $11384, besides cost of collection at 3¼%, is to be raised in a certain town. There are 760 polls assessed at $1.25 each, and the personal property is valued at $124000, and the real estate at $350000. Find the tax rate, make an assessor's table for that rate, and find a person's tax, whose real estate is valued at $6750, personal property at $2500, and who pays for 3 polls.

12. In the above town, how much is B's tax on $15000 real estate, $2750 personal property, and 5 polls?

13. What is C's tax on $9786 and 1 poll?

14. How much tax will a person pay whose property is assessed at $7500, if he pays 1¾% village tax, ⅛% State tax, and 1¼ mills on a dollar school tax?

15. The expense of constructing a bridge was $916.65, which was defrayed by a tax upon the property of the town. The rate of taxation was 2¼ mills on a dollar, and the commission for collecting 3%; what was the assessed valuation of the property of the town?

Note.—Amt. to be raised ÷ by rate = valuation.

686. SYNOPSIS FOR REVIEW.

PERCENTAGE—CONTINUED.

- 17. STOCKS.
  - 1. Defs.
    - 1. *Corporation.* 2. *Charter.* 3. *Capital Stock.* 4. *Certificate of Stock, or Scrip.* 5. *Share.* 6. *Stocks.* 7. *Stockholders.* 8. *Par Value.* 9. *Market Value.* 10. *Premium, Discount, Brokerage.* 11. *Stock Broker.* 12. *Stock jobbing.* 13. *Installment.* 14. *Assessment.* 15. *Dividend.* 16. *Net Earnings.* 17. *Bond.* 18. *Dif. Kinds of U. S. Bonds.* 19. *Coupon.* 20. *Currency.*
  - 2. **648.** 3. **649.** 4. **650.** 5. **651.** 6. **652.** 7. **653.** 8. **654.** To find
    - *Cost.*
    - *No. of Shares.*
    - *Amt. of Investment.*
    - *Rate % Income.*
    - *Price to pay Income.*
    - *Value of Gold in Cur.*
    - *Value of Cur. in Gold.*
    - *Formula.*
- 18. INSURANCE.
  - 1. Defs.
    - 1. *Insurance.* 2. *Insurer or Underwriter.* 3. *Policy.* 4. *Premium.* 5. *Fire Insurance.* 6. *Marine or Inland Insurance.*
  - 2. Corresponding Terms in Percentage.
  - 3. **664.** 4. **665.** 5. **666.** To find
    - *Premium.*
    - *Rate of Insurance.*
    - *Amt. of Insurance.*
    - *Formula.*
- 19. LIFE INS.
  - 1. Defs.
    - 1. *Life Insurance.* 2. *Term Life Policy.* 3. *Whole Life Policy.* 4. *Endowment Policy.* 5. *Dividend.* 6. *Table of Mortality.* 7. *Table of Rates.*
  - 2. **675.** Rule. Formula.
- 20. TAXES.
  - 1. Defs.
    - 1. *Tax.* 2. *Poll Tax.* 3. *Property Tax.* 4. *Real Estate.* 5. *Personal Property.* 6. *Assessor.* 7. *Collector.* 8. *Assessment Roll.* 9. *Rate of Property Tax.*
  - 2. **685.** 3. **686.** To find
    - *Sum to be raised.* Formula.
    - *Amt. of Tax.* Rule, I, II, III.

EXCHANGE

687. ***Exchange*** is the giving or receiving of any sum in one currency for its value in another.

By means of exchange, payments are made to persons at a distance by written orders, called *Bills of Exchange.*

688. ***Exchange*** is of two kinds, *Domestic,* or *Inland,* and *Foreign.*

689. ***Domestic*** or ***Inland Exchange*** relates to remittances made between different places in the same country.

690. ***Foreign Exchange*** relates to remittances made between different countries.

691. A ***Bill of Exchange*** is a written request, or order, upon one person to pay a certain sum to another person, or to his order, at a specified time. An inland bill of exchange is usually called a *Draft.*

692. A ***Set of Exchange*** is a bill drawn in duplicate or triplicate, each copy being valid, until the amount of the bill is paid. These copies are sent by different conveyances, to provide against miscarriage.

693. A ***Sight Draft*** or ***Bill*** is one which requires payment to be made "at sight," that is, at the time it is presented to the person who is to pay it.

694. A ***Time Draft*** or ***Bill*** is one that requires payment to be made at a certain specified time after date, or after sight.

695. The ***Buyer*** or ***Remitter*** of a bill is the person who purchases it. The buyer and payee may be the same person.

696. The ***Acceptance*** of a bill or draft is the agreement by the drawee to pay it at maturity. The drawee thus becomes the *acceptor,* and the bill or draft, an *acceptance.*

1. The drawee accepts by writing the word "accepted" across the face of the bill, and signing it.

2. Three days of grace are usually allowed on bills of exchange, as well as on notes. When a bill is protested for non-acceptance, the drawer is bound to pay it immediately.

697. The ***Par of Exchange*** is the estimated value of the coins of one country as compared with those of another. It is either *intrinsic* or *commercial.*

1. The *Intrinsic Par of Exchange* is the comparative value of the coins of different countries, according to their weight and purity.

2. The *Commercial Par of Exchange* is the comparative value of the coins of different countries, according to their market price.

698. The ***Course*** or ***Rate of Exchange*** is the current price paid in one place for bills of exchange on another place.

This price varies according to the relative conditions of trade and commercial credit at the two places between which the exchange is made. Thus, if New York is largely indebted to London, bills of exchange on London will bear a high price in New York.

699. FORMS OF DRAFTS AND BILLS.

A SIGHT DRAFT.

$500. New York, *July* 1, 1874.

At sight, pay to the order of William Thompson, *five hundred dollars, value received, and charge to the acct. of*

Henry J. Carpenter.

To Harris, Jones & Co.,
Cincinnati, O.

Other drafts have the same form as the above, except that instead of the words "at sight," "—— days after sight," or "—— days after date," are used. When the time is *after sight*, it means after acceptance.

SET OF EXCHANGE.

£700. New York, *August* 1, 1874.

At sight of this First of Exchange (Second and Third of the same tenor and date unpaid), pay to the order of Samuel Monmouth, *Seven Hundred Pounds Sterling*, for value received, and charge the same to the account of

Morton, Bliss & Co.

Morton, Rose & Co., London.

The above is the form of the *first bill;* the *second* requires only the change of "First" into "Second," and instead of "Second and Third of the same tenor," etc., "First and Third." The Third Bill varies similarly.

DOMESTIC OR INLAND EXCHANGE.

The course of exchange for inland bills, or drafts, is always expressed by the rate of premium or discount. Time drafts, however, are subject to bank discount, like promissory notes, for the term of credit given. Hence, their cost is affected by both the course of exchange and the rate of discount for the time.

WRITTEN EXERCISES.

700. What is the *cost*

1. Of a sight draft on New Orleans for \$1750, at 1¼% premium?

OPERATION.—\$1750 × 1.01¼ = \$1771.87½. (**512.**)

FORMULA.—*Cost* = *Face* × $\begin{cases} 1 + \textit{Rate of Premium.} \\ 1 - \textit{Rate of Discount.} \end{cases}$

2. Of a sight draft on Troy for \$1590, at 1⅛% discount?

3. Of a draft on Boston for \$1650, payable in 60 days after sight, exchange being at a premium of 1¾%?

OPERATION.—\$1.0175 = Course of Exchange.
\$.0105 = Bank Dis. on \$1, for 63 da.
\$1.007 = Cost of Exchange, for \$1.
\$1.007 × 1650 = \$1661.55, value of Draft.

4. Of a draft on New York at 30 da. for \$4720, at 1½% premium?

5. Of a draft on New Orleans, at 90 da., for \$5275, int. being 7%, and exchange ½% discount?

6. Find the cost in Philadelphia of a draft on Denver, at 90 da., for \$6400, the course of exchange being 101⅜?

7. What must be paid in New York for a draft on San Francisco, at 90 da., for \$5600, the course of exchange being 102⅛%?

701. Find the *Face*

1. Of a draft on St. Louis, at 90 da., purchased for \$4500, exchange being at 101½%?

OPERATION.—\$1.015 = Course of Exchange.
\$.0155 = Bank Dis. of \$1, for 93 da., at 6%.
\$.9995 = Cost of Exchange of \$1.
\$4500 ÷ .9995 = \$4502.25. (**520.**)

2. Of a draft on Richmond at 60 da. sight, purchased for $797.50, interest 7%, premium 2½%?

3. Of a sight draft bought for $711.90, discount 1⅛%.

4. A commission merchant sold 2780 lb. of cotton at 11½ cents a pound. If his commission is 2½%, and the course of exchange 98⅛%, how large a draft can he buy to remit to his consignor?

5. The Broadway Bank of New York having declared a dividend of 5%, a stockholder in Chicago drew on the bank for the sum due him, and sold the draft at a premium of 1¼%, thus realizing $2283.18¾ from his dividend. How many shares did he own?

6. A man in Rochester purchased a draft on Louisville, Ky., for $5320, drawn at 60 days, paying $5151.10. What was the course of exchange?

7. Received from Savannah 250 bales of cotton, each weighing 520 pounds, and invoiced at 12⅛ cents a pound. Sold it at an advance of 25%, commission 1⅛%, and remitted the proceeds by draft. What was the face of the draft, exchange being ½% discount?

FOREIGN EXCHANGE.

702. ***Money of Account*** consists of the denominations or divisions of money of any particular country, in which accounts are kept.

The Act of March 3, 1873, provides that "the value of foreign coin, as expressed in the money of account of the United States, shall be that of the pure metal of such coin of standard value; and the values of the standard coins in circulation, of the various nations of the world, shall be estimated *annually* by the Director of the Mint, and be proclaimed on the first day of January by the Secretary of the Treasury."

703. *The values in United States money of the pure gold or silver representing respectively the monetary units and standard coins of foreign countries are shown in the following Table published by the Secretary of the Treasury, January 1, 1875.*

COUNTRY.	MONETARY UNIT.	STANDARD.	VALUE IN U. S. MONEY.	STANDARD COINS.
Argentine Republic	Peso fuerte	Gold	$1.00	None.
Austria	Florin	Silver	.45,3	Florin.
Belgium	Franc	Gold and silver	.19,3	5, 10, and 20 francs.
Bolivia	Dollar	Gold and silver	.96,5	Escudo, ½ bolivar, and bolivar.
Brazil	Milreis of 1000 reis	Gold	.54,5	None.
British Possessions in N. Am.	Dollar	Gold	1.00	
Bogota	Peso	Gold	.91,2	
Central America	Dollar	Silver	.91,8	Dollar.
Chili	Peso	Gold	.91,2	Condor, doubloon, and escudo.
Cuba	Peso	Gold	.92,5	
Denmark	Crown	Gold	.26,8	10 and 20 crowns.
Ecuador	Dollar	Silver	.91,8	Dollar.
Egypt	Pound of 100 piasters	Gold	4.97,4	5, 10, 25, and 50 piasters.
France	Franc	Gold and silver	.19,3	5, 10, and 20 francs.
Great Britain	Pound sterling	Gold	4.86,6½	½ sovereign and sovereign.
Greece	Drachma	Gold and silver	.19,3	5, 10, 20, 50, and 100 drachmas.
German Empire	Mark	Gold	.23,8	5, 10, and 20 marks.
Hayti	Dollar	Silver	.95,2	
Japan	Yen	Gold	.99,7	1, 2, 5, 10, and 20 yen.
India	Rupee of 16 annas	Silver	.43,6	
Italy	Lira	Gold and silver	.19,3	5, 10, 20, 50, and 100 lire.
Mexico	Dollar	Silver	.99,8	Peso or dollar, 5, 10, 25, and 50 centavo.
Netherlands	Florin	Silver	.38,5	½ florin, florin, and 2½ florins.
Norway	Crown	Gold	.26,8	10 and 20 crowns.
Peru	Dollar	Silver	.91,8	
Porto Rico	Peso	Gold	.92,5	
Portugal	Milreis of 1000 reis	Gold	1.08,4	2, 5, and 10 milreis.
Russia	Rouble of 100 copecks	Silver	73,4	¼, ½, and 1 rouble.
Sandwich Islands	Dollar	Gold	1.00	
Spain	Peseta of 100 centimes	Gold and silver	.19,3	5, 10, 20, 50, and 100 pesetas.
Sweden	Crown	Gold	.26,8	10 and 20 crowns.
Switzerland	Franc	Gold and silver	.19,3	5, 10, and 20 francs.
Turkey	Piaster	Gold	.04,3	25, 50, 100, 250, and 500 piasters.
Uruguay	Patacon	Gold	.94,9	

704. ***Sterling Bills*** or ***Sterling Exchange*** are bills on England, Ireland, or Scotland. Such bills are negotiated at a rate fixed without reference to the par of exchange.

Formerly such bills were quoted at a certain rate % above the old par value of a pound sterling, which was \$4.44$\frac{4}{9}$. As this was entirely a fictitious value, and always about 9% below the real value, the course of exchange always appeared to be heavily against this country, and thus tended to impair its credit. By the Act of March, 1873, "all contracts made after the first day of January, 1874, based on an assumed par of exchange with Great Britain of fifty-four pence to the dollar, or \$4.44$\frac{4}{9}$ to the sovereign or pound sterling," are declared *null and void*. The *par of exchange* between Great Britain and the United States is fixed at \$4.8665.

705. ***Exchanges with Europe*** are effected chiefly through the following prominent financial circles: London, Paris, Antwerp, Amsterdam, Hamburg, Frankfort, Bremen, and Berlin.

In exchange on Paris, Antwerp, and Switzerland, the unit is the *franc*, and the quotation shows the number of francs and centimes to the dollar, Federal Money. In exchange on Amsterdam, the unit is the *guilder*, quoted at its value in *cents*; on Hamburg, Frankfort, Bremen, and Berlin, the quotation shows the value of *four reichsmarks* (marks) in *cents*.

WRITTEN EXAMPLES.

706. Find the *cost*

1. Of a bill of exchange on London at 3 days' sight, for £393 15s. 6d., exchange being quoted at 4.89½, and gold at 1.10⅛.

OPERATION.

£393 15s. 6d. = £393.775.
\$4.895 × 393 775 = \$1927.529, *gold* value of bill.
\$1927.529 × 1.10⅛ = \$2122.69, value in *currency*.

2. Of a bill of exchange on Liverpool, for £473 5s. 9d. par value, in gold.

3. Of a bill of £625 4s. 3d. sterling, at 4.83½, gold 1.09⅞.

4. Of a bill on *Paris* for 495 francs, at 5.15 francs to the dollar, in gold.

OPERATION.—495 ÷ 5.15 = \$96.12, gold value of the bill.

ANALYSIS.—Since 5.15 francs cost \$1, 495 francs will cost as many dollars as 5.15 francs are contained times in 495 francs, or \$96.12.

5. Of a bill on *Antwerp* for 697½ francs, at 5.17½ francs to the dollar, in gold.

6. Of a bill on *Geneva*, Switzerland, for 1655 francs, at 5.15⅝, in currency, gold being 1.09⅞.

7. Of a bill on *Frankfort* for 650 marks, at 94⅞, in gold.

OPERATION.—\$.94875 ÷ 4 × 650 = \$153.36.

ANALYSIS.—Since \$.94⅞ is the value of 4 marks, 650 marks are worth 650 times ¼ of \$.94⅞, or \$153.36.

8. Of a bill on *Berlin* for 1750 marks, quoted at 96¼, in gold.

9. Of a bill on Hamburg for 2155 marks, at 95⅜, in currency, gold being 1.10¼.

10. Of a bill on *Amsterdam* for 2500 guilders, quoted at 41⅝, brokerage ¼%.

OPERATION.—\$.41625 × 2500 = 1040.625.
\$1040.625 × .00¼ = \$2.60, brokerage.
\$1040.625 + \$2.60 = \$1043.225, cost of bill.

11. Of a bill on Amsterdam for 1950 guilders, at 41⅛.

12. Bought exchange on Amsterdam, at 41¼, for 3750 guilders; on Hamburg, at 95½ for 1000 marks; and on London for £500, at \$4.85. What was the cost of the whole in currency, gold selling at 109¾?

13. What will it cost to remit directly from Boston to Amsterdam, 12560 guilders, at 41¾?

14. What will be the cost of remitting 13550 marks from New York to Frankfort, exchange selling at 94¼, and gold at 109½; brokerage, ⅛%?

707. What will be the *face*

1. Of a bill of exchange on London that can be bought for $5500, in currency, exchange selling at 4.86, and gold at 1.10?

OPERATION.—$5500 currency ÷ 1.10 = $5000, gold, (**519.**)
$5000 ÷ $4.86 = 1028.806 +.
£1028.806 = £1028 16s. 1½d.

2. Of a bill on *Manchester*, England, that can be bought for $7500, gold; rate of exchange, 4.86?

3. Of a bill on *Berlin* that cost $4000 in gold, exchange 93¾?

OPERATION.—($4000 ÷ $.9375) × 4 = 17066⅔ marks.

ANALYSIS.—Since $.98¾ will buy 4 marks, $4000 will buy 4 times as many marks as $.98¾ is contained times in $4000, or 17066⅔ marks.

4. Of a bill on *Hamburg* that cost $550 in gold, exchange 94⅜?

5. Of a bill on *Frankfort* that cost $395.75 in gold, exchange 95⅛?

6. Of a bill on Geneva, Switzerland, that cost $325 in gold, exchange at 5.17?

OPERATION.—5.17 fr. × 325 = 1680.25 francs.

ANALYSIS.—If $1 will buy 5.17 francs, $325 will buy 325 times 5.17 francs, or 1680.25 francs.

7. A merchant in New Orleans gave $6186, currency, for a bill on Paris, at 5.15¼. What was its face?

8. What is the face of a bill on Antwerp, that may be purchased in New York for $2500, exchange at 5.16¼?

ARBITRATION OF EXCHANGE.

708. ***Arbitration of Exchange*** is the process of computing the cost of exchange between two places by means of one or more intermediate exchanges. Such exchange is said to be *indirect* or *circuitous.*

By this computation the relative cost of direct and indirect exchange is ascertained. Sometimes, owing to the course of exchange between different places, it is more advantageous to remit by the latter than by the former.

Arbitration is either *simple* or *compound.*

709. ***Simple Arbitration*** is that in which there is but one intermediate place.

710. ***Compound Arbitration*** is that in which there are several intermediate places.

WRITTEN EXERCISES.

711. 1. I owe 1500 marks to a merchant in Frankfort. Should I remit directly from New York, or through London, exchange on Frankfort being 94, on London, 4.87⅛, and in the latter place on Frankfort 20.75 marks to the pound, and the London brokerage ⅛%?

OPERATION.—\$.94 × 1500 ÷ 4 = \$352.50, cost of direct exchange.
1500 marks ÷ 20.75 marks = £72.29.
£72.29 + ⅛ % = £72.38.
\$4.87⅛ × 72.38 = \$352.85.
\$352.85 − \$352.50 = \$.35, loss by ind. exchange.

2. What will it cost to remit from Boston to Berlin 750 marks, by indirect exchange, through Paris, exchange in New York on Paris being at 5.15, and 4 marks at Paris being worth 4.91 francs, the brokerage being at ⅛%?

3. What will it cost to remit 2500 guilders from New York to Amsterdam, through London and Paris, the rates of exchange being as follows: at New York on London 4.83, at London on Paris 24.75 francs to the pound, and at Paris on Amsterdam 2.09 francs to the guilder, brokerage at London and Paris $\frac{1}{8}$% each?

OPERATION.

$\$ x$ = 2500 guilders.
1 guilder = 2.09 francs.
1 franc (net) = 1.00$\frac{1}{8}$ (with brokerage).
24.75 francs = £1.
£1 (net) = £1.00$\frac{1}{8}$ (with brokerage).
£1 = \$4.83.

Hence, $\dfrac{2500 \times 2.09 \times 1.00\frac{1}{8} \times 1.00\frac{1}{8} \times 4.83}{24.75}$, or

By cancellation, $\dfrac{100 \times 19 \times 1.00\frac{1}{8} \times 1.00\frac{1}{8} \times 1.61}{8} = \$1022.22.$

ANALYSIS.—Since the members of each equation are equal, the product of the corresponding members of any number of equations are equal; hence, the product of all the second members divided by the product of all the first members except one, must give that member, which is the value required.

4. A merchant in St. Louis directs his agent in New York to draw upon Philadelphia at 1% discount, for \$1500 due from the sale of mdse.; he then draws upon the New York agent, at 2% premium, for the proceeds, after allowing the agent to reserve $\frac{1}{2}$% commission. What sum does he realize from his mdse.?

OPERATION.

(x) St. L. = 1500 Philadelphia.
100 Phil. = 99 N. York.
100 N. Y. = 102 St. Louis.
1 = .995 (net proceeds).
By cancellation, .15 × 99 × 102 × .995=\$1507.18

Analysis.—$100 on Philadelphia = $99 on N. Y., and $100 on N. Y. = $102 on St. Louis; and since the agent reserves ½% commission, $1 realized = $.995 net proceeds. Arranging, canceling, and multiplying, we find the result to be $1507.13.

Rule.—I. *Represent the required sum by* (x), *with the proper unit of currency affixed, and place it equal to the given sum on the right.*

II. *Arrange the given rates of exchange so that in any two consecutive equations the same unit of currency shall stand on opposite sides.*

III. *When there is commission for drawing, place* 1 *minus the rate on the left if the cost of exchange is required, and on the right if proceeds are required; and when there is commission for remitting, place* 1 *plus the rate on the right, if cost is required, and on the left, if proceeds are required.*

IV. *Divide the product of the numbers on the right by the product of the numbers on the left, canceling equal factors, and the result will be the required sum.*

Commission for drawing is commission on the *sale* of a draft; commission for remitting is commission on the *purchase price* of a draft.

The above method of operation is sometimes called the *Chain Rule.*

5. If at New York exchange on London is 4.84½, and at London on Paris it is 25.73 francs to the £, what is the arbitrated course of exchange between New York and Paris?

6. If in London exchange on Paris is 25.71, and in New York on Paris it is 5.15¼, what is the arbitrated course of exchange between New York and London?

7. A banker in New York remits $5000 to Liverpool by indirect exchange, through Paris, Hamburg, and Amsterdam, the rates being as follows: in New York on Paris 5.18 fr. to the dollar, in Paris on Hamburg 1.22 fr. to the mark, in Hamburg on Amsterdam 1.70 mark to the guilder, and in Amsterdam 11.83 guilders to the pound sterling. How much sterling will he have in bank at Liverpool, and how much does he gain by indirect exchange, sterling being worth in New York 4.83½ ?

8. A merchant in Philadelphia owes a correspondent in Paris 35000 francs. Direct exchange on Paris is 5.15; but exchange on London is 4.83, and London exchange on Paris is 25.12. Allowing ½% commission for brokerage at London, which is the more advantageous way to remit, and by how much ?

9. An American resident at Amsterdam wishing to obtain funds from the U. S. to the amount of $4500, directs his agent in London to draw on Philadelphia, and remit the proceeds to him in a draft on Amsterdam, exchange on London in Phil. selling at 4.87½, and in London on Amsterdam 11.17½ guilders to the pound sterling. If the agent charges commission at ½% both for drawing and remitting, how much better is this arbitration than to draw directly on the U. S. at 41¼ cents per guilder?

10. A speculator residing in Cincinnati, having purchased 165 shares of railroad stock in New Orleans, at 75%, remits to his agent in N. York a draft purchased at 2% premium, directing the agent to remit the sum due on N. Orleans. Now, if exchange on N. Orleans is at ⅜% discount in N. Y., and the agent's commission for remitting is ⅓%, how much does the stock cost in Cincinnati ?

CUSTOM-HOUSE BUSINESS.

712. A ***Custom-House*** is an office established by government for the transaction of business relating to the collection of customs or duties, and the entry and clearance of vessels.

713. A ***Port of Entry*** is a seaport town in which a custom-house is established.

714. The ***Collector of the Port*** is the officer appointed by government to attend to the collection of duties and to other custom-house business.

715. A ***Clearance*** is a certificate given by the Collector of the port, that a vessel has been entered and cleared according to law.

By the *entry* of a vessel is meant the lodgment of its papers in the custom-house, on its arrival at the port.

716. A ***Manifest*** is a detailed statement, or invoice, of a ship's cargo.

No goods, wares, or merchandise can be brought into the United States by any vessel, unless the master has on board a full manifest, showing in detail the several items of the cargo, the place where it was shipped, the names of the consignees, etc.

717. ***Duties*** or ***Customs*** are taxes levied on imported goods.

The general object of such taxes is the support of government, but they are also designed sometimes to protect the manufacturing industry of a country against foreign competition.

718. A ***Tariff*** is a schedule showing the rates of duties fixed by law on all kinds of imported merchandise

Duties are of two kinds, *Specific* and *Ad Valorem*.

719. A ***Specific Duty*** is a fixed sum imposed on articles according to their weight or measure, but without regard to their value.

720. An ***Ad Valorem Duty*** is an import duty assessed by a percentage of the value of the goods in the country from which they are brought.

Before computing specific duties, certain deductions, or allowances, are made, called *Tare*, *Leakage*, *Breakage*, etc.

721. ***Tare*** is an allowance for the weight of the box, cask, bag, etc., that contains the merchandise.

722. ***Leakage*** is an allowance for waste of liquors imported in casks or barrels.

723. ***Breakage*** is an allowance for loss of liquors imported in bottles.

724. ***Gross Weight*** or ***Value*** is the weight or value of the goods before any allowance is made.

725. ***Net Weight*** or ***Value*** is the weight or value of the goods after all allowances have been deducted.

WRITTEN EXERCISES.

726. Find the *Duty*

1. On 355 yds. of carpeting, invoiced at 11s. 6d. per yd., the duty being 50%.

OPERATION.—11s. 6d. = £.575.
£.575 × 355 = £204.125.
$4.8665 (par value of £1) × 204.125 = $993.37.
$993.37 × .50 = $496.68, duty. (**510.**)

2. On 50 hhd. of sugar, each containing 500 lb., at $5\frac{1}{2}$ cts. per lb.; duty $1\frac{3}{4}$ cts. per lb.

3. On 350 boxes of cigars, each containing 100 cigars, invoiced at $7.50 per box; weight, 12 lb. per 1000; duty, $2.50 per lb., and 25% ad valorem.

4. A wine merchant in New York imported from Havre 100 doz. quart bottles of champagne, at $13 per doz., and 25 casks of sherry wine, each containing 30 gals., at $2.50 per gal. What is the duty, the rate on the champagne being $6 per dozen, and on the sherry 60 cents per gal., and 25% ad valorem?

5. Imported from Geneva 25 watches invoiced at $125 each, and 15 clocks, at $37.50. What was the duty, the rate being on clocks 25%, and on watches, 35% ad valorem?

6. A liquor dealer receives an invoice of 120 dozen pint bottles of porter, rated at $.75 per dozen. If 2½% of the bottles are found broken, what will be the duty at 36 cts. per gallon?

7. H. B. Claflin & Co. imported 20 cases of bleached muslins, each case containing 175 pieces of 24 yards each, 1¼ yards wide. What was the duty at 5¼ cts. per square yard?

8. What was the duty on 10 cases of shawls, average weight of each case 213¼ lb., invoiced at 19375 francs; rate of duty, 50 cts. per lb. and 35% ad valorem? If I pay for the invoice with a bill of exchange bought at 5.15½, and pay charges amounting to $67.50 currency, what do the shawls cost me in currency, gold selling at 1.10?

9. Olmsted & Taylor, of New York, import from Switzerland 1 case of watches, invoiced at 7125 francs; duty, 25%; charges, 13.50 francs; commissions, 2½%. What was the cost of the watches in U. S. gold?

10. Imported from England 5 cases of cloths and cassimeres, net weight, 695 lb.; value as per invoice, £375 10s. What was the duty in American gold, the rate being 50 cts. per lb. and 35% ad valorem?

EQUATION OF PAYMENTS.

727. ***Equation of Payments*** is the process of finding the average time for the payment of several sums of money due at different times, without loss to debtor or creditor.

728. The ***Equated Time*** is the date at which the several debts may be discharged by one payment.

729. The ***Term of Credit*** is the time at the expiration of which a debt becomes due.

730. The ***Average Term of Credit*** is the time at the end of which the several debts due at different dates, may all be paid at once, without loss to debtor or creditor.

ORAL EXERCISES.

731. 1. The interest of $100 for 3 mo. equals the interest of $50 for how many months?

ANALYSIS.—At the same rate, the interest of $100 equals the interest of $50, or *one-half* of $100, for *twice* the time, or 6 mo.

2. The interest of $20 for 4 mo. equals the interest of $10 for how many mo.? Equals the interest of $5 for how many mo.? Of $1? Of $40? Of $100?

3. The interest of $25 for 6 mo. equals the interest of $5 for how many mo.? Of $10? Of $1?

4. The interest of $10 for 6 mo., and of $100 for 2 mo., taken together, equals the interest of $1 for how many months?

5. If I borrow $50 for 3 mo., for how many months should I lend $100 to repay an equal amount of interest?

Analysis.—The interest of $50 for 3 mo. is the same as the interest of $1 for 50 times 3 mo., or 150 mo.; and the interest of $1 for 150 mo. is the same as the interest of $100 for $\frac{1}{100}$ of 150 mo., or 1½ mo.

6. If I lend $200 for 3 mo., for how long a time should I have the use of $50 to balance the favor?

7. If A borrows of B $1000 for 3 mo., what sum should A lend B for 9 mo. to discharge the obligation?

732. **Principle.**—*The interest and rate remaining the same, the greater the principal the less the time, and the less the principal the greater the time.*

WRITTEN EXERCISES.

733. Find the *average term of credit*

1. Of $300 due in cash, $500 due in 3 mo., $750 due in 8 mo., and $950 due in 10 mo.

Operation.

$$\begin{array}{rcrcr}
300 & \times & 0 & = & 0 \\
500 & \times & 3 & = & 1500 \\
750 & \times & 8 & = & 6000 \\
\underline{950} & \times & 10 & = & \underline{9500} \\
2500 & & &) & \underline{17000} \\
 & & & & 6\frac{4}{5}\text{ mo.}
\end{array}$$

Analysis.—On $300, the first payment, there is no interest, since it is due in cash; the int. of $500 for 3 mo., is the same as the int. of $1 for 1500 mo.; the int. of $750 for 8 mo. is the same as that of $1 for 6000 mo.; and the int. of $950 for 10 mo. is the same as the int. of $1 for 9500 mo. Therefore, the whole amt. of int. is that of $1 for 1500 mo. + 6000 mo. + 9500 mo., or 17000 mo.; but the whole debt is $2500; and the int. of $1 for 17000 mo. is equal to the int. of $2500 for $\frac{1}{2500}$ of 17000 mo., or 6⅘ mo.

2. Find the average term of credit of $800 due in 1 mo., $750 due in 4 mo., and $1000 due in 6 mo.

Rule.—I. *Multiply each payment by its term of credit, and divide the sum of the products by the sum of the payments; the quotient is the average term of credit.*

II. (To find the equated time of payment,) *Add the average term of credit to the date at which the several credits begin.*

3. On the first day of December, 1876, a man gave 3 notes, the first for $500 payable in 3 mo.; the second for $750 payable in 6 mo.; and the third for $1200 payable in 9 mo. What was the average term of credit, and the equated time of payment?

4. Bought merchandise Jan. 1, 1875, as follows: $350 on 2 mo., $500 on 3 mo., $700 on 6 mo. What is the equated time of payment?

5. A person owes a debt of $1680 due in 8 months, of which he pays $\frac{1}{3}$ in 3 mo., $\frac{1}{4}$ in 5 mo., $\frac{1}{6}$ in 6 mo., and $\frac{1}{8}$ in 7 mo. When is the remainder due?

6. Bought a bill of goods, amounting to $1500 on 6 months' credit. At the end of 2 mo., I paid $300 on account, and 2 mo. afterward, paid $400 on account, at the same time giving my note for the balance. For what time was the note drawn?

OPERATION.

$$\begin{array}{l}
300 \times 4 = 1200 \\
\underline{400} \times 2 = \underline{800} \\
800 \qquad \overline{)2000} \\
\qquad\qquad\quad 2\frac{1}{2}
\end{array}$$

$$(6 \text{ mo.} - 4 \text{ mo.}) + 2\tfrac{1}{2} \text{ mo.} = 4\tfrac{1}{2} \text{ mo.}$$

Analysis.—$300 paid 4 mo. before it is due, and $400, 2 mo. before it is due, are equivalent to the use of $1 for 2000 months. or the use of $800 (the balance) for 2½ mo. beyond the original time. Hence, the note was drawn for 4½ mo. after the second payment.

7. On a debt of $2500 due in 8 mo. from Feb. 1, the following payments were made: May 1, $350, July 1, $300, and Sept. 1, $500. When is the balance due?

8. Find the average term of credit, and the equated time of payment from Dec. 15, of $225 due in 35 da., $350 due in 60 da., and $750 due in 90 da.

9. Dec. 1, 1874, purchased goods to the amount of $1200, on the following terms: 25% payable in cash, 30% in 3 mo., 20% in 4 mo., and the balance in 6 mo. Find the equated time of payment, and the cash value of the goods, computing discount at 7%.

734. To find the equated time when the terms of credit begin at different dates.

1. J. Prince bought goods of W. Sloan as follows: June 1, 1874, amounting to $350 on 2 mo. credit; July 15, 1874, $400, on 3 mo. credit; Aug. 10, $450, on 4 mo. credit; Sept. 12, $600, on 6 mo. credit. What is the equated time of payment?

OPERATION.

$350	due	Aug. 1,	350	× 0	=	0
400	"	Oct. 15,	400	× 75	=	30000
450	"	Dec. 10,	450	× 131	=	58950
600	"	Mar. 12,	600	× 223	=	133800
			1800		1800)	222750
						123¾

Hence the equated time is 124 da. from Aug. 1, or *Dec. 3.*

ANALYSIS.—Computing the terms of credit from Aug. 1, the *earliest date* at which any of the debts become due, we find the terms of credit to be from Aug. 1 to Oct. 15, 75 da.; to Dec. 10, 131 da., and to March 12, 223 da. The average term of credit is therefore 124 da. from Aug. 1, and the equated time Dec. 8.

PROOF.—Assume as the standard time the latest date, March 12. The operation will then be as follows·

$$\begin{array}{rcrcr} 350 & \times & 223 & = & 78050 \\ 400 & \times & 148 & = & 59200 \\ 450 & \times & 92 & = & 41400 \\ 600 & \times & 0 & = & 0 \\ \hline & & & 1800) & 178650 \\ \hline & & & & 99\frac{1}{4} \end{array}$$

Hence, the equated time is 99 da. previous to March 12, or *Dec. 3.*

2. Peake & Co. sell to Wm. Jones the following bills of goods: March 1, 1875, on 60 da., $800; April 15, on 30 da., $350; May 20, on 4 mo., $3800.

What is the equated time for settlement?

RULE.—I. *Find the date at which each debt becomes due.*

II. *From the earliest of these dates as a standard compute the time to each of the others.*

III. *Then find the average term of credit and equated time as in* **(733).**

PROOF.—*Compute the terms of credit backward from the latest date, and subtract the average time from that date for the equated time.*

If the earliest date is not the *first* of the month, it is more convenient to assume the *first* of the month as the standard date.

3. Bought mdse. as follows: Jan. 15, 1876, on 4 mo., $375; Feb. 3, on 60 da., $550; March 25, on 4 mo., $1100; April 2, on 30 da., $250. Find the equated time.

4. Ira Blunt, of Gadsden, Ala., bought of Opdyke & Co. the following bills of goods on 4 months' credit:

Jan. 1, 1874, $650; Feb. 10, $380; March 12, $900; March 18, $350; April 3, $600.

April 5, he discounted his bills at 2% per month. Find the equated time of payment, and the discount.

5. **James Smith** *to* **Thomas Brown,** *Dr.*

March 10, 1874.	To mdse.	$835.
" 18, "	" "	320.
" 26, "	" "	475.
April 5, "	" "	600.
" 12, "	" "	250.

Allowing 30 days' credit on each of the bills, what is the equated time of payment?

6. Purchased goods as follows:

Sept. 15, 1875,	a bill of	$275,	on 3 mos.
Oct. 10, "	"	351.50,	" 60 da.
" 28, "	"	415.75,	" 30 da.
Nov. 3, "	"	500,	" 4 mos.
Dec. 15, "	"	710,	" 3 mos.

What was due on this account Aug. 10, 1876, computing interest at 7%?

7. I have four notes, as follows: the first for $425, due April 1, 1875; the second for $615, due May 10, 1875; the third for $1500, due May 28, 1875; and the fourth for $750, due June 10, 1875.

At what date should a single note be made payable, to be given in exchange for the four notes?

AVERAGING ACCOUNTS.

735. An ***Account*** is a written statement of debit and credit transactions, with their respective dates.

Debit means what is owed by the person with whom the account is kept; *credit*, what is due to him from the person keeping the account.

736. To ***Average an Account*** is to find, either the *equated time* of paying the balance, or the *cash balance* at any given time.

Each item of a book account should draw interest from the time it becomes due.

WRITTEN EXERCISES.

737. 1. Find the *equated time* of paying the balance of the following account.

Dr. WILLIAM SAMPSON. *Cr.*

1875.			1875.		
Jan. 11	To mdse. . . .	$750	Feb. 10	By draft at 60 da.	$500
Feb. 1	" " at 3 mo.	600	Mar. 3	" cash . . .	700
Mar. 15	" " at 6 mo.	1500	Apr. 15	" " . . .	300
May 3	" " at 4 mo.	900			

OPERATION I. (*Method by Products.*)

Due.	*Amt.*		*Days.*		*Product.*
Jan. 11.	750	×	10	=	7500
May 1.	600	×	120	=	72000
Sept. 15.	1500	×	257	=	385500
" 3.	900	×	245	=	220500
	3750				685500
	1500				125400
	2250				)560100

Paid.	*Amt.*				
Apr. 14.	500	×	103	=	51500
Mar. 3.	700	×	61	=	42700
Apr. 15.	300	×	104	=	31200
	1500				125400

$248\frac{42}{45}$, or 249 da.

Balance due 249 da. from Jan. 1, or *Sept. 7.*

ANALYSIS.—Assuming for convenience Jan. 1 as the standard date, we find as in **734** the term of credit of each debit amount; and, reckoning from the same date, the time to each credit amount. Multiplying each amount by its time in days, and adding the debit and credit products, we find the number of days' interest of $1 due to the debtor, and the number of days' interest of $1 he has already received. The difference, 560100, shows the number of days' interest of $1 still due, and as the balance is $2250, the time must be $\frac{1}{2250}$ of 560100 da., or 249 da. Hence, the *equated time* is 249 da. from Jan. 1, or *Sept. 7.*

OPERATION II. (*Method by Interest.*)

Dr.

$750 to Jan. 11 (from Jan. 1) = 10 da., int. at 1% per mo. $2.50
600 " Feb. 1 + 3 mo. = 4 mo. " " 24.00
1500 " Mar. 15 + 6 mo. = 8 mo. 14 da., " " 127.00
900 " May 3 + 4 mo. = 8 mo. 2 da., " " 72.60
$3750 $226.10

Cr.

$500 to Feb. 10 + 63 da. = 3 mo. 13 da., int. at 1% per mo. $17.17
700 " Mar. 3 = 2 mo. 2 da., " " 14.47
300 " Apr. 15 = 3 mo. 14 da., " " 10.40
$1500 $42.04

$226.10 − $42.04 = $184.06, int. at 1% per mo. due.
Int. of balance, $2250, for 1 mo., at 1% = $22.50.
Hence, $184.06 ÷ $22.50 = 8.18 + mo., or 8 mo. 6 da.
8 mo. 6 da. from Jan. 1, or Sept. 7, *Equated Time.*

In this operation, 12% per annum or 1% per mo. is assumed for convenience; since the int. at 1% per mo. is as many hundredths as there are months, and one-third as many thousandths as there are days. Thus, the int. of $249 for 2 mo. 9 da. is $4.98 + $.747 = $5.727 (**571**).

2. Find the *equated time* of the following:

Dr. WILLIAM SIMPSON. *Cr.*

1874.			1874.		
Aug. 5	To mdse. at 3 mo.	$720	Oct. 10	By cash . . .	$500
Sept. 10	" " " 2 "	850	Dec. 15	" draft at 60 da.	450
Nov. 3	" "	1200	" 25	" cash . . .	900
1875.			1875.		
Jan. 20	" sundr's at 5 mo.	620	Jan. 3	" " . . .	250

RULE 1.—I. *Find the date at which each debit item is due, and each credit item is paid or due.*

II. *Take the first day of the month in the earliest date on either side of the account as a standard date, and mul-*

tiply each sum due or paid by the number of days between its time and the standard date.

III. *Add the products, and their difference divided by the balance due will give the number of days between the standard date and the equated time.* Or,

RULE 2.—*Find the time of each item from the standard date as before, and compute the interest on each at 1% a month. The difference between the amount of interest on each side divided by the interest of the balance at 1% for one month will be the equated time.*

When the terms of credit are long, Rule 2 gives the shorter method.

3. Find the equated time of the following, allowing 60 da. credit on each debit item :

Dr. JOHN DRISCOLL. *Cr.*

1877.			1877.		
June 1	To mdse. . .	$950	Aug. 1	By cash . .	$700
July 6	“ “ . .	300	Sept. 20	“ “ . .	1000
Sept. 8	“ “ . .	1900	Nov. 1	“ “ . .	1200
Oct. 20	“ “ . .	2600			

4. What is the equated time for the payment of the balance of the following account, allowing 4 months' credit on all the debit items ?

Dr. DODD, BROWN & CO. *Cr.*

1878.			1878.		
Jan. 20	To mdse. . .	$570	Feb. 14	By mdse. . .	$490
“ 28	“ “ . .	300	Mar. 1	“ cash . .	1000
Feb. 11	“ “ . .	720	Apr. 2	“ “ . .	1800
“ 26	“ “ . .	835			
Mar. 10	“ “ . .	1150			
“ 28	“ “ . .	960			
Apr. 15	“ “ . .	475			

738. 1. Find the *cash balance* of the following account on the 22d of August, allowing interest at 6%:

Dr. GEORGE HAMMOND. *Cr.*

1875.			1875.		
Mar. 15	To mdse., at 3 mo.	$600	May 10	By cash . .	$300
Apr. 3	" " " 4 mo.	700	July 1	" " . .	400
May 10	" " " 6 mo.	1000	Aug. 15	" " . .	500

OPERATION.—By averaging the account, the equated time for paying the balance, $1100, is found to be *Nov.* 4. (**734.**)

True present worth of $1100 for 74 da. (from Aug. 22 to Nov. 4) is $1086.60, or cash balance Aug. 22.

Or, by *Interest Method,* as follows:

Dr.

Int. of $600, from June 15 to Aug. 22,	68 da.,	$6.71	(**574.**)
" " 700, " Aug. 3 " "	19 da.,	2.19	
		$8.90	

Cr.

Int. of $1000, from Aug. 22 to Nov. 10,	80 da.,	$13.15
" " 300, " May 10 " Aug. 22,	104 da.,	5.13
" " 400, " July 1 " "	52 da.,	3.42
" " 500, " Aug. 15 " "	7 da.,	.58
		$22.28
		8.90
Balance of interest due Hammond,		$13.38

$1100 − $13.38 = $1086.62, Cash Balance, Aug. 22

ANALYSIS.—Charge Hammond with interest on each debit item from the time it is due to date of settlement, and credit him with interest on each sum paid from the date of payment to date of settlement, also on each debit item which becomes due after the date of settlement. Hence, he is entitled to interest on $1000 from Aug. 22 to Nov. 10. As the balance of interest is in favor of Hammond, it must be deducted from the balance of the account, to obtain the cash balance. There is a slight difference in the results, but the interest method is the more accurate. By the use of Interest Tables, it is also the shorter of the two methods.

Rule 1.—I. *Average the account, and find the equated time of payment of the balance.*

II. *If the date of settlement is prior to the equated time, find the present worth of the balance of account for the cash balance; if later, find the interest of the balance of account for the intervening time, and add it to find the cash balance.* Or,

Rule 2.—*Find the interest on each debit and credit item, from the time it is due or paid to the date of settlement, placing on the same side of the account the interest on each item due prior to the date of settlement, and on the opposite side the interest on each item due after the date of settlement. If the balance of interest is on the same side as the balance of the account, add it, if on the other side subtract it; and the result will be the cash balance at the date of settlement.*

2. I owe $1500 due May 1, and $750 due Aug. 15. If I give my note at 30 da. for $450, June 1, and pay $370 in cash July 15, what is the equated time for paying the balance; and what would be due in cash Dec. 10, allowing interest at 7%?

3. When is the balance of the following account due per average?

Dr. O. B. Timpson. *Cr.*

1875.			1875.		
Aug. 10	To mdse. @ 60 da. .	$751.35	Oct. 3	By cash	$300.00
Sept. 5	" " @ 30 da. .	425.00	Nov. 15	" note @ 90 da. .	450.00
Nov. 1	" " @ 90 da. .	927.83	Dec. 20	" cash	500.00
Dec. 5	" " @ 30 da. .	1200.00			

4. What is the cash balance of the above account Jan. 1, 1876, allowing interest at 10%?

5. Find the equated time, and cash balance July 1, of the following, allowing 7% interest:

Dr. THOMAS SMITH. *Cr.*

Jan. 4	To mdse. @ 4 mo.	$1600	Feb. 1	By mdse. @ 4 mo.	$500
" 6	" " @ 3 mo.	1500	Mar. 2	" cash . . .	2000
Apr. 10	" " @ 60 da.	3000	" 25	" "	3150
" 28	" " @ 30 da.	2500	Apr. 16	" "	800

6. Average the following account, and find for what amount a note at 60 days should be given Aug. 1, to pay the balance, interest at 6%.

Dr. ORSON HINMAN. *Cr.*

1875.			1875.		
Apr. 2	To charges .	$87.25	Feb. 25	By mdse. @ 8 mo.	$600
May 15	" " .	35.75	Mar. 3	" " @ 6 "	300
			Apr. 1	" " @ 6 "	500

739. 1. Average the following *Account Sales*, and find when the net proceeds are due. (**543.**)

Account Sales of 1200 bbls. of flour received from Smith, Tyler & Co., Cincinnati.

Date.	Buyer.	Quantity.	Price.	Amount.
1876.				
May 1	J. Brooke	300 bbl.	@ $5.50, 3 mo.	$1650.00
June 5	W. Long	450 "	@ 6.20, 4 mo.	2790.00
" 15	A. Bruce	250 "	@ 6.50, 6 mo.	1625.00
July 1	W. Case	200 "	@ 5.75, 2 mo.	1150.00
				$7215.00

CHARGES.

Apr. 28.	Freight	$674.50	
" "	Cartage	37.50	
May 1.	Storage	191.00	
	Commission on $7215, @ $2\frac{1}{4}\%$ . . .	162.34	
	Total charges		$1065.34
	Net proceeds due per average		$6149.66

OPERATION.

Average of sales, found by the method of Equation of Payments, *Oct. 1*, which is the date at which the commission is due.

Average of charges, including commission (Oct. 1), *May 22.* Equated time of $7215 due Oct. 1, and $1065.34 due May 22, *Oct. 24*, date when the net proceeds are due.

RULE.—I. *Average the sales alone, and the result will be the date to be given to the commission and guaranty.*

II. *Make the sales the credits and the charges the debits, and find the equated time for paying the balance.*

2. Make an account sales, and find the net proceeds and the time the balance is due:

Wm. Brown, of N. York, sold on acct. of J. Berry, of Chicago, June 1, 350 bu. Winter Wheat, @ $1.35, at 60 da.; June 15, 275 bu. Spring Wheat, @ $1.75, at 90 da.; July 3, 1260 bu. Indian Corn, @ $.79, at 6 mo.; and July 10, 375 bu. Rye, @ $1.02, at 3 mo. Paid freight, May 28, $567.50; cartage, May 30, $22.50; insurance, June 5, $56.25; and charged com. at $3\frac{1}{4}\%$, and $1\frac{3}{4}\%$ for guaranty.

3. Sold on account of Brown, Sampson & Co., at 6 mo.: Oct. 1, 1874, 13 hhd. sugar, averaging 1520 lb., @ $.12$\frac{1}{2}$; Oct. 5, 15 chests Hyson Tea, each 95 lb., @ $1.05. Paid charges: Oct. 3, Insurance, $85; Oct. 10, Cooperage, etc., $24.50; Oct. 20, Cartage, $125. Charged commission and guaranty, $4\frac{1}{2}\%$. Make an account sales, and find the equated time for paying the net proceeds.

740. SYNOPSIS FOR REVIEW.

- PERCENTAGE—CONTINUED.
  - 21. EXCHANGE.
    - 1. Defs.
      - 1. *Exchange.* 2. *Domestic Exchange.* 3. *Foreign Exchange.* 4. *Bill of Exchange.* 5. *Set of Exchange.* 6. *Sight Draft or Bill.* 7. *Time Draft or Bill.* 8. *Buyer or Remitter.* 9. *Acceptance.* 10. *Par of Exchange.* 11. *Course or Rate of Exchange.*
    - 2. Forms.
      - 1. *A Sight Draft.* 2. *Set of Exchange.*
    - 3. Inland Exch.
      - 1. **700.** To find *Cost of Draft. Formula.*
      - 2. **701.** To find *Face of Draft.*
    - 4. Foreign Exch'ge.
      - 1. Defs.
        - 1. *Money of Account.*
        - 2. *Sterling Bills, or Exchange.*
      - 2. Exchange with Europe—how effected.
      - 3. **706.** To find *Cost of Bill.*
      - 4. **707.** To find *Face of Bill.*
    - 5. Arbitration of Exch'ge.
      - 1. Defs.
        - 1. *Arbitration of Exchange.*
        - 2. *Simple Arbitration.*
        - 3. *Compound Arbitration.*
      - 2. Rule, I, II, III, IV.
  - 22. CUSTOM-HOUSE BUSINESS.
    - 1. Defs.
      - 1. *Custom House.* 2. *Port of Entry.* 3. *Collector.* 4. *Clearance.* 5. *Manifest.* 6. *Duties or Customs.* 7. *Tariff.* 8. *Specific Duty.* 9. *Ad Valorem Duty.* 10. *Gross W'ght.* 11. *Net Weight.*
    - 2. **726.** To find the Duty.
  - 23. EQUATION OF PAYMENTS.
    - 1. Defs.
      - 1. *Equation of Payments.* 2. *Equated Time.* 3. *Term of Credit.* 4. *Average Term of Credit.*
    - 2. Principle.
    - 3. **733.** Rule, I, II.
    - 4. **734.** Rule, I, II, III. Proof.
  - 24. AVERAGING ACCOUNTS.
    - 1. Defs. 1. *Account.* 2. *To Average an Acct.*
    - 2. **737.** Rule 1, I, II, III. Rule 2.
    - 3. **738.** Rule 1, I, II. Rule 2.
    - 4. **739.** Rule, I, II.

RATIO

ORAL EXERCISES.

741. 1. A father is 30 years old, and his son 6; how many times as old as the son is the father?

2. 30 are how many times 6? $30 \div 6 = ?$

3. What part of $30 are $6? Of 20 cents are 5 cents?

4. What is the relation of 8 to 2? Of 40 rd. to 4 rd.?

5. What relation has 12 to 3? 60 lb. to 20 lb.?

Compare the following, and give their relative values:

6. 75 with 5.
7. 25 with $6\frac{1}{4}$.
8. 1 with 7.
9. $\frac{1}{7}$ with 7.
10. $2\frac{1}{3}$ with $3\frac{1}{2}$.
11. .9 with .3.
12. $.6 with $.2.
13. .42 with .3.
14. $\frac{6}{5}$ with $\frac{3}{4}$.

DEFINITIONS.

742. ***Ratio*** is the relation between two numbers of the same unit value, expressed by the quotient of the first divided by the second. Thus the ratio of 12 to 4 is $12 \div 4 = 3$.

743. The ***Sign*** of ratio is the colon (:), or the sign of division with the line omitted.

Thus, the ratio of 9 to 3 is expressed 9 : 3, or 9÷3, or in the form of a fraction $\frac{9}{3}$, and is read, the ratio of 9 to 3, or 9 divided by 3.

744. The ***Terms*** of a ratio are the two numbers compared.

745. The ***Antecedent*** is the first term, or dividend.

746. The ***Consequent*** is the second term, or divisor.

747. The ***Value*** of a ratio is the *quotient* of the antecedent divided by the consequent, and is an *abstract* number.

Thus, in the ratio \$18 : \$6, \$18 and \$6 are the *terms* of the ratio, \$18 is the *antecedent*; \$6 is the *consequent*; and 3, the quotient of \$18 ÷ \$6, is the *value* of the ratio.

748. A ***Simple Ratio*** is the ratio of two numbers; as 10 : 5.

749. A ***Compound Ratio*** is the ratio of the products of the corresponding terms of two or more simple ratios.

Thus the ratio compounded of the simple ratios,

$$\left.\begin{matrix}8:\ 4\\ 9:12\end{matrix}\right\}\text{ may be expressed }\left\{\begin{matrix}(8:4)\times(9:12)\\ \text{or, }\ 8\times 9 : 4\times 12\end{matrix}\right\} = 72:48:$$

Or, $\frac{8}{4}\times\frac{9}{12}=\frac{3}{2}=3:2.$

When the multiplication is performed the *result* is a *simple* ratio.

750. The ***Reciprocal*** of a ratio is 1 divided by the ratio (**196**), or it is the consequent divided by the antecedent. Thus the ratio of 8 to 9 is 8 : 9, or $\frac{8}{9}$, and its *reciprocal* is $\frac{9}{8}$.

The ratio of two *fractions* is obtained by reducing them to a common denominator, when they are to each other as their numerators (**241**).

If the terms of a ratio are denominate numbers, they must be reduced to the same unit value.

751. From the preceding definitions and illustrations are deduced the following

FORMULAS.—1. *The Ratio = Antecedent ÷ Consequent.*
2. *The Consequent = Antecedent ÷ Ratio.*
3. *The Antecedent = Consequent × Ratio.*

752. Since the antecedent is a dividend, and the consequent a divisor, any change in either or both of the terms of a ratio will affect its value according to the laws of division or of fractions (**200**), which laws become the

GENERAL PRINCIPLES OF RATIO.

1. *Multiplying the antecedent*, or *Dividing the consequent*,	*Multiplies the ratio.*
2. *Dividing the antecedent*, or *Multiplying the consequent*,	*Divides the ratio.*
3. *Multiplying or dividing both antecedent and consequent by the same number*,	*Does not change the ratio.*

753. These principles may be embraced in one

GENERAL LAW.

A change in the antecedent produces a LIKE *change in the ratio; but a change in the consequent produces an* OPPOSITE *change in the ratio.*

EXERCISES.

754. 1. Express the ratio of 11 to 4; of 16 to 2; of 20 to $6\frac{2}{3}$; of \$36 to \$12; of 9 lb. to 27 lb.; of $4\frac{1}{2}$ bu. to 9 bu.

2. Can you express the ratio between \$15 and 5 lb.? The reason?

3. Indicate the ratio of 18 to 20 in *two forms*. What are the *terms* of the ratio? The *antecedent?* The *consequent?* The *kind* of ratio? The *value* of the ratio?

In like manner express, analyze, and give the value,

4. Of 80 to 120; of $12\frac{1}{2}$ to $37\frac{1}{2}$; of $16\frac{1}{8}$ to $\frac{3}{8}$.

5. Of 5.2 to 1.3; of $\frac{3}{4}$ to $\frac{9}{10}$; of $\frac{2 \times 27 \times 42}{12 \times 4 \times 126}$.

6. The antecedents of a ratio are 7 and 10, and the consequents, 5 and 4. What is the value of the ratio?

7. The first terms of a ratio are 18, 12, and 30, the second, 54, 6, and 15. What is the kind of ratio? Express in *three forms.* Find its value in the lowest terms.

Solve, and state the *formula* applied to the following:

8. The consequent is $3\frac{1}{4}$, the antecedent $\frac{13}{28}$; what is the ratio?

9. The antecedent is 60, the ratio 7; what is the consequent?

10. The consequent is 6.12\frac{1}{2}$, the ratio $\frac{13}{28}$; what is the antecedent?

11. The ratio is $2\frac{2}{3}$, the antecedent $\frac{1}{2}$ of $\frac{4}{8}$; what is the consequent?

12. The ratio is 6, the consequent 1 wk. 3 da. 12 hr.; what is the antecedent?

13. Express the ratio of 120 to 80, and give its value in the lowest terms.

14. Make such changes in the last example as will illustrate PRIN. 1.

15. With the same example, illustrate PRIN. 2.

16. Illustrate by the same example PRIN. 3.

17. Find the *reciprocal* of the ratio of 75 to 15.

18. Find the *reciprocal* of the ratio of 2 qt. 1 pt. to 4 gal. 1 qt. 1 pt.

What is the ratio

19. Of 40 bu. 4.5 pk. to 25 bu. 2 pk. 1 qt.

20. Of 6 A. 110 P. to 10 A. 60 P.

21. Of 25 lb. 11 oz. 4 pwt. to 19 lb. 5 oz. 8 pwt.

22. Of $\frac{12\frac{1}{2}}{\frac{1}{4}}$ to $\frac{\frac{2}{3}\text{ of }\frac{3}{4}}{\frac{1}{3}}$.

PROPORTION

ORAL EXERCISES.

755. 1. What is the ratio of 4 to 2? Of 6 to 1? Of 14 to 7? Of 21 to 3?

2. Find two numbers that have the same quotient as 8 ÷ 2. As 27 ÷ 3. As 16 ÷ 4. As 30 ÷ 6. As 4 ÷ ½.

3. Express in the form of a fraction the ratio of 26 to 13. Of 32 to 8.

4. Express in both forms the ratio of two other numbers equal to the ratio of 10 to 2. Of 15 to 5. Of 12 to 3.

5. If 4 stamps cost 12 cents, what will 20 stamps cost at the same rate?

6. What number divided by 12, gives the same quotient as 20 ÷ 4?

7. What number has the same ratio to 12, that 20 has to 4?

8. To what number has 48 the same ratio that 80 has to 5? That 24 has to 3?

9. The ratio of 20 to 5 is the same as the ratio of what number to 4? To 6? To 5½? To 6¼?

10. The ratio of 45 to 9 is the same as the ratio of 15 to what number? Of 30 to what number?

11. 28 is to 7 as 8 is to what number?

12. 56 is to 8 as what number is to 5?

13. 63 ÷ what number equals the ratio of 36 to 4?

DEFINITIONS.

756. A ***Proportion*** is an equation in which each member is a ratio; or it is an equality of ratios.

757. The equality of the two ratios may be indicated by the sign = or by the double colon ::

Thus, we may indicate that the ratio of 8 to 4 is equal to that of 6 to 3, in any of the following ways:

$8:4=6:3$, $8:4::6:3$, $\frac{8}{4}=\frac{6}{3}$, $8\div 4=6\div 3$.

This proportion, in any of its forms, is read, *The ratio of 8 to 4 is equal to the ratio of 6 to 3, or, 8 is to 4 as 6 is to 3.*

758. Since each ratio consists of two terms, every proportion must consist of at least *four terms*. Each ratio is called a *Couplet*, and each term is called a *Proportional.*

759. The ***Antecedents*** of a proportion are the first and third terms, that is, the antecedents of its ratios.

760. The ***Consequents*** are the second and fourth terms, or the consequents of its ratios.

761. The ***Extremes*** are the first and fourth terms.

762. The ***Means*** are the second and third terms.

In the proportion $8:4::10:5$, 8, 4, 10, and 5 are the *proportionals;* $8:4$ is the first *couplet,* $10:5$ the second couplet; 8 and 10 are the *antecedents,* 4 and 5 are the *consequents;* 8 and 5 are the *extremes,* 4 and 10 are the *means.*

Three numbers are proportional, when the ratio of the first to the second is equal to the ratio of the second to the third. Thus the numbers 4, 10, and 25 are proportional, since $4:10=10:25$, the ratio of each couplet being $\frac{2}{5}$.

When three numbers are proportional, the second term is called a *Mean Proportional* between the other two.

The proportion $8 : 4 :: 10 : 5$ may be expressed thus, $\frac{8}{4} = \frac{10}{5}$ (**757**). Reducing these fractions to equivalent ones having a common denominator, $\frac{8 \times 5}{20} = \frac{10 \times 4}{20}$.

Since these fractions are equal, and have a common denominator, their numerators are equal, or $8 \times 5 = 10 \times 4$.

763. PRINCIPLES.—1. *The product of the extremes of a proportion is equal to the product of the means.*

2. *The product of the extremes divided by either mean will give the other mean.*

3. *The product of the means divided by either extreme will give the other extreme.*

EXERCISES.

764. 1. The ratio of 4 to 10 is equal to the ratio of 6 to 15. Express the proportion in all its forms (**757**).

Drill Exercise.—How many terms has a proportion? What are they called? How many ratios? What are they called?

Name the proportionals in example 1; the couplets; the antecedents; the consequents; the extremes; the means. What is the product of the extremes? Of the means? What is the dividend of the first ratio? The divisor of the second ratio? The divisor of the first ratio? The dividend of the second ratio? In the fractional form what are the numerators? The denominators?

2. The ratio of 6 to 15 equals the ratio of 8 to 20.

3. The ratio of $4\frac{1}{2}$ to 18 equals the ratio of 6 to 24.

Change to the form of equations by PRIN. 1:

4. $12 : 1728 :: 1 : 144$.

5. $2\frac{3}{8} : 17 :: 20 : 143\frac{3}{19}$.

6. $27.03 : 9.01 :: 16.05 : 5.35$.

7. $\frac{2}{8} : \frac{3}{7} :: \frac{3}{8} : \frac{9}{14}$.

8. The extremes are 15 and 48, and one of the means is 10. Find the other mean.

9. The means are 25 and 75, and one of the extremes is $12\frac{1}{2}$. Find the other extreme.

The required or omitted term in a proportion, or in an operation, will hereafter be represented by x.

Find the term omitted in each of the following proportions:

11. $8 : 52 :: 20 : x$.
12. $12 : x :: 1 : 144$.
13. $x : 20 :: 120 : 50$.
14. $\$80 : \$4 :: x : 4$.
15. $2.5 : 62.5 :: 5 : x$.
16. $\$175.35 : \$x :: \frac{1}{3} : \frac{3}{4}$.
17. $4\frac{1}{2}$ yd. : x yd. :: $\$9\frac{3}{4} : \$27.25$.
18. $x : 9.01 :: 16.05 : 5.35$.
19. $\frac{3}{8}$ yd. : x yd. :: $\$\frac{7}{8} : \$59.0625$.
20. $\frac{5}{16} : \frac{3}{8} :: x : \frac{3}{4}$.
21. $x : 38\frac{1}{4} :: 8\frac{1}{2} : 76\frac{1}{2}$.
22. $7.5 : 18 :: x$ oz. : $7\frac{1}{15}$ oz.

SIMPLE PROPORTION.

765. A ***Simple Proportion*** is an expression of equality between two simple ratios. It is used to solve problems of which *three* terms are given, and the *fourth* is required.

Of the three given numbers, two must always be of the same kind; and the *third*, of the same kind as the *required term*.

766. A ***Statement*** is the arrangement of these terms in the form of a proportion.

WRITTEN EXERCISES.

767. 1. If 4 tons of coal cost $24, what will be the cost of 12 tons at the same rate?

STATEMENT.

$$4\text{ T.} : 12\text{ T.} :: \$24 : \$x$$

OPERATION.

$$\overline{12 \times 24} \div 4 = \$72$$

OR BY CANCELLATION.

$$\$x = \frac{12 \times \cancel{24}^{6}}{\cancel{4}} = \$72$$

ANALYSIS.—Since 4 tons and 12 tons have the same unit value, they can be compared, and will form one couplet of the proportion.

For the same reason $24 the cost of 4 tons, and $x the cost of 12 tons, will form the other couplet.

Then by PRIN. 3, $\$x = 24 \times 12 \div 4 = \$72$.

Proof.—$4 \times 72 = 12 \times 24$. (**763**, Prin. 1.) In practice, that number which is of the same unit value as the *required* term, is generally made the antecedent of the second couplet or *third* term of the proportion, and the required term, x, the *fourth* term. The terms of the first couplet are so arranged as to have the same ratio to each other, as the terms of the second couplet have to each other, which is easily determined by inspection. The product of the means 12 and 24, divided by the given extreme 4, gives the other extreme, or required term, \$72. (**763**, Prin. 3.)

Drill exercises like the following, will soon make the pupil familiar with the principles and operations of proportion.

2. If 4 horses eat 12 bushels of oats in a given time, how many bushels will 20 horses eat in the same time?

In this example, what two numbers have the same unit value? What do they form? What is the denomination of the third term? Of the required term? What is the antecedent of the second couplet? From the conditions of the question, is the consequent of the second couplet or required term, greater or less than the antecedent? If greater, how must the antecedent and consequent of the first couplet compare with each other? If less, how compare? What is the ratio of the first couplet? Why not 20 to 4? Make the statement. How is the required term found?

3. If 96 cords of wood cost \$240, what will 40 cords cost?
4. If 20 lb. of sugar cost \$1.80, find the cost of 45 lb.
5. If 18 bu. of wheat make 4 barrels of flour, how many barrels will 200 bu. make?

Rule.—I. *Make the statement so that two of the given numbers which are of the same unit value, shall form the first couplet of the proportion, and have a ratio equal to the ratio of the third given term to the required term.*

II. *Divide the product of the means by the given extreme, and the quotient will be the number required.*

CAUSE AND EFFECT.

768. The terms of a proportion have not only the relations of *magnitude*, but also the relations of *cause* and *effect*.

Every problem in proportion may be considered as a comparison of *two causes* and *two effects*.

Thus, if 4 tons as a *cause* will bring when sold, $24 as an *effect*, 12 tons as a *cause* will bring $72 as an *effect*. Or, if 6 horses as a *cause* draw 10 tons as an *effect*, 9 horses as a *cause* will draw 15 tons as an *effect*.

769. Since like causes produce like effects, the ratio of two like causes equals the ratio of two like effects produced by these causes. Hence,

1st cause : 2d cause :: 1st effect : 2d effect.

WRITTEN EXERCISES.

770. 1. If 8 men earn $32 in one week, how much will 15 men earn at the same rate, in the same time?

STATEMENT.

1st cause.		2d cause.		1st effect.		2d effect.
8 men	:	15 men	::	$32	:	$\$x$

OPERATION.

$$\$x = \overline{15 \times 32} \div 8 = \$60$$

ANALYSIS.—In this example an *effect* is required. The first cause is 8 men, the second cause 15 men, and since they are like causes they can be compared.

The effect of the first cause is $32 earned, the effect of the second cause is $\$x$ earned, or the *required* term. Since like effects have the same ratio as their causes (**769**), the causes may form the first couplet, and the effects the second couplet of the proportion. The required term is readily obtained by (**763**, 3).

2. If 20 bushels of wheat produce 6 barrels of flour, how many bushels will be required to produce 24 barrels?

STATEMENT.

1st cause.		2d cause.		1st effect.		2d effect.
20 bu.	:	x bu.	::	6 bbl.	:	24 bbl.

OPERATION.

$$x \text{ bu.} = \overline{20 \times 24} \div 6 = 80 \text{ bu.}$$

Analysis.—In this example a *cause* is required. The first cause is 20 bu., the second cause is x bu. or the *required* term. The effect of the first cause is 6 bbl. of flour, the effect of the second cause is 24 bbl. of flour. Since like causes have the same ratio as their effects (**769**), the statement is made as in Ex. 1, and the required term found by (**763**, 2).

3. If 5 horses consume 10 tons of hay in 8 mo., how many horses will consume 18 tons in the same time?

Drill Exercise.—In this example, what is the first cause? The second cause? The first effect? The second effect? Is the required term a cause or an effect? A mean or an extreme? What is the first couplet? What, the second? Make the statement. How is the required term found?

4. If 8 yards of cloth cost $6, how many yards can be bought for $75?

5. How many men will be required to build 32 rods of wall in the same time that 5 men can build 10 rods?

Rule.—I. *Arrange the terms in the statement so that the ratio of the causes which form the first couplet, shall equal the ratio of the effects which form the second couplet, putting x in the place of the required term.*

II. *If the required term be an extreme, divide the product of the means by the given extreme; if the required term be a mean, divide the product of the extremes by the given mean.*

To shorten the operation, equal factors in the first and second, or in the first and third terms may be *canceled.*

Solve the following by either of the foregoing methods:

6. If 5 sheep can be bought for $20.75, how many sheep can be bought for $398.40?

7. When 10 barrels of flour cost $112.50, what will be the cost of 476 barrels of flour?

8. If a railroad train run 30 miles in 50 min., in what time will it run 260 miles?

9. How many bushels of peaches can be purchased for $454.40, if 8 bushels cost $10.24?

10. If a horse travel 12 miles in 1 hr. 36 min., how far, at the same rate, will he travel in 15 hours?

11. How many days will 12 men require to do a piece of work, that 95 men can do in $7\frac{1}{2}$ days?

12. If $\frac{3}{8}$ of an acre of land cost $60, what will $45\frac{3}{4}$ acres cost?

13. At the rate of 72 yards for £44 16s., how many yards of cloth can be bought for £5 12s.?

14. If $\frac{7}{8}$ of a barrel of cider cost $$1\frac{9}{16}$, what is the cost of $\frac{5}{6}$ of a barrel?

15. If the annual rent of 35 A. 90 P. is $284.50, how much land can be rented for $374.70?

16. What will 87.5 yd. of cloth cost, if $1\frac{3}{4}$ yd. cost $1.26?

17. If by selling $5000 worth of dry goods, a merchant gains $456.25, what amount must he sell to gain $1000?

18. Bought coal at $4.48 per long ton, and sold it at $7.25 per short ton. What was the gain per ton?

19. What will be the cost of a pile of wood 80 ft. long, 4 ft. wide, 4 ft. high, if a pile 18 ft. long, 4 ft. wide, 6 ft. high cost $30.24?

20. If 36 bu. of wheat are bought for $44.50, and sold for $53.50, what is gained on 480 bu. at the same rate?

21. If a business yield $700 net profits in 1 yr. 8 mo., in what time will the same business yield $1050 profits?

COMPOUND PROPORTION.

771. A ***Compound Proportion*** is an expression of equality between two ratios, one or both of which are compound.

All the terms of every problem in compound proportion appear in *couplets*, except *one*, and this is always of the same unit value as the *required* term.

The order of the ratios, and of the terms composing the ratios, is the same as in simple proportion.

WRITTEN EXERCISES.

772. 1. If 18 men build 126 rd. of wall in 60 da., working 10 hr. a day, how many rods will 6 men build in 110 da., working 12 hr. a day?

STATEMENT.

$$\left.\begin{array}{lcl} 18 \text{ men} & : & 6 \text{ men} \\ 60 \text{ days} & : & 110 \text{ days} \\ 10 \text{ hours} & : & 12 \text{ hours} \end{array}\right\} :: \overset{\text{rods.}}{126} : \overset{\text{rods.}}{x}$$

OPERATION.

$$\overset{\text{rods.}}{x} = \frac{\cancel{6} \times \overset{11}{\cancel{110}} \times \cancel{12} \times \overset{42}{\cancel{126}}}{\underset{\cancel{3}}{\cancel{18}} \times \underset{5}{\cancel{60}} \times \cancel{10}} = \frac{462}{5} = 92\tfrac{2}{5} \text{ rd.}$$

Or,

$$\begin{array}{r|l} x & \cancel{6} \\ \cancel{3}\ \cancel{18} & \cancel{110}\ 11 \\ 5\ \cancel{60} & \cancel{12} \\ \cancel{10} & \cancel{126}\ 42 \\ \hline 5 & 462 \\ \hline & 92\tfrac{2}{5} = x \text{ rd.} \end{array}$$

ANALYSIS.—All the terms in this example appear in couplets, except 126 rods, which is of the same unit value as the *required* term, and is made the third term of the proportion, and x rods, the fourth.

The required number of rods depends upon *three* conditions: 1st, the number of men employed; 2d, the number of days they work; and 3d, the number of hours they work each day.

Consider each condition separately, and arrange the terms of the same unit value in couplets, and make the statement as in simple proportion (**767**). Then find the required term by (**763**, 3).

2. If 20 horses consume 36 tons of hay in 9 mo., how many tons will 12 horses consume in 18 months?

Drill Exercise.—In this example, what is the denomination of the required term? What given number has the same unit value? What will be the third term of the proportion? The fourth? How many couplets are there? Name them. What kind of a ratio do they form? How is the antecedent and consequent of each couplet determined? How is a compound ratio reduced to a simple one? Make the statement. Is the required term a mean or an extreme? How is it found? (**763**, 3.)

3. If $320 will pay the board of 4 persons for 8 weeks, for how many weeks will $800 pay the board of 15 persons?

4. If a man walk 192 miles in 6 days, walking 8 hr. a day, how far can he walk in 18 days, walking 6 hr. a day?

5. If 6 laborers can dig a ditch 34 yards long in 10 days, how many days will 20 laborers require to dig a ditch 170 yards long?

RULE.—I. *Form each couplet of the compound ratio from the numbers given, by comparing those which are of the same unit value, arranging the terms of each in respect to the third term of the proportion, as if it were the first couplet of a simple proportion.* (**767.**)

II. *Divide the product of the second and third terms by the product of the first terms, the quotient will be the number required.*

The same preparation of the terms by reduction is to be observed as in simple proportion.

When possible, shorten the operation by *cancellation.* When the vertical line is used, write the factors of the dividend on the right, and the factors of the divisor with x on the left.

CAUSE AND EFFECT.

773. If we regard the conditions of each problem as the comparison of *two causes* and *two effects*, the compound proportion will consist of two ratios, one or both of which may be compound, and the *required* term will be either a *simple* cause, or effect, or a *single element* of a *compound* cause, or effect.

WRITTEN EXERCISES.

774. 1. If 8 men earn $320 in 8 days, how much will 12 men earn in 4 days?

STATEMENT.

1st cause. 2d cause.

$$\left.\begin{array}{lcl} 8 \text{ men} & : & 12 \text{ men} \\ 8 \text{ days} & : & 4 \text{ days} \end{array}\right\} :: \$320 : \$x$$

OPERATION.

$$\$x = \frac{12 \times 4 \times \cancel{\$320}^{\,5}}{\cancel{8} \times \cancel{8}} = \$240$$

Or,

$$\begin{array}{r|r} \$x & 12 \\ \cancel{8} & 4 \\ \cancel{8} & \cancel{\$320}^{\,5} \\ \hline \$x & \$240 \end{array}$$

Analysis. — In this example the first cause is 8 men at work 8 days, the second cause is 12 men at work 4 days; the two form a compound ratio.

The effect of the first cause is $320 earned, the effect of the second cause is $x earned, and is the *required* term; the two effects form a simple ratio.

The value of the *required* term depends upon *two* conditions: 1st, the number of men at work; 2d, the number of days they work. Consider each condition separately, and arrange the terms of the same unit value in couplets, and make a statement in the same manner as in simple proportion. The required term being an *extreme*, is found by (**763**, 3).

2. If it cost $41.25 to pave a sidewalk 5 ft. wide and 75 ft. long, what will it cost to pave a similar walk 8 ft. wide and 566 ft. long?

3. How many days will 21 men require to dig a ditch 80 ft. long, 3 ft. wide, and 8 ft. deep, if 7 men can dig a ditch 60 ft. long, 8 ft. wide, and 6 ft. deep, in 12 days?

STATEMENT.

$$\left.\begin{matrix} 7 : 21 \\ 12 : x \end{matrix}\right\} :: \left\{\begin{matrix} 60 : 80 \\ 8 : 3 \\ 6 : 8 \end{matrix}\right.$$

OPERATION.

$$x = \frac{\cancel{7} \times \cancel{12} \times \overset{8}{\cancel{80}} \times \cancel{3} \times \cancel{8}}{\cancel{21} \times \underset{3}{\cancel{60}} \times \cancel{8} \times \cancel{6}} = \frac{8}{3} = 2\tfrac{2}{3} \text{ da.}$$

Or,

$$\begin{array}{r|l} x & \cancel{7} \\ \cancel{21} & \cancel{12} \\ 3\ \cancel{60} & \cancel{80}\ 8 \\ \cancel{8} & \cancel{3} \\ \cancel{6} & \cancel{8} \\ \hline 3 & 8 \\ \hline \end{array}$$

$$x = 2\tfrac{2}{3} \text{ da.}$$

ANALYSIS.—In this example the causes and the effects each form a compound ratio. The *required* term is an element of the second cause and a *mean*. Hence divide the product of the extremes by the product of the given means, and the quotient is the required factor or term, 2⅔ da. (**763**, 2).

4. If 4 horses consume 48 bushels of oats in 12 days, how many bushels will 20 horses consume in 8 weeks?

RULE.—I. *Of the given numbers, select those which constitute the causes, and those which constitute the effects, and arrange them in couplets as in simple cause and effect, putting x in the place of the required term.*

II. *If the required term, x, be an extreme, divide the product of the means by the product of the given extremes; if x be a mean, divide the product of the extremes by the product of the given means; the quotient will be the required term.*

Solve the following by either of the foregoing methods:

5. What sum of money will produce $300 in 8 mo., if $800 produce $70 in 15 months?

6. If 20 reams of paper are required to print 800 copies of a book containing 230 pages each, 40 lines on a page, how many reams are required to print 3000 copies of 400 pages each, 35 lines on a page?

7. If 10 men can cut 46 cords of wood in 18 da., working 10 hr. a day, how many cords can 40 men cut in 24 da., working 9 hr. a day?

8. What is the cost of 36½ yards of cloth 1½ yd. wide, if 2½ yards 1⅜ yd. wide, cost $3.37½?

9. A contractor employs 45 men to complete a work in 3 months; what additional number of men must he employ, to complete the work in 2½ months?

10. If a vat 16 ft. long, 7 ft. wide, and 15 ft. deep holds 384 barrels, how many barrels will a vat 17½ ft. long, 10½ ft. wide, and 13 ft. deep hold?

11. What is the weight of a block of granite 8 ft. long, 4 ft. wide, and 10 in. thick, if a similar block 10 ft. long, 5 ft. wide, and 16 in. thick, weigh 5200 pounds?

12. If it cost $15 to carry 20 tons 1½ miles, what will it cost to carry 400 tons ½ of a mile?

13. If it take 13500 bricks to build a wall 200 ft. long, 20 ft. high, and 16 in. thick, each brick being 8 in. long, 4 in. wide, and 2 in. thick, how many bricks 10 in. long, 5 in. wide, 3½ in. thick, will be required to build a wall 600 ft. long, 24 ft. high, and 20 ft. thick?

14. What will 15 hogsheads of molasses cost, if 28½ gallons cost $7⅛?

15. At 6¼d. for 1¾ yards of cotton cloth, how many yards can be bought for £10 6s. 8d.?

16. If $750 gain $202.50 in 4 yr. 6 mo., what sum will gain $155.52 in 1 yr. 6 mo.?

17. In what time can 60 men do a piece of work that 15 men can do in 20 days?

18. If 2½ yd. of cloth 6 quarters wide can be made from 1 lb. 12 oz. of wool, how many yards of cloth 4 quarters wide can be made from 70 lb. of wool?

19. If the use of $300 for 1 yr. 8 mo. is worth $30, how long, at the same rate, may $210.25 be retained to be worth $42.891?

20. A farmer has hay worth $18 a ton, and a merchant has flour worth $10 a barrel. If the farmer ask $21 for his hay, what should the merchant ask for his flour?

21. How many men will be required to dig a cellar 45 ft. long, 34.6 ft. wide, and 12.3 ft. deep, in 12 da. of 8.2 hr. each, if 6 men can dig a similar one 22.5 ft. long, 17.3 ft. wide, and 10.25 ft. deep, in 3 da. of 10.25 hr. each?

22. If a bin 8 ft. long, 4⅛ ft. wide, and 2½ ft. deep, hold 67½ bu., how deep must another bin be made, that is 18 ft. long and 3⅝ ft. wide, to hold 450 bu.?

23. What will 120 lb. of coffee cost, if 10 lb. of sugar cost $1.25, and 16 lb. of sugar are worth 5 lb. of coffee?

24. Two men have each a farm. A's farm is worth $48.75, and B's $43⅛; but in trading A values his at $60 an acre. What value should B put upon his?

25. If 6 men in 4 mo., working 26 da. for a month, and 12 hr. a day, can set the type for 24 books of 300 pp. each, 60 lines to the page, 12 words to the line, and an average of 6 letters to the word, in how many months of 24 da. each, and 10 hr. a day, can 8 men and 4 boys set the type for 10 books of 240 pp. each, 52 lines to the page, 16 words to the line, and 8 letters to the word, 2 boys doing as much as 1 man?

PARTNERSHIP

ORAL EXERCISES.

775. 1. If John has 10 marbles, William 15 marbles, and Charles 25 marbles, what part of the whole has each?

2. Two men bought a barrel of flour for $9, the first paying $4 and the second $5. What part of the flour belongs to each?

3. Three men bought 108 sheep, and as often as the first paid $3, the second paid $4, and the third $5. How many sheep should each receive?

4. If $45 be divided between two persons, so that of every $5, one receives $2, and the other $3, how many dollars does each receive?

5. Two men hired a pasture for $36; one put in 2 horses for 3 weeks, the other 3 horses for 4 weeks. What should each pay?

DEFINITIONS.

776. ***Partnership*** is the association of two or more persons under a certain name, for the transaction of business with an agreement to share the gains and losses.

777. A ***Firm***, ***Company*** or ***House*** is any particular partnership association.

778. The ***Capital*** is the money or property invested by the partners, called also *Investment*, or *Joint-Stock*.

779. The ***Resources*** of a firm are the amounts due it, together with the property of all kinds belonging to it; called also *Assets*, or *Effects*.

780. The ***Liabilities*** of a firm are its *debts*.

781. The ***Net Capital*** is the excess of resources over liabilities.

WRITTEN EXERCISES.

782. To apportion gains or losses according to capital invested.

1. A and B engage in trade; A furnishes $400 capital, B $600. They gain $250; what is the profit of each?

1st OPERATION. (*By Fractions.*)

$400, A.'s investment = $\frac{400}{1000}$ = $\frac{2}{5}$ of the whole capital.
600, B.'s " = $\frac{600}{1000}$ = $\frac{3}{5}$ " " "
$1000, whole "
$250 × $\frac{2}{5}$ = $100, A.'s share of the gain.
$250 × $\frac{3}{5}$ = $150, B.'s " " "

2D OPERATION. (*By Proportion.*)

$1000 (whole cap.) : $400 (A.'s inv.) :: $250 (whole gain) : A.'s share.
$1000 (whole cap.) : $600 (B.'s inv.) :: $250 (whole gain) : B.'s share.

3D OPERATION. (*By Percentage.*)

$250 gain is $\frac{250}{1000}$ = 25% of the whole capital.
$400 × .25 = $100, A.'s gain; $600 × .25 = $150, B.'s gain.

ANALYSIS.—(*1st Method.*) Since $400, A.'s investment, is $\frac{400}{1000}$, or $\frac{2}{5}$, of the whole capital, he is entitled to $\frac{2}{5}$ of the gain, or $100; and B is entitled to $\frac{3}{5}$ of the gain, or $150.

2d Method. The ratio of $1000, the whole capital, to $400, A.'s investment, is equal to the ratio of $250, the whole gain, to A.'s share of the gain. Hence the proportions, etc.

3d Method. Since the gain is 25% of the whole capital, each partner is entitled to 25% of his investment as his share of the gain.

The *third method* (by dividend) is that generally adopted by joint-stock companies having numerous shareholders.

2. At the end of the year, Norton, Smith & Co. take an account of stock, and find the amount of merchandise, as per inventory, to be $8400; cash on hand, $4850; due from sundry persons, $5273. Their debts are found to amount to $4223. S. Norton's investment in the business is $5000; R. Smith's, $4000; and C. Woodward's, $2000. Make a statement showing the resources, liabilities, net capital, and net gain: and find each partner's share of the gain.

OPERATION.

Resources.

Mdse. as per inventory, ...	$8400	
Cash on hand, ...	4850	
Debts due the firm, ...	5273	
		$18523

Liabilities.

Debts due to sundry persons, ...	4223
Net capital, ...	$14300

Investments.

S. Norton, ...	$5000	
R. Smith, ...	4000	
C. Woodward, ...	2000	
Total investments, ...		$11000
Net gain, ...		$3300

S. Norton's fractional part, $\frac{5000}{11000} = \frac{5}{11}$ of $3300 = $1500

R. Smith's " " $\frac{4000}{11000} = \frac{4}{11}$ of $3300 = $1200

C. Woodward's " " $\frac{2000}{11000} = \frac{2}{11}$ of $3300 = $ 600

PROOF.—$1500 + $1200 + $600 = $3300, total gain.

RULE 1. *Find what fractional part each partner's investment is of the whole capital, and take such part of the whole gain or loss for his share of the gain or loss.* Or,

2. *State by proportion, as the whole capital is to each partner's investment, respectively, so is the whole gain or loss to each partner's share of the gain or loss.* Or,

3. *Find what per cent. the gain or loss is of the whole capital, and take that per cent. of each partner's investment for his share of the gain or loss, respectively.*

3. A furnishes $4000, B, $2700, and C, $2300, to purchase a house, which they rent for $720. What is each one's share of the rent?

4. Four persons rent a farm of 230 A. 64 P. at $7½ an acre. A puts in 288 sheep, B, 320 sheep, C, 384 sheep, and D, 648 sheep; what rent ought each to pay?

5. Prime & Co. fail in business; their liabilities amount to $22000; their available resources to $8800. They owe A $4275, and B $2175.50: what will each of these creditors receive?

6. Four persons engage in manufacturing, and invest jointly $22500. At the expiration of a certain time, A's share of the gain is $2000, B's $2800.75, C's $1685.25, and D's $1014. How much capital did each put in?

7. An estate worth $10927.60 is divided between two heirs so that one receives ⅛ more than the other. What does each receive?

8. Three persons engage in the lumber trade with a joint capital of $37680. A puts in $6 as often as B puts in $10, and as often as C puts in $14. Their annual gain is equal to C's stock. What is each partner's gain?

9. Ames, Lyon & Co. close business in the following condition: notes due the firm to the amount of $24843.75, cash in hand, $42375.80, due on account, $26500, merchandise per inventory, $175840. Notes against the firm, $14058.75, due from the firm on account, $12375.80. Ames invested $60000, Lyon, $40000, and Clark $25000. Make a statement showing the total amount of resources, liabilities, investments, net capital, net gain, and each partner's share of the gain.

783. To apportion gains or losses according to amount of capital invested, and time it is employed.

1. Three partners, A, B, and C, furnish capital as follows: A, $500 for 2 mo.; B, $400 for 3 mo.; C, $200 for 4 mo. They gain $600; what is each partner's share?

OPERATION.

$$\left.\begin{array}{l} 500 \times 2 = 1000 = \frac{1000}{3000} = \frac{1}{3} \times \\ 400 \times 3 = 1200 = \frac{1200}{3000} = \frac{2}{5} \times \\ 200 \times 4 = \underline{\ 800\ } = \frac{800}{3000} = \frac{4}{15} \times \end{array}\right\} \$600 = \left\{\begin{array}{l} \$200, \text{ A's share.} \\ \$240, \text{ B's} \quad `` \\ \$160, \text{ C's} \quad `` \end{array}\right.$$

$$\qquad\qquad 3000$$

ANALYSIS.—The use of $500 for 2 mo. is the same as the use of $1000 for 1 mo.; the use of $400 for 3 mo. is the same as that of $1200 for 1 mo.; and the use of $200 for 4 mo. is the same as that of $800 for 1 mo. Therefore the whole capital is the use of $3000 for 1 mo.; and as A's investment is $1000 for 1 mo., it is $\frac{1}{3}$ of the capital, and hence he should receive $\frac{1}{3}$ of the gain, or $200. For the same reason, B should receive $\frac{2}{5}$, and C $\frac{4}{15}$ of the gain, or $240 and $160, respectively.

The other methods of operation may be applied by considering the products of investment and time as shares of the capital. Thus, $600 is 20% of $3000; and 20% of $1000, $1200, and $800 will give $200, $240, and $160, respectively, the shares of gain required.

2. Barr, Banks & Co. gain in trade \$8000. Barr furnished \$12000 for 6 mo., Banks, \$10000 for 8 mo., and Butts \$8000 for 11 mo. Apportion the gain.

RULE 1. *Multiply each partner's capital by the time it is invested, and divide the whole gain or loss among the partners in the ratio of these products.* Or,

2. *State by proportion: The sum of the products is to each product, as the whole gain or loss is to each partner's gain or loss.*

3. Jan. 1, 1876, three persons began business with \$1300 capital furnished by A; March 1, B put in \$1000; Aug. 1, C put in \$900. The profits at the end of the year were \$750. Apportion it.

4. In a partnership for 2 years, A furnished at first \$2000, and 10 mo. after withdrew \$400 for 4 mo., and then returned it; B at first put in \$3000, and at the end of 4 mo. \$500 more, but drew out \$1500 at the end of 16 mo. The whole gain was \$3372. Find the share of each.

5. The joint capital of a company was \$5400, which was doubled at the end of the year. A put in $\frac{1}{3}$ for 9 mo., B $\frac{3}{8}$ for 6 mo., and C the remainder for 1 year. What is each one's share of the stock at the end of the year?

6. Crane, Child & Coe, forming a partnership Jan. 1, 1875, invested and drew out as follows: Crane invested \$2000, 4 mo. after \$1000 more, and at the end of 9 mo. drew out \$600. Child invested \$5000, 6 mo. after \$1200 more, and at the end of 11 mo. put in \$2000 more. Coe put in \$6000, 4 mo. after drew out \$4000, and at the end of 8 mo. drew out \$1000 more. The net profits for the year were \$7570. Find the share of each.

ALLIGATION

784. ***Alligation*** treats of mixing or compounding two or more ingredients of different values or qualities.

785. ***Alligation Medial*** is the process of finding the *mean* or *average* value or quality of several ingredients.

786. ***Alligation Alternate*** is the process of finding the proportional quantities to be used in any required mixture.

WRITTEN EXAMPLES.

787. 1. If a grocer mix 8 lb. of tea worth $.60 a pound with 6 lb. at $.70, 2 lb. at $1.10, and 4 lb. at $1.20, what is 1 lb. of the mixture worth ?

OPERATION.

$.60 × 8 = $4.80
.70 × 6 = 4.20
1.10 × 2 = 2.20
1.20 × 4 = 4.80
20) $16.00

ANALYSIS.—Since 8 lb. of tea at $.60 is worth $4.80, and 6 lb. at $.70 is worth $4.20, and 2 lb. at $1.10 is worth $2.20, and 4 lb. at $1.20 is worth $4.80, the mixture of 20 lb. is worth $16. Hence 1 lb. is worth $\frac{1}{20}$ of $16, or $16 ÷ 20 = $.80.

2. If 20 lb. of sugar at 8 cents be mixed with 24 lb. at 9 cents, and 32 lb. at 11 cents, and the mixture is sold at 10 cents a pound, what is the gain or loss on the whole ?

RULE.—*Find the entire cost or value of the ingredients, and divide it by the sum of the simples.*

3. A miller mixes 18 bu. of wheat at $1.44 with 6 bu. at $1.32, 6 bu. at $1.08, and 12 bu. at $.84. What will be his gain per bushel if he sell the mixture at $1.50?

4. Bought 24 cheeses, each weighing 25 lb., at 7¢ a pound; 10, weighing 40 lb. each, at 10¢; and 4, weighing 50 lb. each, at 13¢; sold the whole at an average price of 9⅓¢ a pound. What was the whole gain?

5. A drover bought 84 sheep at $5 a head; 86 at $4.75; and 130 at $5⅓. At what average price per head must he sell them to gain 20%?

788. To find the proportional parts to be used, when the mean price of a mixture and the prices of the simples are given.

1. What relative quantities of timothy seed worth $2 a bushel, and clover seed worth $7 a bushel, must be used to form a mixture worth $5 a bushel?

OPERATION.

$$5\left\{\begin{array}{c|c|c}2&\frac{1}{3}&2\\7&\frac{1}{2}&3\end{array}\right\}\textit{Ans.}$$

ANALYSIS.—Since on every ingredient used whose price or quality is *less* than the mean rate there will be a *gain*, and on every ingredient whose price or quality is *greater* than the mean rate there will be a *loss*, and since the gains and losses must be exactly equal, the relative quantities used of each should be such as represent the unit of *value*. By selling one bushel of timothy seed worth $2, for $5, there is a gain of $3; and to gain $1 would require ⅓ of a bushel, which is placed opposite the 2. By selling one bushel of clover seed worth $7, for $5, there is a loss of $2; and to lose $1 would require ½ of a bushel, which is placed opposite the 7.

In every case, to find the unit of value, divide $1 by the gain or loss per bushel or pound, etc. Hence, if every time ⅓ of a bushel of timothy seed is taken, ½ of a bushel of clover seed is taken, the gain and loss will be exactly equal, and ⅓ and ½ will be the *proportional quantities* required.

To express the proportional numbers in integers, reduce these fractions to a common denominator, and use their numerators, since fractions having a common denominator are to each other as their numerators (**241**); thus, $\frac{1}{3}$ and $\frac{1}{2}$ are equal to $\frac{2}{6}$ and $\frac{3}{6}$, and the proportional quantities are 2 bu. of timothy seed to 3 bu. of clover seed.

2. What proportions of teas worth respectively 3, 4, 7, and 10 shillings a pound, must be taken to form a mixture worth 6 shillings a pound?

OPERATION.

		1	2	3	4	5
6	3	$\frac{1}{3}$		4		4
	4		$\frac{1}{2}$		1	1
	7		1		2	2
	10	$\frac{1}{4}$		3		3

ANALYSIS.—To preserve the equality of gains and losses, always compare two prices or simples, one *greater* and one *less* than the mean rate, and treat each pair or couplet as a separate example. In the given example form two couplets, and compare either 3 and 10, 4 and 7, or 3 and 7, 4 and 10.

We find that $\frac{1}{3}$ of a lb. at 3s. must be taken to *gain* 1 shilling, and $\frac{1}{4}$ of a lb. at 10s. to *lose* 1 shilling; also $\frac{1}{2}$ of a lb. at 4s. to *gain* 1 shilling, and 1 lb. at 7s. to *lose* 1 shilling. These proportional numbers, obtained by comparing the two couplets, are placed in columns 1 and 2. If, now, the numbers in columns 1 and 2 are reduced to a common denominator, and their numerators used, the integral numbers in columns 3 and 4 are obtained, which, being arranged in column 5, give the *proportional quantities* to be taken of each.

It will be seen that in comparing the simples of any couplet, one of which is greater, and the other less than the mean rate, the proportional number finally obtained for either term is the difference between the mean rate and the other term. Thus, in comparing 3 and 10, the proportional number of the former is 4, which is the difference between 10 and the mean rate 6; and the proportional number of the latter is 3, which is the difference between 3 and the mean rate. The same is true of every other couplet. Hence, when the simples and the mean rate are integers, the intermediate steps taken to obtain the final proportional numbers as in columns 1, 2, 3, and 4, may be omitted, and the same results readily found by taking the difference between each simple and the mean rate, and placing it opposite the one with which it is compared.

3. In what proportions must sugars worth 10 cents, 11 cents, and 14 cents a pound be used, to form a mixture worth 12 cents a pound?

4. A farmer has sheep worth \$4, \$5, \$6, and \$8 per head. What number may he sell of each and realize an average price of \$5½ per head?

RULE.—I. *Write the several prices or qualities in a column, and the mean price or quality of the mixture at the left.*

II. *Form couplets by comparing any price or quality less, with one that is greater than the mean rate, placing the part which must be used to gain* 1 *of the mean rate opposite the less simple, and the part that must be used to lose* 1 *opposite the greater simple, and do the same for each simple in every couplet.*

III. *If the proportional numbers are fractional, they may be reduced to integers, and if two or more stand in the same horizontal line, they must be added; the final results will be the proportional quantities required.*

1. If the numbers in any couplet or column have a common factor, it may be rejected.

2. We may also multiply the numbers in any couplet or column by any multiplier we choose, without affecting the equality of the gains and losses, and thus obtain an indefinite number of results, any one of which being taken will give a correct final result.

5. What amount of flour worth \$5½, \$6, and \$7¾ per barrel, must be sold to realize an average price of \$6¼ per barrel?

6. In what proportions can wine worth \$1.20, \$1.80, and \$2.30 per gallon be mixed with water so as to form a mixture worth \$1.50 per gallon?

789. When the quantity of one of the simples is limited.

1. A farmer has oats worth \$.30, corn worth \$.45, and barley worth \$.84 a bushel. To make a mixture worth \$.60 a bushel, and which shall contain 48 bu. of corn, how many bushels of oats and barley must he use?

OPERATION.

$$60\left\{\begin{array}{c|c|c|c|c|c|c} 30 & \frac{1}{30} & & 4 & & 4 & 24 \\ 45 & & \frac{1}{15} & & 8 & 8 & 48 \\ 84 & \frac{1}{24} & \frac{1}{24} & 5 & 5 & 10 & 60 \end{array}\right\}\textit{Ans.}$$

ANALYSIS.—By the same process as in (788), the proportional quantities of each are found to be 4 bu. of oats, 8 of corn, and 10 of barley. But since 48 bu. of corn is 6 times the proportional number 8, to preserve the equality of gain and loss take 6 times the proportional quantity of each of the other simples, or 6 × 4 = 24 bu. of oats, and 6 × 10 = 60 bu. of barley. Hence, etc.

2. A dairyman bought 10 cows at \$40 a head. How many must he buy at \$32, \$36, and \$48 a head, so that the whole may average \$44 a head?

RULE.—*Find the proportional quantities as in* (788). *Divide the given quantity by the proportional quantity of the same ingredient, and multiply each of the other proportional quantities by the quotient thus obtained.*

3. A grocer having teas worth \$.80, \$1.20, \$1.50, and \$1.80 per pound, wishes to form a mixture worth \$1.60 a pound, and use 20 lb. of that worth \$1.50 a pound.

4. Bought 12 yd. of cloth for \$30. How many yards must I buy at \$3½ and \$1¼ a yard, that the average price of the whole may be \$2¾ a yard?

5. How many acres of land worth \$70 an acre must be added to a farm of 75 A., worth \$100 an acre, that the average value may be \$80 an acre?

790. When the quantity of the whole compound is limited.

1. A grocer has sugars worth 6 cents, 7 cents, 12 cents, and 13 cents per pound. He wishes to make a mixture of 108 pounds, worth 10 cents a pound; how many pounds of each kind must he use?

OPERATION.

10	6	$\frac{1}{4}$		3		3	27
	7		$\frac{1}{3}$		2	2	18
	12		$\frac{1}{2}$		3	3	27
	13	$\frac{1}{3}$		4		4	36
						12	108

Analysis.—The proportional quantities of each simple found by (**788**), are 3 lb. at 6 cts., 2 lb. at 7 cts., 3 lb. at 12 cts., and 4 lb. at 13 cts. Adding the proportional quantities, the mixture is but 12 lb., while the required mixture is 108, or 9 times 12. If the whole mixture is to be 9 times as much as the *sum* of the proportional quantities, then the quantity of each simple used must be 9 times as much as its respective proportional, or 27 lb. at 6 cts., 18 lb. at 7 cts., 27 lb. at 12 cts., and 36 lb. at 13 cts.

2. A man paid $330 per week to 55 laborers, consisting of men, women, and boys; to the men he paid $10 a week, to the women $2 a week, and to the boys $1 a week; how many were there of each?

Rule.—*Find the proportional numbers as in* (**788**). *Divide the given quantity by the sum of the proportional quantities, and multiply each of the proportional quantities by the quotient thus obtained.*

3. How much water must be mixed with wine worth $.90 a gallon, to make 100 gal. worth $.60 a gallon?

4. One man and 3 boys received $84 for 56 days' labor; the man received $3 per day, and the boys $½, $¾, and $1¾ respectively; how many days did each labor?

791. SYNOPSIS FOR REVIEW.

- RATIO.
  - 1. DEFS.
    - 1. Ratio. 2. Sign of Ratio. 3. Terms. 4. Antecedent. 5. Consequent. 6. Value of a Ratio. 7. Simple Ratio. 8. Compound Ratio. 9. Reciprocal of a Ratio.
  - 2. FORMULAS, 1, 2, 3.
  - 3. GENERAL PRINCIPLES, 1, 2, 3.
  - 4. GENERAL LAW.
- PROPORTION.
  - 1. DEFS.
    - 1. Proportion. 2. Sign. 3. Couplet. 4. Proportional. 5. Antecedents. 6. Consequents. 7. Extremes. 8. Means. 9. Mean Proportional.
  - 2. PRINCIPLES, 1, 2, 3, 4.
  - 3. SIMPLE PROPORTION.
    - 1. Defs.
      - 1. *Simple Proportion.*
      - 2. *Statement.*
    - 2. Rule, I, II.
    - 3. Cause and Effect.
    - 4. Rule, I, II.
  - 4. COMPOUND PROPORTION.
    - 1. Def. Compound Proportion.
    - 2. Rule, I, II.
    - 3. Cause and Effect.
    - 4. Rule, I, II.
- PARTNERSHIP.
  - 1. DEFS.
    - 1. Partnership. 2. Firm, Company, or House. 3. Capital. 4. Resources. 5. Liabilities. 6. Net Capital.
  - 2. **782.** Rule, 1, 2, 3.
  - 3. **783.** Rule, 1, 2.
- ALLIGATION.
  - 1. DEFS.
    - 1. Alligation. 2. Alligation Medial 3. Alligation Alternate.
  - 2. **787.** Rule.
  - 3. **788.** Rule, I, II, III.
  - 4. **789.** Rule.
  - 5. **790.** Rule.

TEST PROBLEMS.

792. 1. The sum of two numbers is 120, and their difference is equal to $\frac{1}{3}$ of the greater. Find the numbers.

2. E's age is $1\frac{1}{3}$ times the age of D, and F's age is $2\frac{1}{10}$ times the age of both, and the sum of their ages is 124. What is the age of each?

3. If 7 bu. of wheat are worth 10 bu. of rye, and 5 bu. of rye are worth 14 bu. of oats, and 6 bu. of oats are worth $6, how many bushels of wheat will $60 buy?

4. A mechanic was engaged to labor 20 days, on the conditions that he was to receive $5 a day for every day he worked, and to forfeit $2 a day for every day he was idle; at the end of the time he received $86. How many days did he labor?

5. One man can build a fence in 18 da., working 10 hr. a day; another can build it in 9 da., working 8 hr. a day. In how many days can both together build it, if they work 6 hours a day?

6. If 6 boxes of starch and 7 boxes of soap cost $33, and 12 boxes of starch and 10 boxes of soap cost $54, what is the price of 1 box of each?

7. Three men agree to build a barn for $540. The first and second can do the work in 16 da., the second and third in $13\frac{1}{3}$ da., and the first and third in $11\frac{3}{7}$ da. In how many days can all do it working together? In how many days can each do it alone? What part of the pay should each receive?

8. A dealer paid $182 for 20 barrels of flour, giving $10 for first quality, and $7 for second quality. How many barrels were there of each?

9. The hour and minute hands of a clock are together at 12 M. When will they be exactly together the third time after this?

10. Bought 15 bu. of wheat and 30 bu. of oats for $35, and 9 bu. of wheat and 6 bu. of oats for $15. What was the price per bushel of each?

11. If Ames can do as much work in 3 days as Jones can do in 4½ days, and Jones can do as much in 9 days as Smith can do in 12 days, and Smith as much in 10 days as Ray in 8 days, how many days' work done by Ray are equal to 5 days done by Ames?

12. A merchant bought 40 pieces of cloth, each piece containing 25 yd., at $4⅜ per yard, on 9 mo. credit, and sold the same at $4⅝ per yard, on 4 mo. credit. Find his net cash gain, money being worth 6%.

13. There are 70 bu. of grain in 2 bins, and in 1 bin are 10 bu. less than ⅗ as much as there is in the other. How many bushels in the larger bin?

14. Three men can perform a piece of work in 12 hr.; A and B can do it in 16 hr., A and C in 18 hr. What part of the work can B and C do in 9⅗ hours?

15. What per cent. in advance of the cost must a merchant mark his goods, so that after allowing 5% of his sales for bad debts, an average credit of 6 mo., and 4% of the cost of the goods for his expenses, he may make a clear gain of 12½% on the first cost of the goods, money being worth 7%?

16. An elder brother's fortune is 1⅓ times his younger brother's; the interest of ½ of the elder brother's fortune and ⅓ of the younger's for 5 years, at 6%, is $2400. What is the fortune of each?

17. The top of Trinity Church steeple in New York is 268 ft. from the ground; $\frac{3}{4}$ the height of the steeple above the church plus 12 ft. is equal to the height of the church. Find the height of the steeple above the church.

18. Two persons have the same income: A saves $\frac{1}{4}$ of his, but B by spending \$300 a year more than A, at the end of 2 years is \$200 in debt. What is their income?

19. Divide \$2520 among 3 persons, so that the second shall have $\frac{3}{4}$ as much as the first, and the third $\frac{1}{2}$ as much as the other two. What is the share of each?

20. A man owes a debt to be paid in 4 equal installments of 4, 9, 12, and 20 months respectively; a discount of 5% being allowed, he finds that \$1500 ready money will pay the debt. What is the amount of the debt?

21. A quantity of flour is to be distributed among some poor families; if 50 lb. are given to each family, there will be 6 lb. left; if 51 lb. are given to each, there will be wanting 4 lb. What is the quantity of flour?

22. I have three notes payable as follows: one for \$400, due Jan. 1, 1875; another for \$700, due Sept. 1, 1875; and another for \$1000, due April 1, 1876. What is the average of maturity?

23. An estate worth \$123251.82 is left to four sons, whose ages are 19, 17, 13, and 11 years, respectively, and is to be so divided that each part being put out at 7% simple interest, the amounts shall be equal when they become 21 years of age. What are the parts?

24. If a piece of silk cost \$1.20 a yard, at what price must it be marked that it may be sold at 10% less than the marked price, and still make a profit of 20%?

25. A farmer sold 100 geese and turkeys; for the geese

he received $.75 apiece, and for the turkeys $1.25 apiece, and for the whole $104. What was the number of each?

26. A man left his property to three sons; to A ⅓ wanting $180, to B ¼, and to C the rest, which was $590 less than A and B received. What was the whole estate?

27. What is the simple interest and the amount; the compound interest and amount; the present worth and the true discount; the bank discount and the proceeds of $1920, for 2 yr. 5 mo. 12 da., at 6%? Also the face of the note, which when discounted at a bank for the same time, and at the same rate, will produce the same sum?

28. Divide $1500 among 3 persons, so that the share of the second may be ½ greater than that of the first, and the share of the third ½ greater than that of the second.

29. A merchant owes for three bills of goods as follows: $500 due March 1, $800 due June 1, and $600 due Aug. 1. He wishes to give two notes for the amount, one for $1000, payable April 1; what must be the face, and when the maturity, of the other?

30. A man in New York purchased a draft on Chicago for $10640, drawn at 60 da., $10302.18. What was the course of exchange?

31. B. B. Northrop, through his broker, invested a certain sum in U. S. 6's, 5-20, at 107½, and twice as much in U. S. 5's of '81, at 98½, brokerage on each, ¼%. His income from both investments is $1674. How much did he invest in each kind of stock?

32. A, B, and C are under a joint contract to furnish 6000 bu. of corn, at $.48 a bushel; A's corn is worth $.45, B's $.51, and C's $.54; how many bushels must each put into the mixture that the contract may be fulfilled?

33. A cask contains 42¾ U. S. gallons of wine, worth $4½ per gallon. How much less will it cost in U. S. money, at the rate of £1 2s. per the Imperial gallon?

34. A garden 400 ft. long and 300 ft. wide has a walk 20 ft. wide laid off from each of its two sides. What is the ratio between the area of the walk and the area of what remains?

35. A commission merchant in Charleston received into his store on May 1, 1875, 1000 bbl. of flour, paying as charges on the same day, freight $175.48, cartage $56.25, and cooperage $8.37. He sold out the shipment as follows: June 3, 200 bbl. @ $6.25; June 30, 350 bbl. @ $6.50; July 29, 400 bbl. @ $6.12½; Aug. 6, 50 bbl. @ $6. Required, the net proceeds, and the date when they shall be accredited to the owner, allowing commission at 3½%, and storage at 2 cents per week per bbl.

36. Three men engage in manufacturing. L puts in $3840 for 6 mo.; M, a sum not specified for 12 mo.; and N, $2560 for a time not specified. L received $4800 for his capital and profits; M, $9600 for his; and N, $4160 for his. Required, M's capital and N's time.

37. My expenditures in building a house, in the year 1874, were as follows: Jan. 16, $536.78; Feb. 20, $425.36; March 4, $259.25; April 24, $786.36. At the last date I sold the house for exactly what it cost, interest at 6 per cent. on the money expended added, and took the purchaser's note for the amount. What was the face of the note?

38. A man bought a farm for $6000, and agreed to pay principal and interest in 3 equal annual installments. What was the annual payment, interest being 6%?

INVOLUTION

ORAL EXERCISES.

793. 1. What is the product of 3 used twice as a factor?

2. What is the product of 3 used 3 times as a factor?

3. What is the product of 4 used 3 times as a factor?

4. What is the result of using 5 twice as a factor?

5. What is the product of $\frac{1}{2}$ used twice as a factor?

6. What is the result of using $\frac{3}{4}$ twice as a factor? $\frac{2}{3}$, three times as a factor?

7. What number will be produced by using .3 twice as a factor? .7, twice? .4, three times? .05, twice?

8. A room is 9 ft. on each side; how many square feet in the floor?

9. A cubical block of stone is 4 ft. on each edge; how many cubic feet does it contain?

DEFINITIONS.

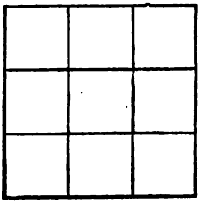
794. A ***Power*** of a number is the product of factors, each of which is equal to that number. Thus, 27 is the third power of 3, since $27 = 3 \times 3 \times 3$.

795. ***Involution*** is the process of finding any power of a number.

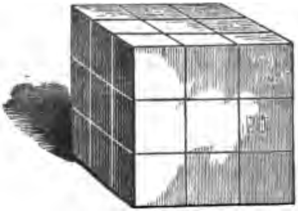
796. The ***Base*** or ***Root*** of a power is one of the equal factors of the power. Thus, 27 is the third power of 3, and 3 is the *base*, or *root*, of that power.

797. The ***Exponent*** of a power is a number placed at the right of the base and a little above it, to show how many times it is used as a factor to produce the power. It also denotes the *degree* of the power. Thus,

3^1 or $3 = 3$, the *first* power of 3.
$3^2 = 3 \times 3 = 9$, the *second* power of 3.
$3^3 = 3 \times 3 \times 3 = 27$, the *third* power of 3.
$3^4 = 3 \times 3 \times 3 \times 3 = 81$, the *fourth* power of 3.


$3^2 = 3 \times 3 = 9$

798. The ***Square*** of a number is its *second* power, so called because the number of superficial units in a square is equal to the second power of the number of linear units in one of its *sides*.


$3^3 = 3 \times 3 \times 3 = 27$

799. The ***Cube*** of a number is its *third* power, so called because the number of units of volume in a cube is equal to the third power of the number of linear units in one of its *edges*.

800. A ***Perfect Power*** is a number which can be resolved into equal factors. Thus, 25 is a perfect power of the *second* degree, and 27 is a perfect power of the *third* degree.

801. PRINCIPLE.—*The sum of the exponents of two powers of the same number is equal to the exponent of the product of those powers.* Thus, $2^2 \times 2^3 = 2^5$; for $2^2 = 2 \times 2$, and $2^3 = 2 \times 2 \times 2$; hence $2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$.

WRITTEN EXERCISES.

802. To find any power of a number.

1. Find the third power of 35.

OPERATION.

$35 = 35^1$; $35 \times 35 = 35^2 = 1225$

$1225 \times 35 = 35^3 = 42875$

ANALYSIS.—Since using any number three times as a factor produces the third power of that number (**797**), $35 \times 35 \times 35 = 35^3 = 42875$.

2. Find the square of 37. Of 42. Of 56. Of 75.

3. Find the cube of 15. Of 18. Of 42. Of 54.

4. What is the value of 63^2? of 48^3? of 32^4? of 12^5?

RULE.—*Find the product of as many factors, each equal to the given number, as there are units in the exponent of the required power.*

5. What is the third power of $\frac{4}{5}$?

OPERATION.—$(\frac{4}{5})^3 = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{4 \times 4 \times 4}{5 \times 5 \times 5} = \frac{4^3}{5^3} = \frac{64}{125}$.

RULE.—*A fraction may be raised to any power by involving each of its terms separately to the required power.*

6. What is the square of $\frac{7}{16}$? The cube of $\frac{12}{31}$?

7. Raise $\frac{9}{14}$ to the 4th power. $2\frac{1}{3}$ to the 5th power.

Find the required power of the following:

8. 25.4^3.
9. 106^3.
10. $(44\frac{1}{4})^2$.
11. $(\frac{11}{15})^4$.
12. $.0342^3$.
13. $.5^6$.
14. 36.02^3.
15. $.40316^3$.
16. $(182\frac{1}{8})^2$.
17. $(4.07\frac{1}{2})^2$.
18. $(1\frac{2}{10})^5$.
19. $.0063^3$.

Find the value of each of the following expressions:

20. $4.6^3 \times 25^3$.

21. $6.75^4 - (7\frac{1}{4})^2$.

22. $\frac{7}{8}$ of $(\frac{4}{5})^3 \times (3\frac{5}{8})^2$.

23. $8^6 \div .4096$.

24. $2.5^3 \times (12\frac{3}{4})^2$.

25. $(7.5)^3 \div (1\frac{1}{2})^3$.

26. $(4^3 \times 5^6 \times 12^3) \div (4^2 \times 10^4 \times 3^2)$.

FORMATION OF SQUARES AND CUBES BY THE ANALYTICAL METHOD.

803. To find the square of a number in terms of its tens and units.

1. Find the square of 27 in terms of its tens and units.

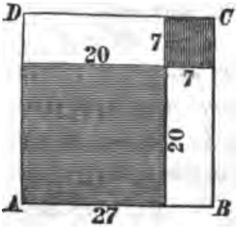
OPERATION.

$$\begin{array}{rr}
27 = & 20+7 \\
27 = & 20+7 \\
\hline
189 = & 20 \times 7 + 7^2 \\
540 = & 20^2 + 20 \times 7 \\
\hline
729 = & 20^2 + (2 \times 20 \times 7) + 7^2
\end{array}$$

ANALYSIS.—The product of $\overline{20+7}$ by 7 is $20 \times 7 + 7^2$, and the product of 20+7 by 20 is $20^2 + (20 \times 7)$; hence $20^2 + (2 \times 20 \times 7) + 7^2$, which is the sum of these partial products, is the square of 20+7 or 27.

PRINCIPLE.—*The square of a number consisting of tens and units, is equal to the sum of the squares of the tens and units increased by twice their product.*

GEOMETRICAL ILLUSTRATION.



Let ABCD be a square, each side of which is 27 feet, and let lines be drawn as represented in the figure. It is evident that the square ABCD (27^2) is equal to the sum of two squares, one of which is the square of tens (20^2), the other the square of the units (7^2), together with two rectangles each of whose areas is 20×7.

2. What is the square of 37?

$$\begin{array}{rr}
30^2 = & 900 \\
2 \times 30 \times 7 = & 420 \\
7^2 = & 49 \\
\hline
37^2 = & 1369
\end{array} \quad (\textbf{803}, \text{Prin.})$$

3. Find the square of 42 in terms of its tens and units.

In like manner find the square

4. Of 48.
5. Of 56.
6. Of 98.
7. Of 125.
8. Of 105.
9. Of 225.
10. Of 197.
11. Of 342.

804. To find the cube of a number in terms of its tens and units.

1. Find the cube of 25 in terms of its tens and units.

OPERATION.

$$\begin{array}{rl}
25^2 = & 20^2 + (2 \times 20 \times 5) + 5^2 \\
25 = & 20 + 5 \\
\hline
25^2 \times 5 = & (20^2 \times 5) + (2 \times 20 \times 5^2) + 5^3 \\
25^2 \times 20 = & 20^3 + (2 \times 20^2 \times 5) + (20 \times 5^2) \\
\hline
25^3 = & 20^3 + (3 \times 20^2 \times 5) + (3 \times 20 \times 5^2) + 5^3
\end{array}$$

Analysis.—The *square* of 25 is $20^2 + (2 \times 20 \times 5) + 5^2$. (803, Prin.) Multiplying this by $20 + 5$ gives the *cube* of 25.

2. Find the cube of 34 in terms of its tens and units.

Principle.—*The cube of a number consisting of tens and units is equal to the cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.*

GEOMETRICAL ILLUSTRATION.

FIG. 1.

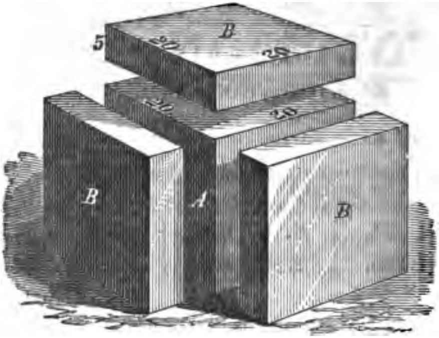


FIG. 2.

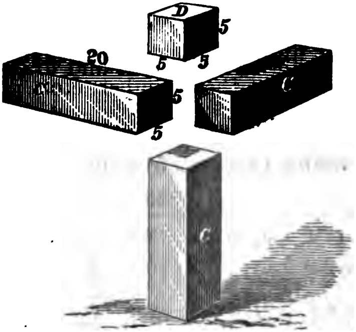
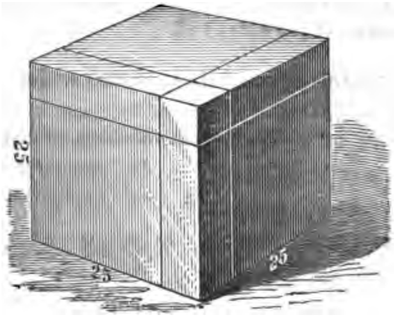


FIG. 3.



The volume of the cube marked A, Fig. 1, is 20^3; the volume of each of the rectangular solids marked B is $20 \times 20 \times 5$, or $20^2 \times 5$; the volume of each o the rectangular solids marked C, in Fig. 2. is $20 \times 5 \times 5$, or 20×5^2; and the volume of the small cube marked D is 5^3. It is evident, that if all these solids are put together as represented in Fig. 3, a cube will be formed, each edge of which is 25.

3. Find the cube of 46.

OPERATION.

$$40^3 = 64000$$
$$40^2 \times 6 \times 3 = 28800$$
$$40 \times 6^2 \times 3 = 4320$$
$$6^3 = 216$$
$$\overline{46^3 = 97336}$$

In like manner find the cube

4. Of 48.
5. Of 64.
6. Of 95.
7. Of 125.

EVOLUTION

805. 1. What are the two equal factors of 25? 36?

2. What are the three equal factors of 27? 64? 125?

3. What are the four equal factors of 16? 81? 256?

4. Of what is 81 the 2d power? The 4th power?

DEFINITIONS.

806. The ***Square Root*** of a number is one of the two equal factors of that number; the ***Cube Root*** is one of the three equal factors of that number, etc.

Thus, 3 is the square root of 9, 2 is the cube root of 8, etc.

807. ***Evolution*** is the process of finding the root of any power of a number.

808. The ***Radical Sign*** is $\sqrt{\ }$. When prefixed to a number, it indicates that some root of it is to be found.

809. The ***Index*** of the root is a small figure placed above the radical sign to denote what root is to be found. When no index is written, the index 2 is understood.

Thus, $\sqrt{100}$ denotes the *square root* of 100; $\sqrt[3]{125}$ denotes the *cube root* of 125; $\sqrt[4]{256}$ denotes the *fourth root* of 256; and so on.

Evolution, or both involution and evolution, may be indicated in the same expression by a *fractional exponent*, the numerator denoting the required *power* of the given number, and the denominator the *root* of that power of the number. Thus,

$9^{\frac{1}{2}}$ is equivalent to $\sqrt{9}$; $64^{\frac{1}{3}}$, to $\sqrt[3]{64}$; and $8^{\frac{2}{3}}$, to the cube root of the second power of 8, equivalent to $\sqrt[3]{8^2}$, etc.

EVOLUTION BY FACTORING.

WRITTEN EXERCISES.

810. To find any root of a number by factoring.

1. Find the cube root of 1728.

OPERATION.

$$\begin{array}{r} 3\underline{)1728} \\ 3\underline{)576} \\ 3\underline{)192} \\ 2\underline{)64} \\ 2\underline{)32} \\ 2\underline{)16} \\ 2\underline{)8} \\ 2\underline{)4} \\ 2 \end{array}$$

ANALYSIS.—A number that is a perfect cube, is composed of *three* equal factors, and one of them is the cube root of that number.

The prime factors of 1728 are 3, 3, 3, 2, 2, 2, 2, 2, 2; hence $1728 = (3 \times 2 \times 2) \times (3 \times 2 \times 2) \times (3 \times 2 \times 2)$; therefore the cube root of 1728 is $(3 \times 2 \times 2)$, or 12.

RULE.—*Resolve the given number into its prime factors; then, to produce the square root, take one of every two equal factors; to produce the cube root take one of every three equal factors; and so on.*

2. Find the square root of 64. Of 256. Of 576. Of 6561.

3. Find the cube root of 729. Of 2744. Of 9261. Of 3375.

GENERAL METHOD OF SQUARE ROOT.

811. A ***Perfect Square*** is a number which has an exact square root.

812. PRINCIPLES.—1. *The square of a number expressed by a single figure contains no figure of a higher order than tens.*

2. *The square of tens contains no significant figure of a lower order than hundreds, nor of a higher order than thousands.*

3. *The square of a number contains twice as many figures as the number, or twice as many less one.* Thus.

$$\begin{array}{rcrlrcr} 1^2 &=& 1, & \qquad & 10^2 &=& 100, \\ 9^2 &=& 81, & & 100^2 &=& 10000, \\ 99^2 &=& 9801, & & 1000^2 &=& 1000000. \end{array}$$

Hence,

4. *If any perfect square be separated into periods of two figures each, beginning with units' place, the number of periods will be equal to the number of figures in the square root of that number.*

If the number of figures in the number is *odd*, the left-hand period will contain only one figure.

WRITTEN EXERCISES.

813. To find the square root of a number.

1. Find the square root of 4356.

OPERATION.

$$\begin{array}{rl} & 43{,}56\ (\,60+6=66 \\ 60^2= & \underline{3600} \\ 120+6=126\,) & 756 \\ & \underline{756} \end{array}$$

ANALYSIS.—Since 4356 consists of two periods, its square root will consist of two figures (**812**, PRIN. 4). Since 56 cannot be a part of the square of the tens (**812**, PRIN. 2), the tens of the root must be found from the first period 43.

The greatest number of tens whose square is contained in 4300 is 6. Subtracting 3600, which is the square of 6 tens, from the given number, the remainder is 756. This remainder is composed of *twice* the product of the tens by the units, and the square of the units (**803**, PRIN.). But the product of tens by units cannot be of a lower order than tens; hence the last figure 6 cannot be a part of twice the product of the tens by the units; this double product must therefore be found in the part 750.

Now, if we double the tens of the root and divide 750 by the result, the quotient 6 will be the units' figure of the root, or a figure greater than the units' figure. This quotient cannot be too small, for the part 750 is at least equal to twice the product of the tens by the units; but it may be too large, for the part 750, besides the double product of the tens by the units, may contain tens arising from the square of the units. (**812,** Prin. 1.) Subtracting $6\times120+6^2$ or $6\times\overline{120+6}$ from 756, nothing remains. Hence 66 is the required root.

1. In this example, 120 is a *partial* or *trial divisor,* and 126 is a *complete divisor.*

2. If the root contains more than two figures, it may be found by a similar process, as in the following example, where it will be seen that the partial divisor at each step is obtained by doubling that part of the root already found.

2. Find the square root of 186624.

OPERATION.

$$
\begin{array}{rl}
 & 18,66,24\ (\,400+30+2=432 \\
 & \underline{16\ 00\ 00} \\
400\times2+30=830\,) & 2\ 66\ 24 \\
 & \underline{2\ 49\ 00} \\
400\times2+30\times2+2=862\,) & 1724 \\
 & \underline{1724}
\end{array}
$$

The ciphers on the right are usually omitted for the sake of brevity. Thus,

$$
\begin{array}{rl}
 & 18,66,24\ (\,432 \\
 & \underline{16\qquad} \\
83\,) & 266,\ \text{etc.}
\end{array}
$$

3. Find the square root of 7225.

4. What is the square root of 58564.

Rule.—I. *Separate the given number into periods of two figures each, beginning at the units' place.*

II. *Find the greatest number whose square is contained in the period on the left; this will be the first figure in the root. Subtract the square of this figure from the period on the left, and to the remainder annex the next period to form a dividend.*

III. *Divide this dividend, omitting the figure on the right, by double the part of the root already found, and annex the quotient to that part, and also to the divisor; then multiply the divisor thus completed by the figure of the root last obtained, and subtract the product from the dividend.*

IV. *If there are more periods to be brought down, continue the operation in the same manner as before.*

1. If a cipher occur in the root, annex a cipher to the trial divisor, and another period to the dividend, and proceed as before.

2. If there is a remainder after the root of the last period is found, annex periods of ciphers and continue the root to as many decimal places as are required.

Find the square root

5. Of 9604.
6. Of 13225.
7. Of 11881.
8. Of 994009.
9. Of 2050624.
10. Of 29855296.

11. Find the square root of $\frac{100}{121}$.

OPERATION.—$\sqrt{\frac{100}{121}} = \frac{\sqrt{100}}{\sqrt{121}} = \frac{10}{11}$.

RULE.—*The square root of a fraction may be found by extracting the square root of the numerator and denominator separately.*

Mixed numbers may be reduced to the decimal form before extracting the root; or, if the denominator of the fraction is a perfect square, to an improper fraction.

In extracting the square root of a number containing a *decimal*, begin at the units' place, and proceed both toward the left and the right to separate into periods, then proceed as in the extraction of the square root of integers.

Extract the square root

12. Of $\frac{169}{196}$.
13. Of $\frac{625}{6561}$.
14. Of $\frac{576}{18225}$.
15. Of .001225.
16. Of 196.1369.
17. Of 2.251521.
18. Of 58.1406$\frac{1}{4}$.
19. Of 17$\frac{3}{4}$.
20. Of 10795.21.

21. What is the square root of 3486784401?
22. What is the square root of 9.0000994009?
23. Find the value of $32^{\frac{1}{2}}$ to 6 decimal places.
24. Find the square root of $2\frac{3}{8}$ to 4 decimal places.
25. Find the square root of $\frac{5}{6}$ to 5 decimal places.
26. Find the value of $.125^{\frac{2}{3}}$ to 5 decimal places.

Find the second member of the following equations:

27. $\sqrt{.1369}+\sqrt{1296}=?$
28. $(36\frac{2}{25})^{\frac{1}{2}}\times\sqrt{.25^2}=?$
29. $2.8^3\div\sqrt{.117649}=?$
30. $\left[\sqrt{\frac{3136}{7056}}-(\frac{225}{2025})^{\frac{1}{2}}\right]\div 2^2=?$
31. $\frac{9^{\frac{1}{2}}}{\sqrt{3^2}}\times\frac{3^2}{\sqrt{9^2}}=?$
32. $\sqrt{2.6896}+.(.3729\times\frac{3}{4}$ of $\sqrt{.256})=?$
33. $(7.2-\sqrt{27.04})^5\div(\frac{2}{3})^2=?$
34. $(\sqrt{81}-16^{\frac{1}{2}})\times(\sqrt{169}+25^{\frac{1}{2}})=?$
35. $\sqrt{264^2}\times 4.41\div(5{,}3361)^{\frac{1}{2}}-(2.3^3\times\sqrt{3.61})=?$

GEOMETRICAL EXPLANATION OF SQUARE ROOT.

814. What is the length of one side of a square whose area is 729 square feet?

FIG. 1.

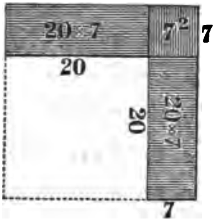
Let Fig. 1 represent a square whose area is 729 square feet. It is required to find the length of one side of this square.

Since the area of a square is equal to the square of one of its sides, a side may be found by extracting the square root of the area.

Since 729 consists of two periods, its square root will consist of two figures. The greatest number of *tens* whose square is contained in 700 is 2. Hence the length of the side of the square is 20 feet plus the units' figure of the root.

Removing the square whose side is 20 feet and whose area is 400 square feet, there remains a surface whose area is 329 square feet (Fig. 2). This remainder consists of two equal rectangles, each of which is 20 feet long, and a square whose side is equal to the width of each rectangle. The units' figure of the root is equal to the width of one of these rectangles.

FIG. 2.



The area of a rectangle is equal to the product of its length and width (**462**); hence, if the area be divided by the length, the quotient will be the width. Now, since the two rectangles contain the greater portion of the 329 square feet, 2 × 20 or 40, the length of the two rectangles, may be used as a trial divisor to find the width. Dividing 329 by 40, the quotient is 8. But this quotient is too large for the width of the rectangles, for if 8 feet is the width, the area of Fig. 2 will be $40 \times 8 + 8^2$ or 384 square feet. Taking 7 feet for the width of the rectangles, the area of Fig. 2 is $40 \times 7 + 7^2$ or 329 square feet. Hence $20 + 7$ or 27 feet is the length of a side of the square whose area is 729 square feet.

PROBLEMS

815. 1. A square field contains 1016064 square feet. What is the length of each side?

2. A square farm contains 361 A. Find the length of one side.

3. A field is 208 rd. long and 13 rd. wide. What is the length of the side of a square containing an equal area?

4. If 251 A. 65 P. of land are laid out in the form of a square, what will be the length of each of its sides?

5. A circular island contains 21170.25 P. of land. What is the length of the side of a square field of equal area?

6. If it cost $312 to enclose a field 216 rd. long and 24 rd. wide, what will it cost to enclose a square field of equal area with the same kind of fence?

CUBE ROOT.

816. A ***Perfect Cube*** is a number which has an exact cube root.

817. The ***Cube Root*** of a number is one of the *three* equal factors of that number. Thus, the cube root of 125 is 5, since $5 \times 5 \times 5 = 125$.

818. PRINCIPLES.—1. *The cube of a number expressed by a single figure contains no figure of a higher order than hundreds.*

2. *The cube of tens contains no significant figure of a lower order than thousands, or of a higher order than hundred thousands.*

3. *The cube of a number contains three times as many figures as the number, or three times as many, less one or two.* Thus,

$$\begin{array}{rlrl} 1^3 = & 1 & 10^3 = & 1,000 \\ 3^3 = & 27 & 100^3 = & 1,000,000 \\ 9^3 = & 729 & 1000^3 = & 1,000,000,000 \\ 99^3 = & 907,299 & 10000^3 = & 1,000,000,000,000 \end{array}$$

4. *If any perfect cube be separated into periods of three figures each, beginning with units' place, the number of periods will be equal to the number of figures in the cube root of that number.*

WRITTEN EXERCISES.

819. To find the cube root of a number.

1. Find the cube root of 405224.

OPERATION.

$$\begin{array}{rl} & 405,224\ (\ 70 + 4 = 74, \text{ cube root.} \\ 70^3 = & 343\ 000 \\ \hline 70^2 \times 3 = 14700\ ) & \underline{62\ 224} \\ 74^3 = & 405\ 224 \end{array}$$

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ANALYSIS.—Since 405224 consists of two periods, its cube root will consist of two figures (**818**, PRIN. 4). Since 224 cannot be a part of the cube of the tens of the root (**818**, PRIN. 2), the first figure of the root must be found from the first period, 405. The greatest number of *tens* whose cube is contained in 405000 is 7. Subtracting the cube of 7 tens from the given number, the remainder is 62224. This remainder is equal to the product of three times the square of the tens of the root by the units, plus three times the product of the tens by the square of the units, plus the cube of the units (**804**, PRIN.). But the product of the square of tens by units cannot be of a lower order than hundreds (**818**, PRIN. 2); hence the number represented by the last two figures, 24, cannot be a part of three times the product of the square of the tens of the root by the units; the triple product must therefore be found in the part 62200. Hence, if 62200 be divided by 3×70^2, the quotient, which is 4, will be the units' figure of the root or a figure greater than the units' figure. Subtracting 74^3 from the given number, the result is 0; hence 74 is the required root.

Instead of cubing 74, the parts which make up the remainder 62224 may be formed and added thus:

$$\begin{array}{r}
3 \times 70^2 \times 4 = 58800 \\
3 \times 70 \times 4^2 = 3360 \\
4^3 = \underline{64} \\
62224;
\end{array}$$

Or, since 4 is a common factor in the three parts which make up the remainder, these parts may be combined thus:

$$\begin{array}{r}
3 \times 70^2 = 14700 \\
3 \times 70 \times 4 = 840 \\
4^2 = \underline{16} \\
15556 \times 4 = 62224.
\end{array}$$

1. In this example, 14700 is a *partial* or *trial divisor*, and 15556 is a *complete divisor*.

2. If the cube root contains more than two figures, it may be found by a similar process, as in the following example, where it will be seen that the partial divisor at each step is equal to three times the square of that part of the root already found.

2. Find the cube root of 12812904.

OPERATION.

			Cube Root.
			12,812,904 ($200+30+4=234$
	$200^3=$		8 000 000
1ST PAR. DIVISOR	3×200^2 $=$	120000	4 812 904
	$3\times200\times30=$	18000	
	$30^2=$	900	4167000
1ST COMPLETE DIVISOR		138900	645904
2D PAR. DIVISOR	3×230^2 $=$	158700	
	$3\times230\times4=$	2760	
	$4^2=$	16	645904
2D COMPLETE DIVISOR		161476	

The operation may be abridged as follows:

			12,812,904 (234
	$2^3=$		8
1ST PARTIAL DIVISOR	3×20^2 $=$	1200	4812
	$3\times20\times3=$	180	
	$3^2=$	9	4167
1ST COMPLETE DIVISOR		1389	645904
2D PAR. DIVISOR	3×230^2 $=$	158700	
	$3\times230\times4=$	2760	
	$4^2=$	16	645904
2D COMPLETE DIVISOR		161476	

RULE.—I. *Separate the given number into periods of three figures each, beginning at the units' place.*

II. *Find the greatest number whose cube is contained in the period on the left; this will be the first figure in the root. Subtract the cube of this figure from the period on the left, and to the remainder annex the next period to form a dividend.*

III. *Divide this dividend by the partial divisor, which is* 3 *times the square of the root already found, considered as tens; the quotient is the second figure of the root.*

IV. *To the partial divisor add* 3 *times the product of the second figure of the root by the first considered as tens, also the square of the second figure, the result will be the complete divisor.*

V. *Multiply the complete divisor by the second figure of the root and subtract the product from the dividend.*

VI. *If there are more periods to be brought down, proceed as before, using the part of the root already found, the same as the first figure in the previous process.*

1. If a cipher occur in the root, annex two ciphers to the trial divisor, and another period to the dividend; then proceed as before, annexing both cipher and trial figure to the root.

2. If there is a remainder after the root of the last period is found, annex periods of ciphers and proceed as before. The figures of the root thus obtained will be *decimals*.

What is the cube root

3. Of 15625? | 5. Of 1030301? | 7. Of 1045678375?
4. Of 166375? | 6. Of 4492125? | 8. Of 4080659192?

9. Find the cube root of $\frac{8}{27}$.

OPERATION.—$\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$.

RULE.—*The cube root of a fraction may be found by extracting the cube root of the numerator and denominator.*

In extracting the cube root of *decimal* numbers, begin at the units' place and proceed both toward the left and the right, to separate into periods of *three* figures each.

Extract the cube root

10. Of $\frac{1000}{1331}$. | 12. Of $2\frac{7}{8}$. | 14. Of .091125.
11. Of $\frac{13824}{15625}$. | 13. Of 39304. | 15. Of 12.812904.

16. What is the cube root of 98867482624?

17. What is the cube root of .000529475129?

18. Find the cube root of $\frac{1}{2}$ correct to 4 decimal places.

Find the second member of the following equations:

19. $1.44^{\frac{1}{2}} + 2.5^{\frac{3}{2}} = ?$

21. $\sqrt[3]{.4096 - .2368} = ?$

20. $\sqrt[3]{\frac{1331}{3375}} \times \sqrt[3]{\frac{125}{512}} = ?$

22. $\sqrt[3]{54.872} - (21.952)^{\frac{1}{3}} = ?$

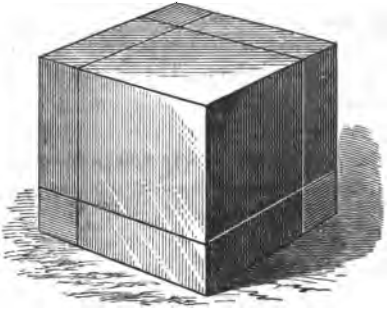
23. $(24.8 + \sqrt[3]{103.823}) \times (.125)^{\frac{1}{3}} = ?$

24. $\sqrt[3]{16^6} \div \sqrt[3]{64} - (4 \times \sqrt[3]{.512}) = ?$

GEOMETRICAL EXPLANATION OF CUBE ROOT.

820. What is the length of the edge of a cube whose volume is 15625 cubic feet?

FIG. 1.



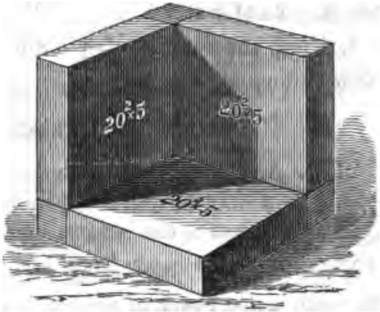
Let Fig. 1 represent a cube whose volume is 15625 cubic feet. It is required to find the length of the edge of this cube.

Since the volume of a cube is equal to the cube of one of its edges, an edge may be found by extracting the cube root of the volume.

Since 15625 consists of two periods, its cube root will consist of two figures. The greatest number of *tens* whose cube is contained in 15000 is 2. Hence, the length of the edge of the cube is 20 feet plus the units' figure of the root. Removing the cube whose edge is 20 feet and whose volume is 8000 cubic feet, there remains a solid whose volume is 7625 cubic feet (Fig. 2). This remainder consists of solids similar to those marked B, C, and D, in Fig. 1 and Fig. 2 of Art. **804.**

$$\begin{array}{rr|l}
 & & 15,625\,(25 \\
 & 2^3 = & 8 \\ \hline
3 \times 20^2 & = 1200 & 7625 \\
3 \times 20 \times 5 & = 300 & \\
 & 5^2 = 25 & \\ \hline
 & 1525 & 7625 \\ \hline
\end{array}$$

FIG. 2.



The volume of a rectangular solid is equal to the product of the area of its base by its height or thickness (**472**); hence, if the volume be divided by the area of the base the quotient will be the thickness. Now, since the three equal rectangular solids, each of which is 20 feet square and whose thickness is the units' figure of the root, contain the greater portion of the 7625 cubic feet, 3×20^2 or 300×2^2 may be used as a trial divisor to find the thickness. Dividing 7625 by 1200 the quotient is 6. But this quotient is too large, for if 6 feet is the thickness, the volume of Fig. 2 will be $3 \times 20^2 \times 6 + 3 \times 20 \times 6^2 + 6^3$, or 9576 cubic feet. Taking 5 feet for the thickness, the volume of Fig. 2 is 7625 cubic feet, for $3 \times 20^2 \times 5 + 3 \times 20 \times 5^2 + 5^3 = (300 \times 2^2 + 30 \times 2 \times 5 + 5^2)\ 5 = 1525 \times 5 = 7625$. Hence, 25 feet is the length of the edge of a cube whose volume is 15625 cubic feet.

PROBLEMS.

821. 1. What is the length of the edge of a cubical box that contains 46656 cu. inches?

2. What must be the length of the edge of a cubical bin that shall contain the same volume as one that is 16 ft. long, 8 ft. wide, and 4 ft. deep?

3. What are the dimensions of a cube that has the same volume as a box 2 ft. 8 in. long, 2 ft. 3 in. wide, and 1 ft. 4 in. deep?

4. How many square feet in the surface of a cube whose volume is 91125 cubic feet?

5. What is the length of the inner edge of a cubical bin that contains 150 bushels?

6. What is the depth of a cubical cistern that holds 200 barrels of water?

7. Find the length of a cubical vessel that will hold 4000 gallons of water.

ROOTS OF HIGHER DEGREE.

822. Any root whose index contains no other factors than 2 or 3 may be extracted by means of the square and cube roots.

If any power of a given number is raised to any required power, the result is that power of the given number denoted by the *product* of the two exponents. (**801.**) Conversely, if two or more roots of a given number are extracted, successively, the result is that root of the given number denoted by the product of the indices.

1. What is the 6th root of 2176782336?

OPERATION.

$$\sqrt[2]{2176782336} = 46656$$
$$\sqrt[3]{46656} = 36$$

Or,

$$\sqrt[3]{2176782336} = 1296$$
$$\sqrt[2]{1296} = 36$$

ANALYSIS.—The index of the required root is $6 = 2 \times 3$; hence extract the square root of the given number, and the cube root of this result, which gives 36 as the 6th or required root. Or, first find the cube root of the given number, and then the square root of the result.

RULE.—*Separate the index of the required root into its prime factors, and extract successively the roots indicated by the several factors obtained; the final result will be the required root.*

2. What is the 4th root of 5636405776?

3. What is the 8th root of 1099511627776?

4. What is the 6th root of 25632972850442049?

5. What is the 9th root of 1.577635?

For further practical applications of Involution and Evolution, see "Mensuration."

PROGRESSIONS

823. An ***Arithmetical Progression*** is a succession of numbers, each of which is greater or less than the preceding one by a constant *difference.*

Thus, 5, 7, 9, 11, 13, 15, is an arithmetical progression.

824. The ***Terms*** of an arithmetical progression are the numbers of which it consists. The first and last terms are called the *Extremes,* and the other terms the *Means.*

825. The ***Common Difference*** is the difference between any two consecutive terms of the progression.

826. An ***Increasing Arithmetical Progression*** is one in which each term is greater than the preceding one.

Thus, 1, 3, 5, 7, 9, 11, is an increasing progression.

827. A ***Decreasing Arithmetical Progression*** is one in which each term is less than the preceding one.

Thus, 15, 13, 11, 9, 7, 5, 3, 1, is a decreasing progression.

828. The following are the quantities considered in arithmetical progression and the abbreviations used for them:

1. The first term, (a).
2. The last term, (l).
3. The common difference, (d).
4. The number of terms, (n).
5. The sum of all the terms, (s).

WRITTEN EXERCISES.

829. To find one of the extremes, when the other extreme, the common difference, and the number of terms are given.

1. The first term of an increasing progression is 8, the common difference 5, and the number of terms 20; what is the last term?

OPERATION.

$20 - 1 = 19$

$\overline{19 \times 5} + 8 = 103 = l.$

ANALYSIS.—The 2d term is 8+5; the 3d term is 8+(5 × 2); the 4th term is 8+(5 × 3); and so on. Hence, 8+(19 × 5) or 103 is the 20th or *last term.*

2. The last term of an increasing progression is 103, the common difference 5, and the number of terms 20; what is the first term?

OPERATION.

$20 - 1 = 19$

$103 - \overline{19 \times 5} = 8 = a$

ANALYSIS.—The 1st term must be a number to which, if 19 × 5 be added, the sum shall be 103; hence, if 19 × 5 is subtracted from 103, the remainder is the *first term.*

3. The first term of a decreasing progression is 203, the common difference 5, and the number of terms 40; what is the last term?

4. The last term of a decreasing progression is 1, the common difference 2, and the number of terms 9; what is the first term?

RULE.—I. *If the given extreme is the less, add to it the product of the common difference by the number of terms less one.*

II. *If the given extreme is the greater, subtract from it the product of the common difference by the number of terms less one.*

FORMULÆ.— $\begin{cases} l = a + (n - 1) \times d. \\ a = l - (n - 1) \times d. \end{cases}$

5. The first term of an increasing progression is 5, the common difference 4, and the number of terms 8; what is the last term?

6. The first term of an increasing progression is 2, and the common difference 3; what is the 50th term?

7. The first term of a decreasing progression is 100, and the common difference 7; what is the 13th term?

8. The first term of an increasing progression is $\frac{3}{8}$, the common difference $\frac{3}{8}$, and the number of terms 20; what is the last term?

830. To find the common difference, when the extremes and number of terms are given.

1. The extremes of a progression are 8 and 103, and the number of terms 20; what is the common difference?

OPERATION.

$\overline{103 - 8} \div 19 = 5 = d$

ANALYSIS.—The difference between the extremes is equal to the product of the common difference by the number of terms less one (**829**); hence the *common difference* is $\frac{95}{19}$, or 5.

2. The extremes of a progression are 1 and 17, and the number of terms 9; what is the common difference?

RULE.—*Divide the difference between the extremes by the number of terms less one.*

FORMULA.—$d = \dfrac{l - a}{n - 1}$.

3. The extremes are 3 and 15, and the number of terms 7; what is the common difference?

4. The extremes are 1 and 51, and the number of terms 76; what is the common difference?

5. The youngest of ten children is 8, and the eldest 44 years old; their ages are in arithmetical progression. What is the common difference of their ages?

6. The amount of $800 for 60 years, at simple interest, is $4160. What is the rate per cent.?

7. The extremes are 0 and 2½, and the number of terms 18; what is the common difference?

831. To find the number of terms, when the extremes and common difference are given.

1. The extremes of a progression are 8 and 103, and the common difference 5; what is the number of terms?

OPERATION.

$\overline{103-8}\div 5=19$

$19+1=20=n$

ANALYSIS.—The difference between the extremes is equal to the product of the common difference by the number of terms less one (**830**); hence the number of terms less one is equal to $\frac{95}{5}$ or 19; therefore 19 + 1 or 20 is the *number of terms.*

2. The extremes of a progression are 1 and 17, and the common difference 2; what is the number of terms?

RULE.—*Divide the difference between the extremes by the common difference, and add one to the quotient.*

FORMULA.—$n=\frac{l-a}{d}+1.$

3. The extremes are 5 and 75, and the common difference is 5; what is the number of terms?

4. The extremes are ½ and 20, and the common difference is 6½; what is the number of terms?

5. A laborer received 50 cents the first day, 54 cents the second, 58 cents the third, and so on, until his wages were $1.54 a day; how many days did he work?

6. In what time will $500, at 7 per cent. simple interest, amount to $885?

832. To find the sum of all the terms, when the extremes and the number of terms are given.

1. The extremes of an arithmetical progression are 2 and 14, and the number of terms is 5; what is the sum of all the terms?

OPERATION.

$$\begin{aligned} s &= 2+5+8+11+14 \\ s &= \underline{14+11+8+5+2} \\ 2s &= 16+16+16+16+16 \\ 2s &= 16\times 5 = (2+14)\times 5 \\ s &= \frac{2+14}{2}\times 5 = 40. \end{aligned}$$

ANALYSIS.—The common difference is found to be 3 (**830**); hence the required sum is equal to 2+5+8+11+14, or 14+11+8+5+2. Adding the corresponding terms of these two progressions, we have 2 times the sum $= 16 \times 5 = (2 + 14) \times 5$; hence the sum is $\frac{2+14}{2} \times 5 = 40$.

2. The extremes of an arithmetical progression are 5 and 75, and the number of terms is 15; what is the sum of all the terms?

RULE.—*Multiply the sum of the extremes by half the number of terms.*

FORMULA.—$s = \frac{n}{2} \times (a + l)$.

3. The extremes are 4 and 40, and the number of terms is 7; what is the sum of all the terms?

4. The extremes are 0 and 250, and the number of terms is 1000; what is the sum of all the terms?

5. How many strokes, beginning at 1 o'clock, does the hammer of a common clock strike in 12 hours?

6. A body will fall $16\frac{1}{12}$ ft. in the first second of its fall, $48\frac{1}{4}$ ft. in the second second, $80\frac{5}{12}$ ft. in the third second, and so on; how far will it fall in one minute?

833. A ***Geometrical Progression*** is a succession of numbers, each of which is greater or less than the preceding one in a constant *ratio.*

Thus, 1, 3, 9, 27, 81, etc., is a geometrical progression.

834. The ***Terms*** of a geometrical progression are the numbers of which the progression consists. The first and last terms are called the *Extremes,* and the other terms the *Means.*

835. The ***Ratio*** of a geometrical progression is the quotient obtained by dividing any term by the preceding one.

836. An ***Increasing Geometrical Progression*** is one in which the ratio is greater than 1.

Thus, 1, 2, 4, 8, 16, etc., is an increasing progression.

837. A ***Decreasing Geometrical Progression*** is one in which the ratio is less than 1.

Thus, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc., is a decreasing progression.

838. An ***Infinite Decreasing Geometrical Progression*** is one in which the ratio is less than 1, and the number of terms *infinite.*

Thus, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, and so on is an infinite decreasing progression.

839. The following are the quantities considered in geometrical progression:

1. The first term (a).
2. The last term (l).
3. The ratio (r).
4. The number of terms (n).
5. The sum of all the terms (s).

WRITTEN EXERCISES.

840. To find one of the extremes, when the other extreme, the ratio, and the number of terms are given.

1. The first term of a progression is 2, the ratio 3, and the number of terms 10; what is the last term?

OPERATION.

$$3^9 = 19683$$
$$\underline{\quad\quad\quad 2}$$
$$39366 = l$$

ANALYSIS.—The 2d term is 2×3; the third term is $2 \times 3 \times 3$ or 2×3^2; the 4th term is 2×3^3; and so on. Hence the 10th or *last term* is 2×3^9 or 39366.

2. The last term of a progression is 39366, the ratio 3, and the number of terms 10; what is the first term?

OPERATION.

$$\frac{39366}{3^9} = 2 = a$$

ANALYSIS.—The first term must be a number, by which if 3^9 be multiplied the product shall be 39366; hence, if 39366 be divided by 3^9, the quotient will be the *first term.*

3. The first term of a progression is 1, the ratio $\frac{1}{2}$, and the number of terms 9; what is the last term?

RULE.—I. *If the given extreme is the first term, multiply it by that power of the ratio whose exponent is one less than the number of terms.*

II. *If the given extreme is the last term, divide it by that power of the ratio whose exponent is one less than the number of terms.*

FORMULÆ.—$l = ar^{n-1}$; $\quad a = \dfrac{l}{r^{n-1}}$.

4. The first term of a geometrical progression is 6, the ratio 4, the number of terms 6; what is the last term?

5. The last term is 192, the ratio 2, and the number of terms 7; what is the first term?

6. A drover bought 20 cows, agreeing to pay $1 for the first, $2 for the second, $4 for the third, and so on; how much did he pay for the last cow?

7. Find the amount of $250 for 4 years at 6 per cent. compound interest.

The first term is 250, the ratio 1.06, and the number of terms 5.

8. If 1 cent had been put at interest in 1634, what would it have amounted to in the year 1874, if it had doubled its value every 12 years?

841. To find the ratio, when the extremes and the number of terms are given.

1. The first term is 2, the last term 512, and the number of terms 5; what is the ratio?

OPERATION.

$\frac{512}{2} = 256$

$\sqrt[4]{256} = 4 = r$

ANALYSIS.—If the 4th power of the ratio be multiplied by 2, the product will be 512 (**840**); hence, if 512 be divided by 2, the quotient, 256, will be the 4th power of the ratio. Hence the *ratio* is the 4th root of 256, or 4.

2. The first term is 1, the last term $\frac{1}{256}$, and the number of terms 9; what is the ratio?

RULE.—*Divide the last term by the first, and extract that root of the quotient whose index is one less than the number of terms.*

FORMULA.—$r = \sqrt[n-1]{\frac{l}{a}}$.

3. The first term is 8, the last term 5000, and the number of terms 5; what is the ratio?

4. The first term is .0112, the last term 7, and the number of terms 5; what is the ratio?

5. The first term is $\frac{1}{48}$, the last term $15\frac{3}{16}$, and the number of terms 7; what is the ratio?

842. To find the number of terms, when the extremes and the ratio are given.

1. The extremes are 2 and 512, and the ratio is 4; what is the number of terms?

OPERATION.

$2\,)\,\underline{512}$

256

$4^4 = 256$

ANALYSIS.—If 512 be divided by 2, the quotient, 256, will be that power of the ratio whose exponent is one less than the number of terms (**841**). But 256 is the 4th power of the ratio 4; hence the *number of terms* is 5.

2. The extremes are 1 and $\frac{1}{256}$, and the ratio is $\frac{1}{2}$; what is the number of terms?

RULE.—*Divide the last term by the first; then the exponent of the power to which the ratio must be raised to produce the quotient is one less than the number of terms.*

FORMULA.—$r^{n-1} = \dfrac{l}{a}$.

3. The extremes are 2 and 1458, and the ratio is 3; what is the number of terms?

4. The extremes are $\frac{1}{640}$ and $\frac{1}{5}$, and the ratio 2; what is the number of terms?

843. To find the sum of all the terms, when the extremes and the ratio are given.

1. The extremes are 2 and 128, and the ratio is 4; what is the sum of all the terms?

OPERATION.

$$\frac{(128 \times 4) - 2}{4 - 1} = \frac{510}{3} = 170 = s$$

$4s = \quad 8 + 32 + 128 + 512$

$s = \underline{2 + 8 + 32 + 128}$

$3s = 512 - 2 = 510$

$\frac{510}{3} = 170 = s$

ANALYSIS.—Subtract the sum from 4 times the sum, and 510 remains, which is 3 times the sum; hence, $\frac{510}{3}$, or 170, is the *sum*.

2. The extremes are 1 and $\frac{1}{16}$, and the ratio is $\frac{1}{2}$; what is the sum of all the terms?

$$\begin{aligned} s &= 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} \\ \tfrac{1}{2}\, s &= \underline{\quad\ \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{32}} \\ \tfrac{1}{2}\, s &= 1 - \tfrac{1}{32} = \tfrac{31}{32};\ \tfrac{31}{32} \div \tfrac{1}{2} = 1\tfrac{15}{16}. \end{aligned}$$

RULE.—*Multiply the last term by the ratio, and divide the difference between the product and the first term by the difference between* 1 *and the ratio.*

FORMULA.—$s = \dfrac{lr - a}{r - 1}$.

3. The extremes are 3 and 384, and the ratio is 2; what is the sum of all the terms?

4. The extremes are $4\frac{4}{5}$ and $\frac{8}{405}$, and the ratio is $\frac{1}{3}$. what is the sum of all the terms?

5. What is the sum of all the terms of the infinite progression 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, ... ?

The *last term* of this progression may be conceived as 0.

6. What is the sum of all the terms of the infinite progression 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, ... ?

7. What is the sum of $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$, etc., to infinity?

8. The first is 7, the ratio 3, and the number of terms 4; what is the sum of all the terms?

First find the last term by Art. **840.**

9. A drover bought 10 cows, agreeing to pay $1 for the first, $2 for the second, $4 for the third, and so on; what did he pay for the 10 cows?

10. If a man were to buy 12 horses, paying 2 cents for the first horse, 6 cents for the second, and so on, what would they cost him?

ANNUITIES

844. An ***Annuity*** is a sum of money payable annually. The term is also applied to a sum of money payable at any equal intervals of time.

845. A ***Certain Annuity*** is one which continues for a definite period of time.

846. A ***Perpetual Annuity*** or ***Perpetuity*** is one which continues forever.

847. A ***Contingent Annuity*** is one which begins or ends, or both begins and ends, on the occurrence of some specified future event or events.

848. An ***Annuity Forborne*** or ***in Arrears*** is one the payments of which were not made when due.

849. The ***Amount*** or ***Final Value*** of an annuity is the sum of all the payments increased by the interest of each payment from the time it becomes due until the annuity ceases.

850. The ***Present Worth*** of an annuity is such a sum of money as will, in the given time, and at the given rate per cent., amount to the final value.

851. An annuity is said to be *deferred* when it does not begin until after a certain period of time; it is said to be *reversionary* when it does not begin until after the occurrence of some specified future event, as the death of a certain person; and it is said to be *in possession* when it has begun, or begins immediately.

ANNUITIES AT SIMPLE INTEREST.

852. All problems in annuities at simple interest may be solved by combining the rules in Arithmetical Progression with those in Simple Interest.

WRITTEN EXERCISES.

853. 1. What is the amount of an annuity of $300 for 5 years, at 6 per cent. simple interest?

OPERATION.

$$\frac{300+372}{2}\times 5 = 1680$$

ANALYSIS.—At the end of the 5th year the following sums were due:

The 5th year's payment = $300,
The 4th year's payment = $300 + $18 = $318,
The 3d year's payment = $300 + $36 = $336,
The 2d year's payment = $300 + $54 = $354,
The 1st year's payment = $300 + $72 = $372.

These sums form an arithmetical progression, in which the first term is the annuity, $300, the common difference is the interest of the annuity for 1 year, and the number of terms is the number of years. The sum of all the terms of this progression is $1680 (**832**), which is the amount of the annuity.

2. A father deposits annually for the benefit of his son, beginning with his tenth birthday, such a sum that on his 21st birthday the first deposit, at simple int., amounts to $210, and the sum due his son is $1860. Find the annual deposit, and at what rate per cent. it is deposited.

OPERATION.

$$6 \times (\text{1st term} + 210) = 1860. \quad (\mathbf{832.})$$
$$\text{Hence, 1st term} = 310 - 210 = 100 = a.$$
$$(210 - 100) \div (12 - 1) = \tfrac{110}{11} = 10 = d. \quad (\mathbf{830.})$$

ANALYSIS.—Here $210, the first deposit, is the last term; 12, the number of deposits, is the number of terms;

and $1860, the final value of the annuity, is the sum of all the terms. Using the principle of **832**, we find the *first term* to be $100, which is the annual deposit. By **830**, the *common difference* is found to be $10; hence 10 per cent. is the required rate.

3. What is the amount of an annuity of $150 for 5½ years, payable quarterly, at 1½ per cent. per quarter?

4. What is the present worth of an annuity of $300 for 5 years, at 6 per cent.?

5. What is the present worth of an annuity of $500 for 10 years, at 10 per cent.?

6. In what time will an annual pension of $500 amount to $3450, at 6 per cent. simple interest?

7. Find the rate per cent. at which an annuity of $6000 will amount to $59760 in 8 years, at simple interest.

8. A man works for a farmer 1 yr. 6 mo., at $20 per month, payable monthly; and these wages remain unpaid until the expiration of the whole term of service. What is due the workman, allowing simple interest at 6 per cent. per annum?

ANNUITIES AT COMPOUND INTEREST.

854. All problems in annuities at compound interest may be solved by combining the rules in Geometrical Progression with those in Compound Interest.

WRITTEN EXERCISES.

1. What is the amount of an annuity of $300 for 5 years, at 6 per cent. compound interest?

OPERATION.

$$\frac{300 \times 1.06^5 - 300}{.06} = 1691.13$$

ANALYSIS.—At the end of the 5th year the following sums are due:

The 5th year's payment $= \$300$,
The 4th year's payment + interest for 1 year $= \$300 \times 1.06$,
The 3d year's payment + compound int. for 2 years $= \$300 \times 1.06^2$,
The 2d year's payment + compound int. for 3 years $= \$300 \times 1.06^3$,
The 1st year's payment + compound int. for 4 years $= \$300 \times 1.06^4$.

These sums form a geometrical progression, in which the first term is the annuity, \$300, the ratio is the amount of \$1 for 1 year, and the number of terms is the number of years. The sum of all the terms of this progression is \$1691.13 (**843**), which is the amount of the annuity.

2. What is the present worth of an annuity of \$300 for 5 years, at 6 per cent. compound interest?

OPERATION.

$$\frac{1691.13}{1.338226} = 1263.71$$

ANALYSIS.—The amount of this annuity is \$1691.13. The amount of \$1 for 5 years, at 6 per cent. compound interest, is \$1.338226 (**587**). Hence, the present worth of the annuity is $\frac{\$1691.13}{1.338226}$, or \$1263.71.

3. Find the annuity whose amount for 25 years, at 6 per cent. compound interest, is \$16459.35.

4. What is the present worth of an annuity of \$700 for 7 years, at 6 per cent. compound interest?

5. An annuity of \$200 for 12 years is in reversion 6 years. What is its present worth, compound interest at 6%?

6. A man bought a tract of land for \$4800, which was to be paid in installments of \$600 a year; how much money, at 6 per cent. compound interest, would discharge the debt at the time of the purchase?

7. What is the present value of a reversionary lease of \$100, commencing 14 years hence, and to continue 20 years, compound interest at 5 per cent.?

855. SYNOPSIS FOR REVIEW.

INVOLUTION.

- 1. DEFS. 1. A Power. 2. Involution. 3. Base, or Root. 4. Exponent. 5. Square. 6. Cube. 7. Perfect Power.
- 2. PRINCIPLE.
- 3. **802.** RULE. 1. For Integers. 2. For Fractions.
- 4. **803.** 1. Principle. 2. Geometrical Illustration.
- 5. **804.** 1. Principle. 2. Geometrical Illustration.

EVOLUTION.

- 1. DEFS. 1. Square Root. 2. Cube Root, etc. 3. Evolution 4. Radical Sign. 5. Index.
- 2. **810.** Rule.
- 3. **812.** Principles, 1, 2, 3, 4.
- 4. **813.** Rule, I, II, III. For Fractions.
- 5. **814.** Geometrical Illustration.
- 6. **818.** Principles, 1, 2, 3, 4.
- 7. **819.** Rule, I, II, III, IV, V, VI. For Fractions.
- 8. **820.** Geometrical Illustration.
- 9. **822.** Roots of a Higher Degree. *Rule.*

PROGRESSIONS.

- 1. DEFS. 1. *Arithmetical Progression.* 2. *Terms.* 3. *Common Difference.* 4. *Increasing Arithmetical Progression.* 5. *Decreasing Arithmetical Progression.*
- 2. Quantities considered.
- 3. **829.** Rule, I, II. *Formulæ.*
- 4. **830.** Rule. *Formula.*
- 5. **831.** Rule. *Formula.*
- 6. **832.** Rule. *Formula.*

- 1. DEFS. 1. *Geometrical Progression.* 2. *Terms.* 3. *Ratio.* 4. *Increasing Geom. Prog.* 5. *Decreasing Geom. Prog.* 6. *Infinite Decreasing Geom. Prog.*
- 2. Quantities considered.
- 3. **840.** Rule, I, II. *Formulæ.*
- 4. **841.** Rule. *Formula.*
- 5. **842.** Rule. *Formula.*
- 6. **843.** Rule. *Formula.*

ANNUITIES.

- 1. DEFS. 1. Annuity. 2. Certain Annuity. 3. Perpetuity. 4. Contingent Annuity. 5. Annuity in Arrears. 6. Amount. 7. Present Worth of an Annuity 8. Deferred Annuity. 9. Reversionary Annuity. 10. Annuity in Possession.
- 2. ANNUITIES AT SIMPLE INTEREST. } Problems, how solved.
- 3. ANNUITIES AT COMP. INTEREST. } Problems, how solved.

MENSURATION

856. ***Mensuration*** is the process of finding the number of units in extension.

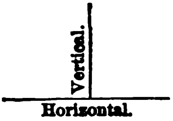
LINES.

857. A ***Straight Line*** is a line that does not change its direction. It is the shortest distance between two points.

858. A ***Curved Line*** changes its direction at every point.

859. ***Parallel Lines*** have the same direction; and being in the same plane and equally distant from each other, they can never meet.

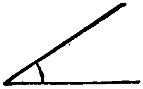
860. A ***Horizontal Line*** is a line parallel either to the horizon or water level.



861. A ***Perpendicular Line*** is a straight line drawn to meet another straight line, so as to incline no more to the one side than to the other.

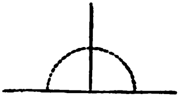
A perpendicular to a horizontal line is called a *vertical* line.

ANGLES.

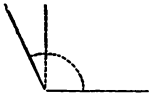


862. An ***Angle*** is the difference in the direction of two lines proceeding from a common point, called the *vertex*.

Angles are measured by degrees. (**301.**)



863. A ***Right Angle*** is an angle formed by two lines perpendicular to each other.



864. An ***Obtuse Angle*** is greater than a right angle.

865. An ***Acute Angle*** is less than a right angle.

All angles except right angles are called *oblique angles.*

PLANE FIGURES.

866. A ***Plane Figure*** is a portion of a plane surface bounded by straight or curved lines.

867. A ***Polygon*** is a plane figure bounded by straight lines.

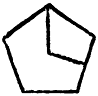
868. The ***Perimeter*** of a polygon is the sum of its sides.

869. The ***Area*** of a plane figure is the surface included within the lines which bound it. (**460.**)

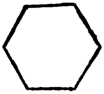
A *regular polygon* has all its sides and all its angles equal.

The *altitude* of a polygon is the perpendicular distance between its *base* and a side or angle opposite.

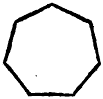
A polygon of three sides is called a *trigon*, or triangle ; of four sides, a *tetragon*, or quadrilateral ; of five sides, a *pentagon*, etc.



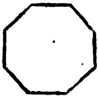
Pentagon.



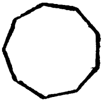
Hexagon.



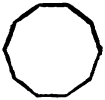
Heptagon.



Octagon.



Nonagon.

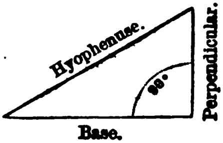


Decagon.

TRIANGLES.

870. A ***Triangle*** is a plane figure bounded by three sides, and having three angles.

871. A ***Right-Angled Triangle*** is a triangle having one right angle.



872. The ***Hypothenuse*** of a right-angled triangle is the side opposite the right angle.

873. The ***Base*** of a triangle, or of any plane figure, is the side on which it is supposed to stand.

874. The ***Perpendicular*** of a right-angled triangle is the side which forms a right angle with the base.

875. The ***Altitude*** of a triangle is a line drawn from the angle opposite perpendicular to the base.

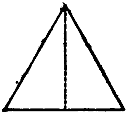
1. The dotted lines in the following figures represent the *altitude*.
2. Triangles are named from the relation both of their sides and angles.

876. An ***Equilateral Triangle*** has its three sides equal.

877. An ***Isosceles Triangle*** has only two of its sides equal.

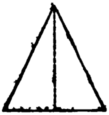
878. A ***Scalene Triangle*** has all of its sides unequal.

FIG. 1.



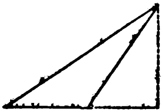
Equilateral.

FIG. 2.



Isosceles.

FIG. 3.



Scalene.

879. An ***Equiangular Triangle*** has three equal angles. (Fig. 1.)

880. An ***Acute-angled Triangle*** has three acute angles. (Fig. 2.)

881. An ***Obtuse-angled Triangle*** has one obtuse angle. (Fig. 3.)

PROBLEMS.

882. The base and altitude of a triangle being given to find its area.

1. Find the area of a triangle whose base is 26 ft. and altitude 14.5 feet.

OPERATION.—$\overline{14.5\times26}\div2=188\frac{1}{2}$ sq. ft. Or, $26\times\frac{14.5}{2}=188\frac{1}{2}$ square feet, *area*.

2. What is the area of a triangle whose altitude is 10 yards and base 40 feet?

RULE.—1. *Divide the product of the base and altitude by 2.* Or,
2. *Multiply the base by one-half the altitude.*

Find the area of a triangle

3. Whose base is 12 ft. 6 in and altitude 6 ft. 9 in.

4. Whose base is 25.01 chains and altitude 18.14 chains.

5. What is the cost of a triangular piece of land whose base is 15.48 ch. and altitude 9.67 ch., at $60 an acre?

6. At $.40 a square yard, find the cost of paving a triangular court, its base being 105 feet and its altitude 21 yards?

7. Find the area of the gable end of a house that is 28 ft. wide, and the ridge of the roof 15 ft. higher than the foot of the rafters.

883. The area and one dimension being given to find the other dimension.

1. What is the base of a triangle whose area is 189 square feet and altitude 14 feet?

OPERATION.—$(189 \text{ sq. ft.} \times 2) \div 14 = 27$ ft., *base.*

2. Find the altitude of a triangle whose area is 20¼ square feet and base 3 yards.

RULE.—*Double the area, then divide by the given dimension.*

Find the other dimension of the triangle

3. When the area is 65 sq. in. and the altitude 10 inches.
4. When the base is 42 rods and the area 588 sq. rods.
5. When the area is 6½ acres and the altitude 17 yards.
6. When the base is 12.25 chains and the area 5 A. 33 P.
7. Paid $1050 for a piece of land in the form of a triangle, at the rate of $5¼ per square rod. If the base is 8 rd., what is its altitude?

884. The three sides of a triangle being given to find its area.

1. Find the area of a triangle whose sides are 30, 40, and 50 ft.

OPERATION.—$(30+40+50) \div 2 = 60$; $60-30 = 30$; $60-40 = 20$; $60-50 = 10$. $\sqrt{60 \times 30 \times 20 \times 10} = 600$ ft., *area.*

2. What is the area of an isosceles triangle whose base is 20 ft., and each of its equal sides 15 feet?

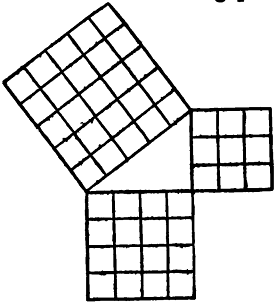
RULE.—*From half the sum of the three sides, subtract each side separately; multiply the half-sum and the three remainders together; the square root of the product is the area.*

3. Find the area of a triangle whose sides are 25, 36, and 49 in.

4. How many acres in a field in the form of an equilateral triangle whose sides each measure 70 rods?

5. The roof of a house 30 ft. wide has the rafters on one side 20 ft. long, and on the other 18 ft. long. How many square feet of boards will be required to board up both gable ends?

885. The following principles relating to *right-angled* triangles have been established by Geometry:



PRINCIPLES.—1. *The square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

2. *The square of the base, or of the perpendicular, of a right-angled triangle is equal to the square of the hypothenuse diminished by the square of the other side.*

886. To find the hypothenuse.

1. The base of a right-angled triangle is 12, and the perpendicular 16. What is the length of the hypothenuse?

OPERATION.—$12^2+16^2=400$ (PRIN. 1). $\sqrt{400}=20$, *hypothenuse.*

2. The foot of a ladder is 15 feet from the base of a building, and the top reaches a window 36 feet above the base. What is the length of the ladder?

RULE.—*Extract the square root of the sum of the squares of the base and the perpendicular; the result is the hypothenuse.*

3. If the gable end of a house 40 ft. wide is 16 ft. high, what is the length of the rafters?

4. A park 25 chains long and 23 chains wide has a walk running through it from opposite corners in a straight line. What is the length of the walk?

5. A room is 20 ft. long, 16 ft. wide, and 12 ft. high. What is the distance from one of the lower corners to the opposite upper corner?

887. To find the base or perpendicular.

1. The hypothenuse of a right-angled triangle is 35 feet, and the perpendicular 28 feet. Find the base.

OPERATION.—$35^2-28^2=441$ (PRIN. 2). $\sqrt{441}=21$ ft., *base.*

2. The hypothenuse of a right-angled triangle is 53 yards and the base 84 feet. Find the perpendicular.

Rule.—*Extract the square root of the difference between the square of the hypothenuse and the square of the given side; the result is the required side.*

3. Find the width of a house, whose rafters are 12 ft. and 15 ft. long, and that form a right angle at the point in which they meet.

4. A line reaching from the top of a precipice 120 feet high, on the bank of a river, to the opposite side is 380 feet long. How wide is the river?

5. A ladder 52 ft. long stands against the side of a building. How many feet must it be drawn out at the bottom that the top may be lowered 4 feet?

QUADRILATERALS.

888. A ***Quadrilateral*** is a plane figure bounded by four straight lines.

There are three kinds of quadrilaterals, the *Parallelogram*, *Trapezoid*, and *Trapezium*.

889. A ***Parallelogram*** is a quadrilateral which has its opposite sides parallel.

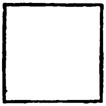
There are four kinds of parallelograms, the *Square*, *Rectangle*, *Rhomboid*, and *Rhombus*.

890. A ***Rectangle*** is any parallelogram having its angles right angles.

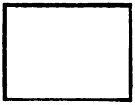
891. A ***Square*** is a rectangle whose sides are equal.

892. A ***Rhomboid*** is a parallelogram whose opposite sides *only* are equal, and whose angles are not right angles.

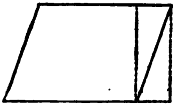
893. A ***Rhombus*** is a parallelogram whose sides are *all* equal, but whose angles are not right angles.



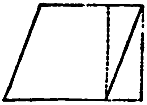
Square.



Rectangle.



Rhomboid.



Rhombus.

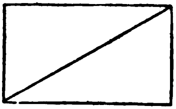
894. A ***Trapezoid*** is a quadrilateral, two of whose sides are *parallel.*

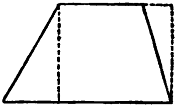
895. A ***Trapezium*** is a quadrilateral having no two sides parallel.

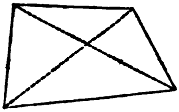
896. The ***Altitude*** of a parallelogram or of a trapezoid is the perpendicular distance between its *parallel* sides.

The dotted vertical lines in the figure represent the *altitude.*

897. A ***Diagonal*** of a plane figure is a straight line joining the vertices of two angles not adjacent.


Parallelogram.


Trapezoid.


Trapezium.

PROBLEMS.

898. To find the area of any parallelogram.

1. Find the area of a parallelogram whose base is 16.25 feet and altitude 7.5 feet.

OPERATION.—16.25 ft. × 7.5 = 121.875 sq. feet, *area.*

2. The base of a rhombus is 10 feet 6 inches, and its altitude 8 feet. What is its area?

RULE.—*Multiply the base by the altitude.*

3. How many acres in a piece of land in the form of a rhomboid, the base being 8.75 ch. and altitude 6 chains?

899. To find the area of a trapezoid.

1. Find the area of a trapezoid whose parallel sides are 23 and 11 feet, and the altitude 9 feet.

OPERATION.—$\overline{23 \text{ ft.} + 11 \text{ ft.}} \div 2 = 17$ ft.; 17 ft. × 9 = 153 sq. ft., *area.*

2. Required the area of a trapezoid whose parallel sides are 178 and 146 feet, and the altitude 69 feet.

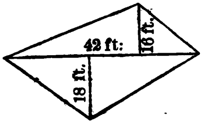
RULE.—*Multiply one-half the sum of the parallel sides by the altitude.*

3. How many square feet in a board 16 ft. long, 18 inches wide at one end and 25 inches wide at the other end?

4. One side of a quadrilateral field measures 38 rods; the side opposite and parallel to it measures 26 rods, and the distance between the two sides is 10 rods. Find the area.

900. To find the area of a trapezium.

1. Find the area of a trapezium whose diagonal is 42 feet and perpendiculars to this diagonal, as in the diagram, are 16 feet and 18 feet.



Operation.—$(\overline{18\text{ ft.}+16\text{ ft.}}\div 2)\times 42 = 714$ sq. feet, *area.*

2. Find the area of a trapezium whose diagonal is 35 ft. 6 in., and the perpendiculars to this diagonal 9 feet and 3 feet.

Rule.—*Multiply the diagonal by half the sum of the perpendiculars drawn to it from the vertices of opposite angles.*

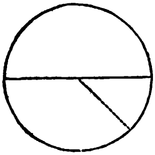
3. How many acres in a quadrilateral field whose diagonal is 80 rd. and the perpendiculars to this diagonal 20.453 and 50.832 rd.?

To find the area of any *regular* polygon, multiply its perimeter, or the sum of its sides, by one-half the perpendicular falling from its center to one of its sides.

To find the area of an *irregular* polygon, divide the figure into triangles and trapeziums, and find the area of each separately. The sum of these areas will be the area of the whole polygon.

THE CIRCLE.

901. A ***Circle*** is a plane figure bounded by a curved line, called the *circumference*, every point of which is equally distant from a point within called the *center.*



902. The ***Diameter*** of a circle is a line passing through its center, and terminated at both ends by the circumference.

903. The ***Radius*** of a circle is a line extending from its center to any point in the circumference. It is one-half the diameter.

PROBLEMS.

904. When either the diameter or the circumference of a circle is given, to find the other dimension of it.

1. Find the circumference of a circle whose diameter is 20 inches.

OPERATION.—20 in. × 3.1416 = 62.832 in. = 5 ft. 2.832 in., *circum*

2. Find the diameter of a circle whose circumference is 62.832 ft.

OPERATION.—62.832 ft. ÷ 3.1416 = 20 ft., *diameter.*

3. Find the diameter of a wheel whose circumference is 50 feet.

RULE.—1. *Multiply the diameter by* 3.1416; *the product is the circumference.*

2. *Divide the circumference by* 3.1416; *the quotient is the diameter.*

4. What is the diameter of a tree whose girt is 18 ft. 6 in.?

5. What is the radius of a circle whose circumference is 31.416 ft.?

6. Find the circumference of the greatest circle that can be drawn with a string 14 inches long, used as a radius.

905. To find the area of a circle, when both its diameter and circumference are given, or when either is given.

1. What is the area of a circle whose diameter is 10 feet and circumference 31.416 feet?

OPERATION.—31.416 ft. × $\overline{10 \div 4}$ = 78.54 sq. ft., *area.*

2. Find the area of a circle whose diameter is 10 feet.

OPERATION.—10 ft.^{2} × .7854 = 78.54 sq. feet, *area.*

3. Find the area of a circle whose circumference is 31.416 feet.

OPERATION.—31.416 ft. ÷ 3.1416 = 10 ft., *diam.*; (10 ft.)^2 × .7854 = 78.54 sq. feet, *area.*

RULES.—To find the area of a circle:

1. *Multiply* ¼ *of its diameter by the circumference.*

2. *Multiply the square of its diameter by* .7854.

4. What is the area of a circular pond whose circumference is 200 chains?

5. The distance around a circular park is 1¼ miles. How many acres does it contain?

906. To find the diameter or the circumference of a circle, when the area is given.

1. What is the diameter of a circle whose area is 1319.472?

OPERATION.—$1319.472 \div .7854 = 1680$; $\sqrt{1680} = 40.987+$, *diameter.*

2. What is the circumference of a circle whose area is 19.635?

OPERATION.—$19.635 \div 3.1416 = 6.25$; $\sqrt{6.25} = 2.5$, *radius*; $2.5 \times 2 \times 3.1416 = 15.708$, *circumference.*

RULE.—1. *Divide the area by* .7854 *and extract the square root of the quotient; the result is the diameter.*

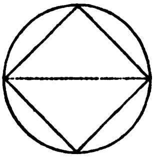
2. *Divide the area by* 3.1416 *and extract the square root of the quotient; the result is the radius. The circumference is obtained by Art.* **904.** Or,

3. *Divide the area by* .07958 *and find the square root of the quotient.*

3. The area of a circular lot is 38.4846 square rods. What is its diameter?

4. The area of a circle is 286.488 square feet. Required the diameter and the circumference.

907. To find the side of an inscribed square when the diameter of the circle is known.



1. What is the side of a square inscribed in a circle whose diameter is 6 rods?

OPERATION.—$6^2 \div 2 = 18$; $\sqrt{18} = 4.24$ rods, *side of square.*

2. The diameter of a circle is 200 feet. Find the side of the inscribed square.

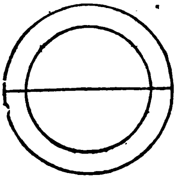
RULE.—1. *Extract the square root of half the square of the diameter.* Or,

2. *Multiply the diameter by* .7071.

3. The circumference of a circle is 104 yards. Find the side of the inscribed square.

4. The area of a circle is 78.54 square feet. Find the side of the inscribed square.

908. To find the area of a circular ring formed by two concentric circles.



1. Find the area of a circular ring, when the diameters of the circles are 20 and 30 feet.

OPERATION.—$(\overline{30+20} \times \overline{30-20}) \times .7854 =$ 392.7 sq. ft., *area.*

2. Find the area of a circular ring formed by two concentric circles, whose diameters are 7 ft. 9 in. and 4 ft. 3 in.

RULE.—*Multiply the sum of the two diameters by their difference, and the product by* .7854; *the result is the area.*

3. Two diameters are 35.75 and 16.25 ft.; find the area of the ring.

4. The area of a circle is 1 A. 154.16 P. In the center is a pond of water 10 rd. in diameter; find the area of the land and of the water.

909. To find a mean proportional between two numbers.

1. What is a mean proportional between 3 and 12?

OPERATION.—$\sqrt{12 \times 3} = 6$, the *mean proportional.*

When three numbers are proportional, the product of the extremes is equal to the square of the mean.

RULE.—*Extract the square root of the product of the two numbers.*

Find a mean proportional between

2. 42 and 168. | 3. 64 and 12.25. 4. $\frac{36}{49}$ and $\frac{4}{81}$.

5. A tub of butter weighed 36 lb. by the grocer's scales; but weighing it in the other scale of the balance, it weighed only 30 pounds. What was the true weight of the butter?

SIMILAR PLANE FIGURES.

910. ***Similar Plane Figures*** are such as have the same *form*, viz., equal angles, and their like dimensions proportional.

All circles, squares, equiangular triangles, and regular polygons of the same number of sides are similar figures.

The like dimensions of circles are their radii, diameters, and circumferences.

PRINCIPLES.—1. *The like dimensions of similar plane figures are proportional.*

2. *The areas of similar plane figures are to each other as the squares of their like dimensions.* And conversely,

3. *The like dimensions of similar plane figures are to each other as the square roots of their areas.*

The same principles apply also to the *surfaces* of all similar figures, such as triangles, rectangles, etc.; the surfaces of similar *solids*, as cubes, pyramids, etc.; and to similar *curved surfaces*, as of cylinders, cones, and spheres. Hence,

4. *The surfaces of all similar figures are to each other as the squares of their like dimensions.* And conversely,

5. *Their dimensions are as the square roots of their surfaces.*

PROBLEMS.

1. A triangular field whose base is 12 ch. contains 2 A. 80 P. Find the area of a field of similar form whose base is 48 chains.

OPERATION.—$12^2 : 48^2 :: 2$ A. 80 P. : x P. $=6400$ P. $=40$ A., *area.* (PRIN. 2.)

2. The side of a square field containing 18 acres is 60 rods long. Find the side of a similar field that contains $\frac{1}{3}$ as many acres.

OPERATION.—18 A. : 6 A. :: $60^2 : x^2 = 1200$; $\sqrt{1200} = 34.64$ rd. +, *side.* (PRIN. 3.)

3. Two circles are to each other as 9 to 16; the diameter of the less being 112 feet, what is the diameter of the greater?

OPERATION.—$9 : 16 :: 112^2 : x^2 = 3 : 4 :: 112 : x = 149$ ft. 4 in., *diameter.* (PRIN. 2.)

4. A peach orchard contains 720 square rods, and its length is to its breadth as 5 to 4; what are its dimensions?

OPERATION.—The area of a rectangle 5 by 4 equals 20 (**898**).

$$20 : 720 :: 5^2 : x^2 = 900;\quad \sqrt{900} = 30 \text{ rd., } \textit{length.}$$

$$20 : 720 :: 4^2 : x^2 = 576;\quad \sqrt{576} = 24 \text{ rd., } \textit{width.}$$

5. It is required to lay out 283 A. 107 P. of land in the form of a rectangle, so that the length shall be 3 times the width. Find the dimensions.

6. A pipe 1.5 in. in diameter fills a cistern in 5 hours; find the diameter of a pipe that will fill the same cistern in 55 min. 6 sec.

7. The area of a triangle is 24276 sq. ft., and its sides in proportion to the numbers 13, 14, and 15. Find the length of its sides in feet.

8. If it cost $167.70 to enclose a circular pond containing 17 A. 110 P., what will it cost to enclose another ⅕ as large?

9. If 63.39 rods of fence will enclose a circular field containing 2 acres, what length will enclose 8 acres in circular form?

REVIEW OF PLANE FIGURES.

PROBLEMS.

911. 1. How much less will the fencing of 20 acres cost in the square form than in the form of a rectangle whose breadth is ⅕ the length, the price being $2.40 per rod?

2. A house that is 50 feet long and 40 feet wide has a square or pyramidal roof, whose height is 15 ft. Find the length of a rafter reaching from a corner of the building to the vertex of the roof.

3. Find the diameter of a circular island containing 1¼ sq. miles.

4. What is the value of a farm, at $75 an acre, its form being a quadrilateral, with two of its opposite sides parallel, one 40 ch. and the other 22 ch. long, and the perpendicular distance between them 25 chains?

5. Find the cost, at 18 cents a square foot, of paving a space in the form of a rhombus, the sides of which are 15 feet, and a perpendicular drawn from one oblique angle will meet the opposite side 9 feet from the adjacent angle.

6. A goat is fastened to the top of a post 4 ft. high by a rope 50 ft. long. Find the area of the greatest circle over which he can graze.

7. How much larger is a square circumscribing a circle 40 rods in diameter, than a square inscribed in the same circle?

8. What is the value of a piece of land in the form of a triangle, whose sides are 40, 48, and 54 rods, respectively, at the rate of $125 an acre?

9. The radius of a circle is 5 feet; find the diameter of another circle containing 4 times the area of the first.

10. How many acres in a semi-circular farm, whose radius is 100 rods?

11. What must be the width of a walk extending around a garden 100 feet square, to occupy one-half the ground?

12. An irregular piece of land, containing 540 A. 36 P. is exchanged for a square piece of the same area; find the length of one of its sides? If divided into 42 equal squares, what is the length of the side of each?

13. A field containing 15 A. is 30 rd. wide, and is a plane inclining in the direction of its length, one end being 120 ft. higher than the other. Find how many acres of horizontal surface it contains.

14. If a pipe 3 inches in diameter discharges 12 hogsheads of water in a certain time, what must be the diameter of a pipe which will discharge 48 hogsheads in the same time?

SOLIDS.

912. A ***Solid*** or ***Body*** has three dimensions, length, breadth, and thickness.

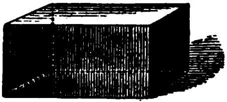
The planes which bound it are called its *faces*, and their intersections, its *edges*.

913. A ***Prism*** is a solid whose ends are equal and parallel, similar polygons, and its sides parallelograms.

Prisms take their names from the *form* of their bases, as *triangular*, *quadrangular*, *pentagonal*, etc.

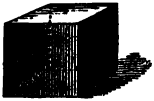
914. The ***Altitude*** of a prism is the perpendicular distance between its bases.

915. A ***Parallelopipedon*** is a prism bounded by six parallelograms, the opposite ones being parallel.



Parallelopipedon.

916. A ***Cube*** is a parallelopipedon whose faces are all equal squares.

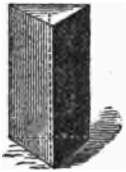


Cube.

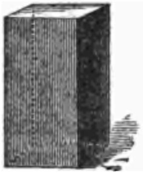
917. A ***Cylinder*** is a body bounded by a uniformly curved surface, its ends being equal and parallel circles.

1. A cylinder is conceived to be generated by the revolution of a rectangle about one of its sides as an axis.

2. The line joining the centers of the bases, or ends, of the cylinder is its *altitude*, or *axis*.

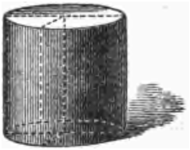


Triangular Prism.



Quadrangular Prism.

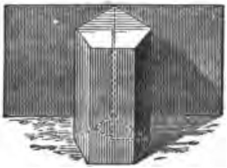
Pentagonal Prism.



Cylinder.

PROBLEMS.

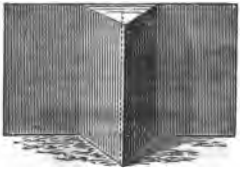
918. To find the convex surface of a prism or cylinder.



1. Find the area of the convex surface of a prism whose altitude is 7 ft., and its base a pentagon, each side of which is 4 feet.

OPERATION.—4 ft. × 5 = 20 ft., *perimeter.*

20 ft. × 7 = 140 sq. ft., *convex surface.*



2. Find the area of the convex surface of a triangular prism, whose altitude is 8½ feet, and the sides of its base 4, 5, and 6 feet, respectively.

OPERATION.—4 ft. + 5 ft. + 6 ft. = 15 ft., *perimeter.*

15 ft. × 8½ = 127½ sq. ft., *convex surface.*

3. Find the area of the convex surface of a cylinder whose altitude is 2 ft. 5 in. and the circumference of its base 4 ft. 9 in.

OPERATION.—2 ft. 5 in. = 29 in.; 4 ft. 9 in. = 57 in.

57 in. × 29 = 1653 sq. in. = 11 sq. ft. 69 sq. inches, *convex surface.*

RULE.—*Multiply the perimeter of the base by the altitude.*

To find the *entire* surface, add the area of the bases or ends.

4. If a gate 8 ft. high and 6 ft. wide revolves upon a point in its center, what is the entire surface of the cylinder described by it?

5. Find the superficial contents, or entire surface of a parallelopipedon 8 ft. 9 in. long, 4 ft. 8 in. wide, and 3 ft. 3 in. high.

6. What is the entire surface of a cylinder formed by the revolution about one of its sides of a rectangle that is 6 ft. 6 in. long and 4 ft. wide?

7. Find the entire surface of a prism whose base is an equilateral triangle, the perimeter being 18 ft., and the altitude 15 ft.

919. To find the volume of any prism or cylinder.

1. Find the volume of a triangular prism, whose altitude is 20 ft., and each side of the base 4 feet.

OPERATION.—The area of the base is 6.928 sq. ft. (**882**).

6.928 sq. ft. × 20 = 138.56 cu. ft., *volume.*

2. Find the volume of a cylinder whose altitude is 8 ft. 6 in., and the diameter of its base 3 feet.

OPERATION.—$3^2 \times .7854 = 7.0686$ square feet, *area of base* (**905**).

7.0686 sq. ft. × 8.5 = 60.083 cubic feet, *volume.*

RULE.—*Multiply the area of the base by the altitude.*

3. Find the solid contents of a cube whose edges are 6 ft. 6 in.

4. Find the cost of a piece of timber 18 in. square and 40 ft. long, at $.30 a cubic foot.

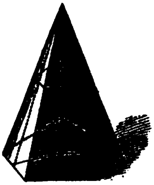
5. Required the solid contents of a cylinder whose altitude is 15 ft. and its radius 1 ft. 3 in.

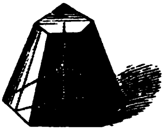
6. What is the value of a log 24 ft. long, of the average circumference of 7.9 ft., at $.45 a cubic foot?

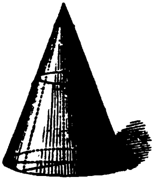
PYRAMIDS AND CONES.

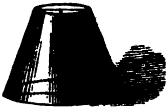
920. A ***Pyramid*** is a body having for its base a polygon, and for its other faces three or more triangles, which terminate in a common point called the *vertex.*

Pyramids, like prisms, take their names from their bases, and are called *triangular*, *square*, or *quadrangular*, *pentagonal*, etc.


Pyramid.


Frustum.


Cone.


Frustum.

921. A ***Cone*** is a body having a circular base, and whose convex surface tapers uniformly to the *vertex.*

It is a body conceived to be formed by the revolution of a right-angled triangle about one of its sides containing the right angle, as an immovable axis.

922. The ***Altitude*** of a pyramid or of a cone is the perpendicular distance from its vertex to the plane of its base.

923. The ***Slant Height*** of a *pyramid* is the perpendicular distance from its vertex to one of the sides of the base; of a *cone*, is a straight line from the vertex to the circumference of the base.

924. The ***Frustum*** of a pyramid or of a cone is that part which remains after cutting off the top by a plane parallel to the base.

PROBLEMS.

925. To find the convex surface of a pyramid or of a cone.

1. Find the convex surface of a triangular pyramid, the slant height being 16 ft., and each side of the base 5 feet.

OPERATION.—$(5\text{ ft.} + 5\text{ ft.} + 5\text{ ft.}) \times \overline{16 \div 2} = 120$ sq. ft., *conv. surf.*

2. Find the convex surface of a cone whose diameter is 17 ft. 6 in., and the slant height 30 feet.

RULE.—*Multiply the perimeter or circumference of the base by one-half the slant height.*

To find the *entire* surface, add to this product the area of the base.

3. Find the entire surface of a square pyramid whose base is 8 ft. 6 in. square, and its slant height 21 feet.

4. Find the entire surface of a cone the diameter of whose base is 6 ft. 9 in. and the slant height 45 ft.

6. Find the cost of painting a church spire, at $.25 a sq. yd., whose base is a hexagon 5 ft. on each side, and the slant height 60 feet.

926. To find the volume of a pyramid or of a cone.

1. What is the volume, or solid contents, of a square pyramid whose base is 6 feet on each side, and its altitude 12 feet?

OPERATION.—$\overline{6 \times 6} \times \overline{12 \div 3} = 144$ cu. ft., *volume.*

2. Find the volume of a cone, the diameter of whose base is 5 ft. and its altitude 10½ feet?

OPERATION.—$(5^2\text{ ft.} \times .7854) \times \overline{10\tfrac{1}{2} \div 3} = 68.72\tfrac{1}{4}$ cu. ft., *volume.*

RULE.—*Multiply the area of the base by one-third the altitude.*

3. Find the solid contents of a cone whose altitude is 24 ft., and the diameter of its base 30 inches.

4. What is the cost of a triangular pyramid of marble, whose altitude is 9 ft., each side of the base being 3 ft., at $2½ per cu. foot?

5. Find the volume and the entire surface of a pyramid whose base is a rectangle 80 feet by 60 feet, and the edges which meet at the vertex are 130 feet.

927. To find the convex surface of a frustum of a pyramid or of a cone.

1. What is the convex surface of a frustum of a square pyramid, whose slant height is 7 feet, each side of the greater base 4 feet, and of the less base 18 inches?

OPERATION.—The *perimeter* of the greater base is 16 ft., of the less 6 feet.

$$\overline{16\text{ ft.} + 6\text{ ft.}} \times \overline{7 \div 2} = 77\text{ sq. ft., } \textit{convex surface.}$$

2. Find the convex surface of a frustum of a cone whose slant height is 15 feet, the circumference of the lower base 30 feet, and of the upper base 16 feet.

RULE.—*Multiply the sum of the perimeters, or of the circumferences, by one-half the slant height.*

To find the *entire* surface, add to this product the area of both ends, or bases.

3. How many square yards in the convex surface of a frustum of a pyramid, whose bases are heptagons, each side of the lower base being 8 feet, and of the upper base 4 feet, and the slant height 55 feet?

928. To find the volume of a frustum of a pyramid or of a cone.

1. Find the volume of the frustum of a square pyramid whose altitude is 10 feet, each side of the lower base 12 feet, and of the upper base 9 feet.

OPERATION.—$12^2 + 9^2 = 225$; $(225 + \sqrt{144 \times 81}) \times \overline{10 \div 3} = 1110$ cu. feet, *volume.*

2. How many cubic feet in the frustum of a cone whose altitude is 6 feet, and the diameters of its bases 4 feet and 3 feet?

RULE.—*To the sum of the areas of both bases add the square root of the product, and multiply this sum by one-third of the altitude.*

3. How many cubic feet in a piece of timber 30 ft. long, the greater end being 15 inches square, and that of the less 12 inches?

4. How many cubic feet in the mast of a ship, its height being 50 ft., the circumference at one end 5 feet and at the other 3 feet?

THE SPHERE.

929. A *Sphere* is a body bounded by a uniformly curved surface, all the points of which are equally distant from a point within, called the *center*.

930. The *Diameter* of a sphere is a straight line passing through the center of the sphere, and terminated at both ends by its surface.

931. The *Radius* of a sphere is a straight line drawn from the center to any point in the surface.

932. To find the surface of a sphere.

1. Find the surface of a sphere whose diameter is 9 in.

OPERATION.—9 in. × 3.1416 = 28.2744 in., *circumference.*
28.2744 in. × 9 = 254.4696 sq. in., *surface.*

RULE.—*Multiply the diameter by the circumference of a great circle of the sphere.*

2. What is the surface of a globe 3 feet in diameter?
3. Find the surface of a globe whose radius is 1 foot.

933. To find the volume of a sphere.

1. Find the volume of a sphere whose diameter is 18 inches.

OPERATION.—18 in. × 3.1416 = 56.5488 in., *circumference.*
56.5488 in. × 18 = 1017.8784 sq. in., *surface.*
1017.8784 sq. in. × $\overline{18 \div 6}$ = 3053.6352 cu. in., *volume.*

RULE.—*Multiply the surface by ⅙ of the diameter, or ⅓ of the radius.*

2. Find the volume of a globe whose diameter is 30 in.
3. Find the solid contents of a globe whose radius is 5 yards.

934. To find the three dimensions of a rectangular solid, the volume and the ratio of the dimensions being given.

1. What are the dimensions of a rectangular solid, whose volume is 4480 cu. ft., and its dimensions are to each other as 2, 5, and 7?

OPERATION.—$\sqrt[3]{4480 \div (2 \times 5 \times 7)} = 4$; 4 ft. × 2 = 8 ft., *height;* 4 ft. × 5 = 20 ft., *width;* 4 ft. × 7 = 28 ft., *length.*

RULE.—I. *Divide the volume by the product of the terms proportional to the three dimensions, and extract the cube root of the quotient.*

II. *Multiply the root thus obtained by each proportional term; the products will be the corresponding sides.*

2. What are the dimensions of a rectangular box whose volume is 3000 cu. ft., and its dimensions are to each other as 2, 3, and 4?

3. A pile of bricks in the form of a parallelopiped contains 30720 cu. feet, and the length, breadth, and height are to each other as 3, 4, and 5. What are the dimensions of the pile?

SIMILAR SOLIDS.

935. ***Similar Solids*** are such as have the same *form*, and differ from each other only in volume.

PRINCIPLES.—1. *The volumes of similar solids are to each other as the cubes of their like dimensions.*

1. If the volume of a cube 3 inches on each side is 27 cu. in., what is the volume of one 7 inches on each side?

OPERATION.—$3^3 : 7^3 :: 27$ cu. in. $: x = 343$ cu. in., *volume.*

2. *The like dimensions of similar solids are to each other as the cube roots of their volumes.*

3. If the diameter of a ball whose volume is 27 cu. in. is 3 in., what is the volume of one 7 inches on each side?

OPERATION.—$\sqrt[3]{27} : \sqrt[3]{343} :: 3 : x = 7$ in., *diameter.*

REVIEW OF SOLIDS.

PROBLEMS.

936. 1. What is the edge of a cube whose entire surface is 1050 sq. feet, and what is its volume?

2. What must be the inner edge of a cubical bin to hold 1250 bu of wheat?

3. How many gallons will a cistern hold, whose depth is 7 ft., the bottom being a circle 7 feet in diameter and the top 5 feet in diameter?

4. What is the value of a stick of timber 24 ft. long, the larger end being 15 in. square, and the less 6 in., at 28 cents a cubic foot?

5. If a cubic foot of iron were formed into a bar ½ an inch square, without waste, what would be its length?

6. If a marble column 10 in. in diameter contains 27 cu. ft., what is the diameter of a column of equal length that contains 81 cu. ft.?

7. How many board feet in a post 11 ft. long, 9 in. square at the bottom, and 4 in. square at the top?

8. The surface of a sphere is the same as that of a cube, the edge of which is 12 in. Find the volume of each.

9. A ball 4.5 in. in diameter weighs 18 oz. Avoir.; what is the weight of another ball of the same density, that is 9 in. in diameter?

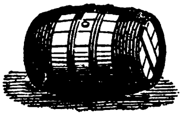
10. In what time will a pipe supplying 6 gal. of water a minute, fill a tank in the form of a hemisphere, that is 10 ft. in diameter?

11. The diameter of a cistern is 8 feet; what must be its depth to contain 75 hhd. of water?

12. How many bushels in a heap of grain in the form of a cone, whose base is 8 ft. in diameter and altitude 4 feet?

GAUGING.

937. ***Gauging*** is the process of finding the capacity or volume of casks and other vessels.



A cask is equivalent to a cylinder having the same length and a diameter equal to the *mean diameter* of the cask.

To find the *mean diameter* of a cask (*nearly*),

Add to the head diameter $\frac{2}{3}$, *or, if the staves are but little curved*, .6, *of the difference between the head and bung diameters.*

To find the *volume* of a cask in gallons,

Multiply the square of the mean diameter by the length (both in inches) and this product by .0034.

1. How many gallons in a cask whose head diameter is 24 inches, bung diameter 30 in., and its length 34 inches?

OPERATION.—$24 + (\overline{30 - 24} \times \frac{2}{3}) = 28$ in., *mean diameter.*

$28^2 \times 34 \times .0034 = 90.63$ gal., *capacity.*

2. What is the volume of a cask whose length is 40 inches, the diameters 21 and 30 in., respectively?

3. How many gallons in a cask of slight curvature, 3 ft. 6 in. long, the head diameter being 26 in., the bung diameter 31 in.?

938. CIRCLES.

1. The ***Diameter***	× 3.1416 ÷ .3183	= *the circumference.*
	× .8862 ÷ 1.1284	= *the side of an equal square.*
	× .8660 ÷ 1.1547	= *the side of an inscribed equilateral triangle.*
	× .7070 ÷ 1.4142	= *the side of an inscribed square*
2. The ***Circumference***	× .3183 ÷ 3.1416	= *the diameter.*
	× .2821 ÷ 3.5450	= *the side of an equal square.*
	× .2756 ÷ 3.6276	= *the side of an inscribed equilateral triangle.*
	× .2251 ÷ 4.4428	= *the side of an inscribed square.*
	× .15915 ÷ 6.28318	= *the radius.*
3. The ***Area***	÷ 3.1416	= *the square of the radius.*
	× 1.2732 ÷ .7854	= *the square of the diameter.*
	× 12.5663 ÷ .07958	= *the sq're of the circumference.*

939. SPHERES.

1. The ***Surface*** =	*Circumference* × *its diam.* $\textit{Radius}^2 \times 12.5664.$ $\textit{Diameter}^2 \times 3.1416.$ $\textit{Circumference}^2 \times .3183.$
2. The ***Volume*** =	$\textit{Surface} \times \frac{1}{6}$ *its diameter.* $\textit{Radius}^3 \times 4.1888.$ $\textit{Diameter}^3 \times .5236.$ $\textit{Circumference}^3 \times .0169.$
3. The ***Diameter*** =	$\sqrt{\textit{Of surface}} \times .5642.$ $\sqrt[3]{\textit{Of volume}} \times 1.2407.$
4. The ***Circumference*** =	$\sqrt{\textit{Of surface}} \times 1.77255.$ $\sqrt[3]{\textit{Of volume}} \times 3.8978.$
5. The ***Radius*** =	$\sqrt{\textit{Of surface}} \times .2821.$ $\sqrt[3]{\textit{Of volume}} \times .6204.$
6. The ***Side of Inscribed Cube*** =	*Radius* × 1.1547. *Diameter* × .5774.

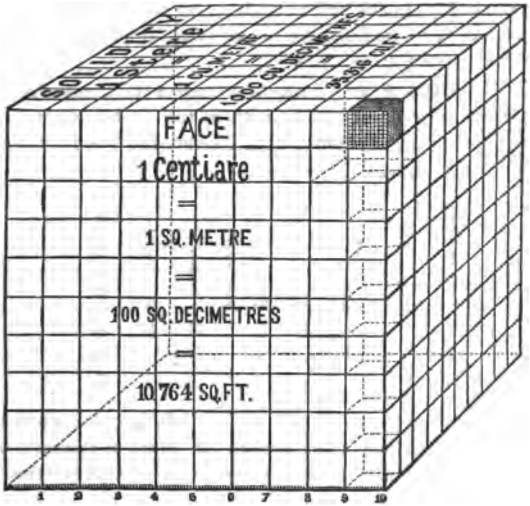
940. SYNOPSIS FOR REVIEW.

MENSURATION.

- 1. DEFINITION. 2. LINES. 3. ANGLES. 4. PLANE FIGURES.
- 5. TRIANGLES.
  - 1. Defs.
    - 1. *Triangle.* 2. *Right-angled Tri.* 3. *Hypothenuse* 4. *Base.* 5. *Perpendicular.* 6. *Altitude.* 7. *Equilateral Triangle.* 8. *Isosceles Triangle.* 9. *Scalene Triangle.* 10. *Equiangular Triangle.* 11. *Acute angled Triangle.* 12. *Obtuse-angled Triangle.*
  - 2. Problems. To find … Rule.
    - 882. *Area of Triangle.*
    - 883. *Either Dimension.*
    - 884. *Area of a Triangle.*
    - 886. *The Hypothenuse.*
    - 887. *The Base or Perp.*
- 6. QUADRILATERALS.
  - 1. Defs.
    - 1. *Quadrilateral.* 2. *Parallelogram.* 3. *Rectangle.* 4. *Square.* 5. *Rhomboid.* 6. *Rhombus.* 7. *Trapezoid.* 8. *Trapezium.* 9. *Altitude.*
  - 2. Problems. To find area of … Rule.
    - 898. *Parallelogram.*
    - 899. *Trapezoid.*
    - 900. *Trapezium.*
- 7. CIRCLE.
  - 1. Defs. 1. *Circle.* 2. *Diameter.* 3. *Radius.*
  - 2. Problems. To find
    - 904. *Diam. or Circum.* Rule, 1, 2.
    - 905. *Area.* Rule, 1, 2.
    - 906. *Diam. or Circ.* Rule, 1, 2, 3.
    - 907. *Side of Ins. Square.* Rule, 1, 2.
    - 908. *Area of Circular Ring.* Rule.
    - 909. *Mean Proportional.* Rule.
- 8. SIMILAR PLANE FIGURES. 1. Defs. 2. Prin. 1, 2, 3, 4, 5.
- 9. SOLIDS.
  - 1. Defs.
    - 1. *Solid or Body.* 2. *Prism.* 3. *Altitude.* 4. *Parallelopipedon.* 5. *Cube.* 6. *Cylinder.* 7. *Pyramid.* 8. *Cone.* 9. *Altitude of Pyramid or Cone.* 10. *Slant Height.* 11. *Frustum.* 12. *Sphere.* 13. *Diameter.* 14. *Radius.*
  - 2. Problems. To find
    - 918. *Conv. Surf. of Prism or Cyl.* Rule.
    - 919. *Volume* " " Rule.
    - 925. *Conv. Surf. of Pyr. or Cone.* Rule.
    - 926. *Volume* " " Rule.
    - 927. *Conv. Surf. of Frustum.* Rule.
    - 928. *Volume* " " Rule.
    - 932. *Surface of Sphere.* Rule.
    - 933. *Volume* " " Rule.
    - 934. *Dim. of Rectang. Solid.* Rule.
  - 3. Similar Solids. 1. *Defs.* 2. *Principles,* 1, 2.
- 10. GAUGING. 1. Definitions. 2. Rules.

METRIC SYSTEM.

The edges of this cube are each 1 *Me'ter*, or 10 *Dec'i-me'ters*, or 100 *Cen'ti-me'ters*, in length.



SCALE, $\frac{1}{10}$ of the *Exact Size*.

941. The ***Metric System*** of weights and measures is based upon the *decimal notation*, and is so called because its *primary unit* is the ***Me'ter***.

942. The ***Me'ter*** (*m.*) is the *base* of the system, and is the *one ten-millionth* part of the distance on the earth's surface from the equator to either pole, or 39.37079 inches.

Me'ter means *measure*; and the *three principal units* are units of *length*, *capacity* or *volume*, and *weight*.

943. The ***Multiple Units,*** or higher denominations, are named by prefixing to the name of the *primary* units the Greek numerals, *Dek'a* (10), *Hek'to* (100), *Kil'o* (1000), and *Myr'ia* (10000).

Thus, 1 dek'a-me'ter (*Dm.*) denotes 10 me'ters (*m.*); 1 hek'to-me'ter (*Hm.*), 100 me'ters; 1 kil'o-me'ter (*Km.*), 1000 me'ters; and 1 myr'ia-me'ter (*Mm.*), 10000 meters.

944. The ***Sub-multiple Units,*** or lower denominations, are named by prefixing to the names of the *primary* units the Latin ordinals, *Dec'i* ($\frac{1}{10}$), *Cen'ti* ($\frac{1}{100}$), *Mil'li* ($\frac{1}{1000}$).

Thus, 1 dec'i-me'ter (*dm.*) denotes $\frac{1}{10}$, or .1 of a me'ter; 1 cen'ti-me'ter (*cm.*), $\frac{1}{100}$, or .01 of a me'ter; 1 mil'li-me'ter (*mm.*), $\frac{1}{1000}$, or .001 of a me'ter.

Hence, it is apparent from the *name* of a unit whether it is *greater* or *less* than the standard unit, and also *how many times.*

945. The *Metric System* being based upon the *decimal scale,* the *denominations* correspond to the *orders* of the Arabic Notation; and hence are written like United States Money, the lowest denomination at the right. Thus,

Tens of Thou.	Thousands.	Hundreds.	Tens.	Units.	*Dec. Point.*	Tenths.	Hundredths.	Thousandths.
6	7	0	1	5	.	6	3	8
Mm.	*Km.*	*Hm.*	*Dm.*	*m.*		*dm.*	*cm.*	*mm.*

The number is read, 67015.638 *me'ters.* It may be expressed in other denominations by placing the decimal point at the right of the required denomination, and writing the name or abbreviation after the figures.

Thus, the above may be read, 670.15638Hm.; or 67.015638 Km.; or 670156.38 dm.; or 6701563.8 cm.; or it may be read,

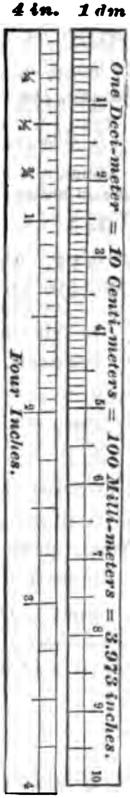
6 Mm. 7 Km. 0 Hm. 1 Dm. 5 m. 6 dm. 3 cm. 8 mm.

Write 3672.045 *me'ters,* and read it in the several orders; read it in kil'o-me'ters; in hek'to-me'ters; in dek'a-me'ters; in dec'i-me'ters; in cen'ti-me'ters.

The names *mill, cent, dime,* used in United States Money, correspond to *mil'li, cent'i, dec'i,* in the Metric System. Hence the *eagle* might be called the *dek'a-dollar,* since it is 10 dollars; the dime, a *dec'i-dollar,* since it is $\frac{1}{10}$ of a dollar, etc.

MEASURES OF LENGTH.

946. The ***Me'ter*** is the *unit of length*, and is equal to 39.37 in. or, 1.0936 yd. +.


4 in.
1 dm
One Deci-meter = 10 Centi-meters = 100 Milli-meters = 3.973 inches.
Four Inches.
¼
½
¾
1
2
3
4
1
2
3
4
5
6
7
8
9
10

Metric Denominations.			U. S. Value.
		1 Mil'li-me'ter =	.03937 in.
10 Mil'li-me'ters,	*mm.*	= 1 Cen'ti-me'ter =	.3937 in.
10 Cen'ti-me'ters,	*cm.*	= 1 Dec'i-me'ter =	3.937 in.
10 Dec'i-me'ters,	*dm.*	= 1 ***Me'ter*** =	39.37 in.
10 ME'TERS,	*m.*	= 1 Dek'a-me'ter =	32.809 ft.
10 Dek'a-me'ters,	*Dm.*	= 1 Hek'to-me'ter =	19.8842 rd.
10 Hek'to-me'ters,	*Hm.*	= 1 Kil'o-me'ter =	.6213 mi.
10 Kil'o-me'ters,	*Km.*	= 1 Myr'ia-me'ter =	6.2138 mi.

Units of long measure form a scale of *tens;* hence, in writing numbers expressing *length, one* decimal place must be allowed for each denomination.

Thus, 9652 mm. may be written 965.2 cm., or 96.52 dm., or 9.652 m., or .9652 Dm.

1. The *Me'ter* is used in measuring cloths and *short* distances.

2. The *Kil'o-me'ter* is commonly used for measuring *long* distances, and is about ⅝ of a common mile.

3. The *Cent'i-me'ter* and *Mil'li-me'ter* are used by mechanics and others for *minute* lengths.

[illegible] expressed in *Cent'i-* [illegible]

[illegible] *yr'ia-me'ter* are [illegible] *Kil'o-me'ters.*

Read the [illegible]

8.0 m [illegible]	[illegible] Hm.	
[illegible] mm.	[illegible] Km.	13.043 Km.
12[illegible]	[illegible] Dm.	500.032 m.
6.57 dm.	[illegible] Km.	31045.7 cm.

Change the following to *me′ters:*

327 Dm. 947 cm. 0.72 Km. 30674 mm.
28 Hm. 236 dm. 1.73 Hm. 83.062 cm.
16.8 Km. 43.5 cm. 35.4 Dm. 4000.5 dm.

1. Write 6 kilometers 6 dekameters 6 meters 6 decimeters 6 centimeters. *Ans.* 6.06666 Km., or 60.6666 Hm., or 606.666 Dm., etc.

Write the following, expressing each in *three* denominations:

2. 24379 dm.; 15032036 cm.; 2475064 mm.; 30471 Dm.

3. 6704 Hm.; 85 Km.; 120000 m.; 780109 cm.; 75 m.

Similar examples should be given, until the pupil is familiar with the reduction of higher to lower, and of lower to higher denominations, by changing the place of the decimal point and using the proper abbreviations.

947. To add, subtract, multiply, and divide Metric Denominations.

1. What is the sum of 314.217 m., 53.062 Hm., and 225 cm.?

OPERATION. 314.217 m. + 5306.2 m. + 2.25 m. = 5622.667 m., *Ans.*

2. Find the difference between 4.37 Km. and 1246 m.

OPERATION. 4.37 Km. — 1.242 Km. = 3.128 Km., *Ans.*

3. How much cloth in 8¼ pieces, each containing 43.65 m.?

OPERATION. 43.65 m. × 8.25 = 384.8625 m., *Ans.*

4. How many garments, each containing 3.5 m., can be made from a piece of cloth containing 43.75 Dm.?

OPERATION. 437.5 m. ÷ 3.5 m. = 125 times; hence, 125 garments, *Ans.*

RULE.—*Reduce the given numbers to the same denominations, when necessary; then proceed as in the corresponding operations with whole numbers and decimals.*

EXERCISES.

1. Add 7.6 m., 36.07 m., 125.8 m., and 9.127 m.

2. Express as meters and add 475 dm., 3241 cm., and 725 mm.

3. Add 56.07 m., 1058.2 dm., 430765 cm., 6034.58 m., and express the result in kilometers.

4. From 8.125 Km. take 3276.4 m. *Ans.* 4.8486 Km.

5. The distance round a certain square is 3.15 Km. How many *meters* will a man travel who walks around it 4 times?

6. How many meters of ribbon will be required to make 32 badges, each containing 40 centimeters? *Ans.* 12.8 m.

7. What will be its cost, at 15 cents a meter?

8. Find the difference between 25.3 Km. and 425.25 m.

9. If an engine runs 36.8 Km. in an hour, how far does it run between 8 o'clock and 12 o'clock?

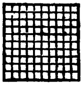
10. In what time will a train run from Boston to Albany, at the rate of 46.55 Km. per hour, the distance being about 325.85 Km.?

11. From a piece of cloth containing 45.75 m., a tailor cut 5 suits, each containing 7.5 m. How much remained?

12. A wheel is 3.6 m. around. How many times will it revolve in rolling a distance of 1.08 Km.? *Ans.* 300.

MEASURES OF SURFACE.

948. The *units* of square measure are *squares*, the sides of which are equal to a unit of long measure.



1 *sq. cm., Exact Size.*

100 Sq. Mil'li-me'ters (*sq. mm.*)	=	1 *sq. cm.*	=	0.155 sq. in.
100 Sq. Cen'ti-me'ters	=	1 *sq. dm.*	=	15.5 sq. in.
100 Sq. Dec'i-me'ters	=	1 *sq. m.* 1 Centar (*ca.*)	=	10.764 sq. ft. 1.196 sq. yd.
100 Sq. Me'ters	=	1 *sq. Dm.* 1 ***Ar.*** (*a.*)	=	3.954 sq. rd. .0247 acre.
100 Sq. Dek'a-me'ters	=	1 *sq. Hm.* 1 Hektar (*Ha.*)	=	2.471 acres.
100 Sq. Hek'to-me'ters	=	1 *sq. Km.*	=	.3861 sq. mi.

Units of square measure form a scale of *hundreds;* hence, in writing numbers expressing *surface*, *two* decimal places must be allowed for each denomination.

Thus, 36 sq. m. 4 sq. dm. 27 sq. cm. are written 36.0427 sq. m.; and 6 Ha. 5 a. 3 ca. are written 6.0503 Ha., or 605.03 a., etc.

1. The *Square Me'ter* is the *unit* for measuring ordinary surfaces of small extent, as floors, ceilings, etc.

2. The *Ar*, or *Square Dek'a-me'ter*, is the *unit of land measure*, and is equal to 119.6 sq. yd., or 3.954 sq. rd., or .0247 acre.

EXERCISES.

1. Read 36145 sq. m., naming each denomination. *Ans.* 3 sq. Hm. 61 sq. Dm. 45 sq. m.

2. Write in one number 4 of each denomination from sq. Hm. to sq. mm., expressed in sq. Hm. *Ans.* 4.0404040404 sq. Hm.

3. Express the following, each in *three* denominations:
6 sq. Km. 6 sq. Hm. 24 sq. Dm. 5 sq. m.;
16 sq. Dm. 8 sq. m. 4 sq. dm. 15 sq. cm.

4. In 15 sq. Hm. how many square meters?

5. What is the surface of a floor 12 m. long and 7 m. wide?

6. Add 8 times 4 Ha., 7 times 9 a., and 12 times 14 ca.

7. What is the area of a piece of land 42 Dm. long and 36 Dm. wide? *Ans.* 1512 sq. Dm., or 15.12 Ha.

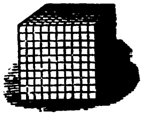
8. Divide 125000 ca. into 8 equal parts.

9. How many times is 2.50 sq. m. contained in 5 Ha.?

10. How many meters of carpeting 0.6 m. wide will cover a floor 8 m. long and 5.7 m. wide? *Ans.* 76 m.

11. At 15 cents a sq. m., what is the cost of painting a surface 20.5 m. long and 6.8 m. wide? *Ans.* $20.91.

12. A man having 5 Ha 8 a. 7 ca. of land, sold .3 of it, at $25 an ar. What did he receive for what he sold?



1 *cu. cm.*, *Exact Size.*

MEASURES OF VOLUME.

949. The *units* of cubic measure are *cubes*, the edges of which are equal to a unit of long measure.

1000 Cu. Mil′li-me′ters (*cu. mm.*)	=	1 *cu. cm.*	=	.061 cu. in.
1000 Cu. Cen′ti-me′ters	=	1 *cu. dm.* 1 Li′ter (*l.*)	=	.0353 cu. ft. 1.0567 li. qt.
1000 Cu. Dec′i-me′ters	=	1 *cu. m.* 1 Ster (*s.*)	=	35.3165 cu. ft. .2759 cord.

Units of cubic measure form a scale of *thousands*; hence, in writing numbers expressing *volume*, *three* decimal places must be allowed for each denomination.

Thus, 42 cu. m. 31 cu. dm. 5 cu. cm. are written 42.031005 cu. m.

The *cubic dec′i-me′ter*, when used as a unit of liquid or dry measure, is called a *li′ter*.

WOOD MEASURE.

1000 Cu. Dec′i-me′ters (*cu. dm.*) } = { 1 *cu. m.* } = { .2759 cord.
10 Dec′i-sters (*ds.*) } = { 1 *Ster, s.* } = { 35.3165 cu. ft.
10 Sters = 1 Dek′a-ster, *Ds.* = 2.759 cord.

Units of wood measure form a scale of *tens;* hence, but *one* decimal is required for each denomination.

Thus, 9 Ds. 4 s. 7 ds. are written 94.7 s.; or 9.47 Ds.

1. The *Cubic Me′ter* is the *unit* for measuring ordinary solids; as excavations, embankments, etc.

2. *Cubic Cen′ti-me′ters* and *Mil′li-me′ters* are used for measuring minute bodies.

3. The *Cubic Me′ter* when used as a unit of measure for wood or stone is called a *Ster.*

4. The *common Cord* is about the same as 3.6 *sters*, or 36 *dec′i-sters.*

EXERCISES.

1. Write 30 Ds. 6 s. 8 ds. *Ans.* 30.68 Ds.

2. Express in cu. m., 3 cu. m. 3 cu. dm. 3 cu. cm. 3 cu. mm. *Ans.* 3.003003003 cu. m.

3. Write and read the following, each in cu. dm., in cu. cm., and in cu. mm.:

16 cu. m. 275 cu. dm.; 204 cu. m. .016 cu. dm. .024 cu. cm.; 10 cu. m. 324 cu. dm. .016 cu. cm. 3244 cu. cm.

4. Express in cu. meters and add: 7 cu. m., 55 cu. dm., 12 cu. m., 6 cu. dm., 15 cu. cm., 10532 cu. cm. *Ans.* 19.071547 m.

5. From 36 cu. m. subtract 8 times 42 cu. dm. *Ans.* 35.664 m.

6. How many cubic meters of brick in a wall 16 m. long, 3 m. high, and 8 dm. thick? *Ans.* 38.4 cu. m.

7. How many cu. meters of earth must be removed in digging a cellar 16.5 m. long, 8.2 m. wide, and 3.2 m. deep?

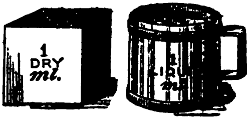
8. In a pile of wood 9.3 m. long, 2.8 m. high, and 1.5 m. wide, how many sters? *Ans.* 39.06 s.

9. At $2.25 a ster, what would be the cost of a pile of wood 5.6 m. long, 3.4 m. wide, and 2.5 m. high?

10. If a cu. centimeter of silver is worth $.75, what is the value of a brick of silver 12.4 cm. long, 3.6 cm. wide, and 2.5 cm. thick?

MEASURES OF CAPACITY.

950. The ***Li'ter*** is the *unit of capacity*, both of Liquid and of Dry Measures, and is equal in volume to *one cu. dec'i-me'ter*, equal to 1.0567 qt. Liquid Measure, or .908 qt. Dry Measure.



		Dry M.	*Liquid M.*
10 Mil'li-li'ters, *ml.*	= 1 Cen'ti-li'ter	= .61 cu. in.	= .338 fl'd oz.
10 Cen'ti-li'ters, *cl.*	= 1 Dec'i-li'ter	= 6.10 " "	= .845 gi.
10 Dec'i-li'ters, *dl.*	= 1 ***Li'ter***	= .908 qt.	= 1.0567 qt.
10 LI'TERS, *L.*	= 1 Dek'a-li'ter	= 9.081 "	= 2.64175 gal.
10 Dek'a-li'ters, *Dl.*	= 1 Hek'to-li'ter	= 2.837 bu.	= 26.4175 "
10 Hek'to-li'ters, *Hl.*	= 1 Kil'o-li'ter or Ster	= 28.37 bu. 1.308 cu. yd	= 264.175 "
10 Kil'o-li'ters, *Kl.*	= 1 Myr'ia-li'ter (*Ml.*)	= 283.72 bu.	= 2641.75 "

1. The *Li'ter* is used in measuring liquids in moderate quantities.

2. The *Hek'to-li'ter* is used for measuring grain, fruit, roots, etc., in large quantities, also wine in casks.

3. Instead of the *Kil'o-li'ter* and *Mil'li-me'ter*, the *Cubic Me'ter* and *Cubic Cen'ti-me'ter*, which are their equals, may be used.

EXERCISES.

1. Write 5 kiloliters 5 liters 5 deciliters 5 centiliters.
Ans. 5.00555 Kl., or 5005.55 l.

2. Read, naming *each* denomination, the following:
45624 cl.; 306721 ml.; 76031 dl.; 89764 l.

3. In 3846 l. how many cl.? How many Dl.? Kl.? dl.? ml.?

4. Find the sum of 175 l., 25 Hl., 42 cl., and 16 dl.

5. From 6 times 25 Hl. take 15 times 36 l.

6. Divide 5 Hl. of corn equally among 25 persons. *Ans.* 20 l.

7. From a cask of wine containing 2 Hl. of wine, 125 l. were drawn out. How much remained?

8. How many Hl. of wheat can be put into a bin 3 m. long, 2 m. wide, and 1.5 m. deep? *Ans.* 90 Hl.

9. What must be the length of a bin 1.5 m. wide, 1 m. deep, to contain 7500 liters of grain? *Ans.* 5

MEASURES OF WEIGHT.

951. The ***Gram*** is the ***unit of weight***, and is equal to the weight of a *cu. cen'ti-me'ter* of distilled water.

A *Gram* is equal to 15.432 gr. Troy, or .03527 oz. Avoir.

10 Mil'li-grams,	*mg.*	= 1 Cen'ti-gram	=	.1543 +	gr. Tr.
10 Cen'ti-grams,	*cg.*	= 1 Dec'i-gram	=	1.5432 +	" "
10 Dec'i-grams,	*dg.*	= 1 ***Gram***	=	15.432 + .03527 +	" " oz. Av.
10 GRAMS,	*g.*	= 1 Dek'a-gram	=	.3527 +	" "
10 Dek'a-grams,	*Dg.*	= 1 Hek'to-gram	=	3.5274 +	" "
10 Hek'to-grams,	*Hg.*	= 1 Kil'o-gram, or ***Kil'o***	=	2.6792 2.2046 +	lb. Tr. lb. Av.
10 Kil'os,	*Kg.*	= 1 Myr'ia-gram	=	22.046 +	" "
10 Myr'ia-grams, *Mg.*, or 100 Kil'o-grams		= 1 Quin'tal	=	220.46 +	" "
10 Quin'tals, *Q.*, or 1000 KIL'OS, *K.*		= 1 Tonneau, or ***Ton***	=	2204.62 + 1.1023 +	" " tons.

1. The *Gram* is used for weighing letters, gold, silver, medicines, and all small, or costly articles.

2. The *Kil'o-gram* or *Kil'o* is the weight of a *cu. dm.* of water, and is the unit of common weight in trade, being a trifle less than 2¼ lb. Avoir.

3. The *Ton* is the weight of a *cu. m.* of water, and is used for weighing very heavy articles, being about 204½ lb. more than a common ton.

4. The *Avoir. oz.* is about 28 *g.*; the *pound* is a little less than ½ a *kilo*.

EXERCISES.

1. Read 340642 cg. in grams; in hectograms; in kilograms.
2. Change 16.5 T. to kilos; to grams; to decigrams.
3. If coffee is $.80 a kilo, what will 5 quintals cost?
4. How many boxes containing 1 gram each, will be required to hold 1 kilo of quinine? *Ans.* 1000.
5. If a letter weighs 3.5 g., how many such letters will weigh 1.015 Kg.? *Ans.* 290.
6. A car weighing 6.577 T. contains 125 barrels of salt, each weighing 102.15 K. What is the weight of the car and contents?
7. Find the difference in the weight of the car and its contents?

952. To change the Metric to the Common System.

1. In 3.6 Km., how many feet?

OPERATION.

3.6 Km. × 1000 = 3600 m.
39.37 in. × 3600 = 141732 in.
141732 in. ÷ 12 = 11811 ft., *Ans.*

ANALYSIS.—The *meter* is the principal unit of the table; hence, reduce the kilometers to meters. Since there are 39.37 inches in 1 meter, in 3600 m. there are 3600 times 39.37 in., or 141732 in. = 11811 ft. Therefore, 3.6 Km. are equal to 11811 ft.

RULE.—*Reduce the metric number to the denomination of the principal unit of the table; then multiply by the equivalent, and reduce the product to the required denomination.*

EXERCISES.

2. How many feet in 472 centimeters? *Ans.* 15.485 ft.
3. How many cubic feet in 2000 sters?
4. How many gallons, liquid measure, in 325 deciliters?
5. How many gallons in 108.24 liters? *Ans.* 28 gal. 2.77 qt.
6. How many bushels in 3262 kiloliters?
7. How many acres in 436 ars? *Ans.* 10.774 A.
8. In 942325 centiliters, how many bushels?
9. In 456 kilograms, how many pounds? *Ans.* 1005.024 lb.
10. In 42 ars, how many square rods?
11. Change 75.5 hektars to acres. *Ans.* 186.56 A.
12. How many gallons in $24\frac{1}{2}$ liters of wine?
13. How many pounds of butter in 124 kilos?
14. In 28 sters, how many cords? *Ans.* 7.725 C.
15. In 72 kilometers, how many miles?
16. Change 148 grams to ounces Avoirdupois. *Ans.* 5.22 oz.
17. Change 150.75 kilos to pounds.
18. How many sq. rods in 5 a. 85 ca.? *Ans.* 23.13 sq. rd.
19. What is the weight of 24 cu. dm. 148 cu. cm. of silver, if a cu. centimeter weighs 11.4 g.? *Ans.* 737.556 lb. Tr.

953. To change the Common to the Metric System.

1. In 10 lb. 4 oz. Troy, how many kilograms?

OPERATION.

10 lb. 4 oz. = 10.25 lb.
10.25 lb. × 5760 = 59040 gr.
59040 gr. ÷ 15.432 gr. = 3825.75 g.
3825.75 g. ÷ 1000 = 3.82575 Kg., *Ans.*

ANALYSIS.—The *gram*, the principal unit of the table, is expressed in grains; hence, reduce the pounds and ounces to grains. Since 15.432 gr. make 1 gram, there are as many grams in 59040 gr. as 15.432 gr. is contained times in 59040 gr., or 3825.75 g. And since there are 1000 grams in a kilogram, dividing 3825.75 g. by 1000 g., the quotient is 3.82575. Therefore, there are 3.82575 Kg. in 10 lb. 4 oz.

RULE.—*Reduce the given quantity to the denomination in which the equivalent of the principal unit of the metric table is expressed; divide by this equivalent, and reduce the quotient to the required denomination.*

EXERCISES.

2. In 6172.9 lb av., how many kilograms? *Ans.* 2800.009 Kg.

3. How many ars in a square mile?

4. How many cu. decimeters in 1892 cu. feet?

5. In 892 gr., how many grams? *Ans.* 57.8 g.

6. In 2 mi. 272 rd. 5 yd., how many kilometers? *Ans.* 4.59 Km.

7. How many sters in 264.4 cu. feet?

8. How many liters in 3 bu. 1 pk.? *Ans.* 114.5 l.

9. How many grams in 6 lb. Troy? In 6 lb. Avoir.?

10. How many meters in 3 mi. 272 rd.?

11. In 1828 cu. yd. how many cu. meters? *Ans.* 1397.52 cu. m.

12. In 3588 sq. yards, how many sq. meters?

13. Bought 454 bu. of wheat, at $3 a bushel, and sold the same at $8.75 per hektoliter; how many hektoliters did I sell? Did I gain or lose, and how much? *Ans.* 160 Hl.; gain, $38.

14. In 13 gal. 3 qt. 2 pt. 3 gi., how many liters? *Ans.* 53.35 l. +.

15. How many sq. meters of plastering in a room 18 ft. 6 in. long, 14 ft. wide, and 9 ft. 6 in. high? *Ans.* 81.427 sq. m. +.

TEST PROBLEMS.

954. 1. Find the weight of a barrel of flour (196 lb.) in Kg.?

2. What is the cost of a carpet for a room 10.5 m. long, and 8.4 m. wide, if the carpet is 84 cm. wide and costs $2.75 a meter? *Ans.* $288.75.

3. A farmer sold 540 Hl. of wheat, at $2 a bushel, and invested the proceeds in coal at $7 per ton. How many tons did he buy? *Ans.* 437.785 T.+.

4. What is the cost of a building lot 75 m. long and 62 m. wide, at $40 an ar? *Ans.* $1860.

5. A bushel of wheat weighs 60 lb. What is the weight of 5 Hl. of wheat, in kilograms? *Ans.* 386.05 Kg.

6. What will be the cost of a pile of wood 15.7 m. long, 3 m. high, and 7.52 m. wide, at $1.50 a ster?

7. The new silver dollar weighs 412½ gr. Troy. How many grams does it weigh? *Ans.* 26.73 g.

8. How many acres of land in 24.6 Km. of a highway, which is 20 m. wide? *Ans.* 121.573 A.

9. A bin is 4.2 m. long, 2.8 m. wide, and 1.5 m. deep. What will be the cost of filling it with charcoal, at 25 cts. a hektoliter?

10. A merchant bought 300 m. of silk in Lyons, at 12.5 francs a meter; he paid 75 cents a yard for duty and freight, and sold it in New York at $5 a yard. What was his gain? *Ans.* $670.61.

11. What price per pound is equivalent to $2.50 per Hg.?

12. If a man buys 5000 g. of jewels, at 35 francs a gram, and sells them at $15 a pennyweight, what was his gain or loss?

13. If a field produces 40 Hl. of oats to the hektar, how many bushels is that to the acre? *Ans.* 45.93 bu.

14. What price per peck is equivalent to 80 cts. a dekaliter?

15. What will be the cost of excavating a cellar 18.3 m. long, 10.73 m. wide, and 3.4 m. deep, at 20 cents per ster?

16. How many pounds Avoir. are there in 96.4 kilos of salt?

17. How many liters will a cistern hold that measures on the inside 5.5 ft. long, 4 ft. 6 in. wide, and 4 ft. deep? *Ans.* 2803.383 l.

18. How many meters of lining that is 60 cm. wide will line 15 m. of silk that is 75 cm. wide? *Ans.* 18,75 cm.

19. A lady bought 40.5 m. of silk in Paris. What would be its value in Boston, at $4.75 per yard?

20. A bin is 4 m. long, 2.3 m. wide. How deep must it be to contain 40 Hl. of grain? *Ans.* 4.347 + dm.

21. How many sters of wood can be piled in a shed 8.5 m. long, 5.8 m. wide, and 4.2 m. high? What would be its value at $3.25 a cord? *Ans.* 207.06 s.; $185.665.

22. A dray is loaded with 60 bags of grain, each bag holding 8 Dl.; allowing 75 K. of grain to the hectoliter, what is the weight of the load in metric tons? *Ans.* 3.6 T.

23. How many meters of shirting, at $.18 per meter, must be given in exchange for 250 Hl. of oats, at $1.20 per hectoliter?

24. A merchant shipped to France 50 barrels of sugar, each containing 250 lb., paying $2 per cwt. for transportation. He sold the sugar at $.34 per kilogram, and invested the proceeds in broadcloth, at $4 per meter. How many *yards* did he purchase?

25. A cu. decimeter of copper weighs 8.8 Kg. What is the value of a bar of the same metal 15 dm. long, 9.6 cm. broad, and 6.4 cm. thick, at $1.30 a kilogram? *Ans.* $105.43.

26. How many bricks, each 20 cm. long and 10 cm. wide, will pave a walk 95.4 m. long and 2.1 m. wide; and what will they cost, at $1.75 per hundred? *Ans.* 10017 bricks; $175.297.

27. What is the value of a pile of wood 40 ft. 6 in. long, 4 ft. broad, and 6 ft. 6 in. high, at $6.50 per dekastere?

28. What will be the cost of building a wall 96 Dm. 6 m. 8 dm. long, 1 m. 6 dm. thick, and 2 m. 4 cm. high, at $6.75 a cu. meter?

29. A wine merchant imported to Boston 1000 dekaliters of wine, at a cost of $.75 a liter, delivered. At what price per gallon must he sell the same to clear $2000 on the shipment? *Ans.* $3.596.

30. How many gallons of water will a cistern contain that is 3 m. deep, 2 m. long, and 1.5 m. wide; and what will be its weight in metric tons? *Ans.* 2377.575 gals.; 9 T.

TABLE OF EQUIVALENTS.

955. The equivalents here given agree with those that have been established by Act of Congress for use in legal proceedings and in the interpretation of contracts.

1 inch = 2.540 centimeters.	1 centimeter = 0.3937 inch.
1 foot = 3.048 decimeters.	1 decimeter = 0.328 foot.
1 yard = 0.9144 meters.	1 meter = 1.0936 yds. = 39.37 in.
1 rod = 0.5029 dekameters.	1 dekameter = 1.9884 rods.
1 mile = 1.6093 kilometers.	1 kilometer = 0.62137 mile.
1 sq. in. = 6.452 sq. centimeters.	1 sq. centimeter = 0.1550 sq. in.
1 sq. ft. = 9.2903 sq. decimeters.	1 sq. decimeter = 0.1076 sq. ft.
1 sq. yard = 0.8361 sq. meter.	1 sq. meter = 1.196 sq. yards.
1 sq. rd. = 25.293 sq. meters.	1 ar = 3.954 sq. rods.
1 acre = 0.4047 hektar.	1 hektar = 2.471 acres.
1 sq. mile = 2.590 sq. kilometers.	1 sq. kilometer = 0.3861 sq. mi.
1 cu. in. = 16.387 cu. centimeters.	1 cu. centimeter = 0.0610 cu. in.
1 cu. ft. = 28.317 cu. decimeters.	1 cu. decimeter = 0.0353 cu. ft.
1 cu. yard = 0.7645 cu. meter.	1 cu. meter = 1.308 cu. yards.
1 cord = 3.624 sters.	1 ster = 0.2759 cord.
1 liquid quart = 0.9463 liter.	1 liter = 1.0567 liquid quarts.
1 gallon = 0.3785 dekaliters.	1 dekaliter = 2.6417 gallons.
1 dry quart = 1.101 liters.	1 liter = 0.908 dry quart.
1 peck = 0.881 dekaliter.	1 dekaliter = 1.135 pecks.
1 bushel = 3.524 dekaliters.	1 hectoliter = 2.8375 bushels.
1 ounce av. = 28.35 grams.	1 gram = 0,03527 ounce Av.
1 pound av. = 0.4536 kilogram.	1 kilogram = 2.2046 pounds Av.
1 T. (2000 lbs.) = 0.9072 met. ton.	1 metric ton = 1.1023 tons.
1 grain Troy = 0.0648 gram.	1 gram = 15.432 grains Troy.
1 ounce Troy = 31.1035 grams.	1 gram = 0.03215 ounce Troy.
1 pound Troy = 0.3732 kilogram.	1 kilogram = 2.679 pounds Troy

VERMONT RULE FOR PARTIAL PAYMENTS.

956. The General Statutes of Vermont provide the following RULE for computing interest on notes, when partial payments have been made:

"*On all notes, bills, or other similar obligations, whether made payable on demand or at a specified time,* WITH INTEREST, *when payments are made, such payments shall be applied: first, to liquidate the interest that has accrued at the time of such payments; and, secondly, to the extinguishment of the principal.*

"*On all notes, bills, or other similar obligations, whether made payable on demand or at a specified time,* WITH INTEREST ANNUALLY, *the annual interests that remain unpaid shall be subject to simple interest, from the time they become due to the time of final settlement; but if in any year, reckoning from the time such annual interest began to accrue, payments have been made, the amount of such payments at the end of such year, with interest thereon from the date of payment, shall be applied: first, to liquidate the simple interest that has accrued upon the unpaid annual interests; secondly, to liquidate the annual interests that have become due; and thirdly, to the extinguishment of the principal.*"

EXERCISES.

$3458. BRADFORD, VT., Sept. 13, 1869.

1. *For value received, I promise to pay E. W. Colby or order three thousand four hundred and fifty-eight dollars, on or before the first day of January, 1878, with interest.* SAMUEL S. GREEN.

Indorsed as follows: Dec. 16, 1870, $100; May 1, 1871, $1000; Jan. 13, 1874, $85; April 13, 1876, $450.75.

What was due Jan. 1, 1878? *Ans.* $3239.90.

$872. ST. JOHNSBURY, VT., Nov. 22, 1868.

2. *For value received, I promise to pay James Ferguson or order eight hundred and seventy-two dollars, on demand, with interest annually.* SYLVANUS E. BOYLE.

Indorsed as follows: April 4, 1869, $28; July 10, 1872, $94.40; Dec. 10, 1874, $6.72; Jan. 14, 1877, $396.

What was due Dec. 28, 1878?

OPERATION.

	Int. on Int.	*Yearly Int.*	*Prin.*
Int. of prin. to Nov. 22, 1869		$52.32	$872
Am't of 1st payment		29.06	
Bal. of unpaid yearly int.		23.26	
Int. of prin. to Nov. 22, 1872		156.96	
Int. on 1 year's int. 3 years	$9.42		
Int. on bal. of unpaid yearly int. 3 years .	4.19	13.61	
		193.83	
Am't of 2d payment		96.48	
Bal. of unpaid yearly int.		97.35	
Int. of prin. to Nov. 22, 1875		156.96	
Int. on 1 year's int. 3 years	9.42		
Int. on bal. of unpaid yearly int. 3 years .	17.52		
	26.94	254.31	
Am't of 3d payment	7.10		
Bal. of int. on int.	19.84		
Int of prin. to Nov. 22, 1877		104.64	
Int. on 1 year's int. 1 year	3.14		
Int. on bal. of unpaid yearly int. 2 years .	30.52	53.50	412.45
			1284.45
Am't of 4th payment			416.33
New principal			868.12
Int. of new prin. to Dec. 28, 1878			57.30
Int. on 1 year's int. 1 mo. 6 d.			.31
Due, Dec. 28, 1878			$925.73

EXPLANATION.—We compute the interest for one year from the date of the note, as a payment is made within that year, and deduct the amount of the payment at the end of the year from the interest due. The balance of interest bears interest till Nov. 22, 1872. The amount of the payment at the end of this year exceeds the interest on interest due. We therefore deduct the amount of the payment from the total interest due, and have a balance of unpaid yearly interest, $97.35, which bears simple interest till Nov. 22, 1875. At this date the amount of the payment is less than the interest on interest due. We therefore deduct the amount of the payment from the amount of interest on interest, and have a remainder of $19.84, which is without interest. The amount of unpaid yearly interest at this date bears simple interest till the next balance.

The amount of the fourth payment, Nov. 22, 1877, exceeds the total interest due. We therefore deduct it from the sum of the interest and principal. The remainder forms a new principal, which bears simple interest to the settlement of the note, Dec. 28, 1878, and one year's interest on the same bears interest from Nov. 22, 1878, to Dec. 28, 1878, which interest, added to the new principal, gives the amount due Dec. 28, 1878—$925.73.

In cases of annual interest with partial payments, like the above example, observe the following notes:

1. To avoid compounding interest, keep the principal, unpaid yearly interests, and interest on yearly interest, in separate columns.

2. Deduct the amount of the payment or payments at the end of the year from the interest on the unpaid yearly interest, when it does not *exceed* this interest. The remainder never draws interest, but is liquidated by the first payment that equals or exceeds it.

3. Deduct the amount of the payment or payments at the end of the year from the *sum* of the unpaid yearly interests and the interest on the unpaid yearly interests, when this amount exceeds the interest on the interest, but is less than such sum. The remainder is a balance of unpaid yearly interest which draws *simple* interest until canceled by a payment.

4. Deduct the amount of the payment or payments at the end of the year from the sum of the total interest due and the principal, when it exceeds the total interest due. The remainder forms a new principal, with which proceed as with the original principal.

$5000. NEWPORT, VT., Oct. 19, 1862.

3. For value received, we jointly and severally promise to pay John Smith or bearer five thousand dollars, sixteen years after date, with interest annually.

GEO. S. LEAZER.
E. D. CRAWFORD.

Indorsed as follows: Jan. 13, 1866, $393; Sept. 24, 1866, $48; July 10, 1869, $493.47; Oct. 14, 1873, $100; Dec. 12, 1877, $3200; April 15, 1878, $65.

What was due Oct. 19, 1878? *Ans.* $7056.17.

$420. BURLINGTON, VT., March 23, 1872.

4. For value received, I promise to pay Jas. B. Vinton or order four hundred and twenty dollars, six years from date, with interest annually.

GEO. A. BANCROFT.

Indorsed as follows; Oct. 3, 1873, $40.23; March 1, 1874, $8; Sept. 13, 1875, $33.38.

What was due March 23, 1878? *Ans.* $494.62.

$639. **Barton, Vt. Aug. 20, 1872.**

5. For value received, I promise to pay E. J. Baxter or order six hundred and thirty-nine dollars, on demand, with interest annually.

Samuel Macomber.

Indorsed as follows: Oct. 14, 1877, $10; Dec. 24, 1878, $20.

What was due March 30, 1879? ***Ans.*** **$904.58.**

TABLE.

Showing amount of $1.00 from 1 to 20 years, at 4, 5, 6, 7 and 8 per cent., Annual Interest.

Years.	4 per cent.	5 per cent.	6 per cent,	7 per cent.	8 per cent.	Years.
1 .	$1.0400	$1.0500	$1.0600	$1.0700	$1.0800	. 1
2 .	1.0816	1.1025	1.1236	1.1449	1.1664	. 2
3 .	1.1248	1.1575	1.1908	1.2247	1.2592	. 3
4 .	1.1696	1.2150	1.2616	1.3094	1.3584	. 4
5 .	1.2160	1.2750	1.3360	1.3990	1.4640	. 5
6 .	1.2640	1.3375	1.4140	1.4935	1.5760	. 6
7 .	1.3136	1.4025	1.4956	1.5929	1.6944	. 7
8 .	1.3648	1.4700	1.5808	1.6972	1.8192	. 8
9 .	1.4176	1.5400	1.6696	1.8064	1.9504	. 9
10 .	1.4720	1.6125	1.7620	1.9205	2.0880	. 10
11 .	1.5280	1.6875	1.8580	2.0395	2.2320	. 11
12 .	1.5856	1.7650	1.9576	2.1634	2.3824	. 12
13 .	1.6448	1.8450	2.0608	2.2922	2.5392	. 13
14 .	1.7056	1.9275	2.1676	2.4259	2.7024	. 14
15 .	1.7680	1.0125	2.2780	2.5645	2.8720	. 15
16 .	1.8320	2.1000	2.3920	2.7080	3.0480	. 16
17 .	1.8976	2.1900	2.5096	2.8564	3.2304	. 17
18 .	1.9648	2.2825	2.6308	3.0097	3.4192	. 18
19 .	2.0336	2.3775	2.7556	3.1679	3.6144	. 19
20 .	2.1040	2.4750	2.8840	3.3100	3.8160	20

VERMONT METHOD OF ASSESSING TAXES.

957. The *Grand List* is the base on which all taxes are assessed; it is 1% of the appraised value of the real estate and personal property, together with the poll list.

The *Poll List* is $2.00 for every male inhabitant, from 21 to 70 years of age, except such as are specially exempt by law.

The General Statutes of Vermont provide that the listers in each town shall make a list of all the real estate and personal property, and the number of taxable polls in such town, and that the said list shall contain the following particulars:

"*First.* The name of each taxable person.

"*Second.* The number of polls and the amount at which the same are set in the list.

"*Third.* The quantity of real estate owned or occupied by such person.

"*Fourth.* The value of such real estate.

"*Fifth.* In the fifth column the full value of all taxable personal estate owned by such person.

"*Sixth.* In the sixth column shall be set the one per centum on the value of all personal and real estate, together with the amount of the polls, which sum shall be the amount on which all taxes shall be made or assessed.

The *State* and *County* Taxes are assessed by the Legislature.

The minimum of the State School and Highway Taxes is fixed by law, and a higher rate left optional with the town.

A *Town* Tax is assessed by vote of the town, a *Village* Tax by vote of the village, and a *School District* Tax by vote of the district.

EXERCISES.

1. The town of Montpelier voted a town tax of $2.60 on each dollar of the grand list. The appraised value of the real estate was $702727, and of the personal property $309987, and there were 740 taxable polls. What was the grand list of the town? How much money was raised by this vote? What was John Hammond's town tax, who was 30 years of age, and whose property was appraised at $8927.75?

OPERATION.

$702727 + $309987 = $1012714, assessed value of the property.

$1012714 × .01 = $10127.14, 1% of the assessed value.

$2.00 × 740 = $1480, the poll list.

$10127.14 + $1480 = $11607.14, the grand list.

$2.60 × 11607.14 = $30178.56, amount of money raised.

$8927.75 × .01 = $89.28, 1% of the assessed value of John Hammond's property.

$89.28 + $2.00, his poll list = $91.28, John Hammond's grand list

$2.60 × 91.28 = $237.33, John Hammond's town tax.

2. The appraised value of property, both real and personal, in the town of Rutland, for the year 1878, was $3415264. The number of taxable polls was 2066. The town voted to raise a tax of $28713.48. What was the tax on a dollar of the grand list?
Ans. $0.75.

3. The appraised value of the real estate in the city of Burlington was $2542373; of the personal property, $399937. There were 2040 taxable polls. The city voted to raise $60305.58 city tax. What was the amount of Henry Cook's tax, a resident, who was 73 years of age, and whose real estate was appraised at $750, and his personal property at $475.50? *Ans.* $22.06.

4. The grand list in the town of Chelsea was $4403.74. The appraised value of all the property was $368774. How many taxable polls were there in that town? *Ans.* 358.

5. The estimated cost of schools in school district No. 8, in the town of Cabot, for one year, was $765. The amount of public money received from the town was $71.50. The appraised value of the real estate in the district was $48545; of the personal estate $15428.75; the number of taxable polls in the district 103. How much tax on a dollar of the grand list must the district vote, to pay its expenses? *Ans.* $0.82.

6. James Bell resides in Hardwick; he is 44 years of age; his property, both real estate and personal, is appraised at $8975.50. Hardwick voted a town tax of $1.60 on a dollar of the grand list. The highway tax is $0.40: the state tax is $0.45; the state school tax is $0.09; the school tax is $0.86; and the county tax $0.04, on the dollar. What is the amount of his taxes? *Ans.* $315.64.

FRENCH AND SPANISH MEASURES.

958. The old ***French Linear***, and ***Land Measure***, is still used to some extent in Louisiana, and in other French settlements in the United States.

Table.

12 Lines = 1 Inch.
12 Inches = 1 Foot.
6 Feet = 1 Toise.
32 Toises = 1 Arpent.
900 Square Toises = 1 Square Arpent.

The *French Foot* equals 12.8 inches, American, nearly.

The *Arpent* is the old French name for *Acre*, and contains nearly $\frac{5}{6}$ of an English acre.

In Texas, New Mexico, and in other Spanish settlements of the United States, the following denominations are still used:

Table.

1000000 Square Varas = 1 Labor = 177.136 Acres (American).
25 Labors = 1 League = 4428.4 Acres "

The *Spanish Foot* = 11.11 + in. (Am.); 1 Vara = 33$\frac{1}{3}$ in. (Am.); 108 Varas = 100 Yards, and 1900.8 Varas = 1 Mile.

Other Denominations in Use.

5000 Varas Square = 1 Square League.
1000 Varas Square = 1 Labor, or $\frac{1}{25}$ League.
5645.376 Square Varas = 4840 Square Yards = 1 Acre.
23.76 Square Varas = 1 Square Chain = $\frac{1}{10}$ Acre.
1900.8 Varas Square = 1 Section = 640 Acres.

TABLE FOR INVESTORS.

959. *The following Table shows the rate per cent. of Annual Income from Bonds bearing* 5, 6, 7, *or* 8 *per cent. interest, and costing from* 40 *to* 125.

Purchase Price.	5%.	6%.	7%.	8%.	Purchase Price.	5%.	6%.	7%.	8%.
40	12.50	15.00	17.50	20.00	83	6.02	7.22	8.43	9.63
41	12,20	14.64	17.08	19.52	84	5.95	7.14	8.33	9.52
42	11.90	14.28	16.66	19.04	85	5 88	7.05	8.23	9.41
43	11.63	13.95	16.28	18.61	86	5.81	6.97	8.13	9.30
44	11.36	13.63	15.90	18.18	87	5.74	6.89	8.04	9.19
45	11.11	13.32	15.56	17.78	88	5.68	6.81	7.94	9.09
46	10.86	13.04	15.21	17.39	89	5.61	6.74	7.86	8.98
47	10.63	12.77	14.90	17.02	90	5.55	6.66	7.77	8.88
48	10.41	12.50	14.53	16.66	91	5.49	6.59	7.69	8.79
49	10.20	12.25	14.29	16.33	92	5.43	6.52	7.60	8.69
50	10.00	12.00	14.00	16.00	93	5.37	6.45	7.52	8.60
51	9.80	11.76	13.72	15.68	94	5.31	6.38	7.44	8.51
52	9.61	11.53	13.46	15.38	95	5.26	6 31	7.36	8.42
53	9.43	11.32	13.20	15.09	96	5.20	6.25	7.29	8.33
54	9.25	11.11	12.96	14.81	97	5.15	6.18	7.21	8.24
55	9.09	10.90	12.72	14.54	98	5.10	6.12	7.14	8.16
56	8.92	10.70	12.50	14.28	99	5.05	6.06	7.07	8.08
57	8.77	10.52	12.27	14.03	100	5.00	6.00	7.00	8.00
58	8.62	10.34	12.06	13.79	101	4.95	5.94	6.93	7.92
59	8.47	10.16	11.86	13.55	102	4.90	5.88	6.86	7.84
60	8.33	10.00	11.66	13.33	103	4.85	5.82	6.79	7.76
61	8.19	9.83	11.47	13.11	104	4.80	5.76	6.72	7.69
62	8.06	9.67	11.29	12.90	105	4.76	5.71	6.66	7.61
63	7.93	9.52	11.11	12.69	106	4.71	5.66	6.60	7.54
64	7.81	9.37	10.93	12.50	107	4.67	5.60	6.54	7.47
65	7.69	9.23	10.76	12.30	108	4.62	5.55	6.48	7.40
66	7.57	9.09	10.60	12.12	109	4.58	5.50	6.42	7.33
67	7.46	8.95	10.44	11.94	110	4.54	5.45	6.36	7.27
68	7.35	8.82	10.29	11.76	111	4.50	5.40	6.30	7.20
69	7.24	8.69	10.14	11.59	112	4 46	5.35	6.25	7.14
70	7.14	8.57	10.00	11.43	113	4.42	5.30	6.19	7.07
71	7.04	8.45	9.85	11.26	114	4.38	5.26	6.14	7.01
72	6.94	8.33	9.72	11.11	115	4.35	5.21	6.08	6.95
73	6.84	8.21	9.58	10.95	116	4.31	5.17	6.03	6.89
74	6.75	8.10	9.45	10.80	117	4.27	5.12	5.98	6.83
75	6.66	8.00	9.33	10.66	118	4.23	5.08	5.93	6.77
76	6.57	7.89	9.21	10.52	119	4.20	5.04	5.88	6.72
77	6.49	7.79	9.00	10.38	120	4.16	5 00	5.83	6.66
78	6.41	7.69	8.97	10.25	121	4.13	4.95	5.78	6.61
79	6.32	7.59	8.86	10.12	122	4.09	4.91	5.73	6.55
80	6.25	7.50	8.75	10.00	123	4.06	4.87	5.69	6.5
81	6.17	7.40	8.64	9.87	124	4.03	4.83	5.65	6.4
82	6.09	7.31	8.53	9.75	125	4.00	4.80	5.60	6.4

ANSWERS.

The answers to the introductory and more simple examples of many of the articles have been omitted.

Art. 77.

1. $5.78.
2. $39.18.
3. $137.87.
4. $247.78.
5. $38.58.
6. $27.78.
7. $189.75.
8. $17.67.

Art. 79.

2. 1646.
3. 1619.
4. $65.94.
5. $287.67.
6. $376.71.
7. 4491.
8. 7504 lb.
9. 75686.
10. 72447.
11. $696.87.
12. $18.12.
13. $80.87.
14. $149.18.
15. 105233.
16. $220.84.
17. 181776.
18. 11965.
19. $944.66.
20. $7193.28.
21. $3554.05.
22. 1547164.
23. $6692.23.
24. 15873478.
25. $104560.

Art. 91.

10. $14.11.
11. 3231.
12. $51.24.
13. 2123 tons.
14. 2324 ft.
15. 2324 days.
16. $41.28.
17. $230.48.
18. $202.12.
19. 224113.
20. 721220.
21. 210532.
22. 4175.
23. 151.
24. 5113.
25. $15.21.
26. $22.10.
27. $25.26.
28. 2710.
29. 34213.
30. $212.20.
31. $6746.
32. 221533.

Art. 93.

2. 1848.
3. 3883.
4. 1318.
5. 4195.
6. 28286 miles.
7. 26762 acres.
8. 228670 ft.
9. $240.81.
10. $95.58.
11. $38.08.
12. $6.16.
13. 32358.
14. $64.84.
15. $135.28.
16. $157.63.
17. 8728 rd.
18. 45736 tons.
25. 12336.
26. 37588.
27. 69356.
28. 4800.
29. $3323.59.
30. $1264.50.
31. 33798.
32. 35555.
33. $291.35.
34. $222.75.
35. $3015.05.
36. $5524.77.
37. 10386.
38. $695.79.
39. $5351.84.
40. $101.10.
41. 474889.

Art. 95.

1. 332650.
2. $895.66.
3. 30443.
4. 6132.
5. 2517.
6. $15.22.
7. 4190 miles.
8. $3640.
9. 78388 sq. mi.
10. 3572 ft.
11. $53945.
12. $9505.67.
13. 1909609.
14. $5044.25.
15. $16948.50.
16. $1417.16.
17. 702.
18. $36.50.
19. 8346.
20. 16552.

Art. 105.

12. $4743.
13. $1956.
14. $6190.
20. $40.50.
21. $30.59.
22. $622.50.
23. $16120.

Art. 107.

2. 12771.
3. 25830.
4. 34104.
5. $1239.30 ;
$1713.15.
6. $3885.75 ;
$4521.60.
7. $2209.32 ;
$2383.74.
8. 482400 ;
430944 ;
874752.
9. 2953216 ;
5606496 ;
7083104.
10. $85692.24 ;
$279759.96 ;
$171384.48.
11. $2529.25.
12. $319192.
13. $14064.
14. $264958.
15. 404914.
16. 186516.
17. 241768.
18. $51188.62.
19. 17902976.
20. $154037.36.
21. 15704325 da.
22. 2082600 cts.
23. 1508741097.
24. 1587862270.
25. 3654860576.
26. 8198473608.

27. 982275037.
28. 3363731415.
29. $2715413.50.
30. $21718.16.
31. 416304.
32. 0.
33. 947363302.
34. 5395144320.
35. 72618.
36. $3594.24.
37. $4101.25.
38. 51408;
$7454160.
39. 277536;
$49956.48.

Art. 109.

2. $3505.92.
3. 3605472.
4. 3906168.
5. $19789.44.
6. 84338.28.
7. 16810320.
8. 54793296.
9. $109804.80.
10. $9212.
11. $430.08.
12. $19234.82.

Art. 110.

5. $472.
6. $1824.
7. $840000.
8. 600000.
9. 12600000.
10. 104000000.
11. 126930871800.
12. 350310024000.
13. 96000.
128000.
268800.
14. $400000.

Art. 113.

1. $1617.30.
2. $50.19.
3. $829.56.
4. $3023.75.
5. 17920.
6. 2878.
7. 37200.
8. 151218.
9. $7198.75.
10. $18801,
Whole.
$10938,
Farm.
$4617, Stock

11. $8232.
12. $25 loss.
13. 77050.
14. 92500.
15. $1714.50.
16. $43187.32.

Art. 133.

15. 1887;
7303;
2883.
16. $47208\frac{1}{7}$;
2754;
$131181\frac{4}{7}$.
17. $48475\frac{5}{9}$;
$67297\frac{2}{9}$;
$115458\frac{4}{9}$.
18. $172.65.
19. $5801\frac{1}{8}$ lb.
20. $2584\frac{1}{8}$ days.
21. $820.50.
22. $71474\frac{4}{7}$ mi.
23. 8219 men.
24. $20116\frac{1}{8}$ A.
25. 63362 rd.
26. 1592 bbl.;
$883\frac{1}{8}$ "
27. 9375 bu.
28. $108 50.
29. 93 oranges.
30. 91 yd.
31. $256.

Art. 136.

3. $2340\frac{2}{14}$;
$2047\frac{10}{14}$;
$1424\frac{10}{33}$;
$1170\frac{2}{38}$.
4. $2248\frac{12}{58}$;
$2070\frac{16}{63}$;
$1610\frac{16}{81}$;
$1762\frac{38}{74}$.

Art. 138.

9. $9.58.
10. $14.89.
11. $25.21.
12. 354 times.
13. 416 "
14. 672 "
15. 1763 "
16. 3300 "
17. $18\frac{20}{105}$ "
18. $44\frac{48}{700}$ "
19. $642\frac{2}{47}$;
$592\frac{41}{47}$;
$2201\frac{27}{47}$.
20. $1083\frac{21}{185}$;
$25414\frac{35}{185}$.
21. $$1823\frac{86}{515}$.
22. $97.
23. $76.
24. 475 acres.
25. 37 horses;
$110 left.
26. 394.
27. 5482.
28. 7198.
29. 31416.
30. 7071.
31. 8723.
32. 610.
33. $28004\frac{1752}{2694}$.
34. $1172\frac{464}{736}$.
35. 4321.
36. 2036.
37. $3645\frac{3867}{3076}$.
38. 7500.
39. 43785.
40. 4629.
41. 346

Art. 139.

2. 173.
3. 285.
4. 4175.
5. 437.
7. $1931\frac{42}{73}$.
8. $76671\frac{61}{91}$.
9. 4175.
10. $544\frac{82}{135}$.
11. $1640\frac{176}{343}$.

Art. 140.

8. $279\frac{8305}{18000}$.
9. $2824\frac{861}{2640}$.
10. $545\frac{7160}{14800}$.
11. $43.
12. 20 lots.
13. $84\frac{20}{470}$.

Art. 145.

1. 4920.
2. 9 times.
3. 9 "
4. 394950.
5. 538.
6. $443\frac{30}{174}$.
7. $10.78.
8. 16399.
9. $28.15.
10. 3000 lb.
11. $7.50.
12. 42588.
13. 718284.
14. 7 years.
15. 55552.
16. 50496.
17. 7325.
18. 826776.
20. 76 cts.
21. $107.
22. $$2123\frac{3}{5}$.
23. 42 weeks.
24. $367.
25. $1806.
26. $30247.
28. 3823;
1849.
29. $720;
$530.
30. 2008;
1781.

31. \$16550.
\$11925.
32. 24 boxes.
33. 356 cords.
\$4 cost.
34. 288.
35. 2.
36. 10.
37. 1476.
38. 469.

Art. 165.

2. $2, 3, 5^2, 7$.
3. $2^2, 3, 5, 19$.
4. 3, 5, 163.
5. $2, 7, 13^2$.
6. $3^2, 5, 7^2$.
7. 2, 3, 5, 7, 11.
8. 3, 5, 7, 11.
9. $2, 3^2, 163$.
10. $2^2, 3^2, 5^2, 7$.
11. $3^2, 5, 7^3$.
12. 11, 31, 41.
13. $2^6, 5, 101$.
14. $2^{10}, 3, 7$.
15. $3^2, 5^2, 7, 19$.
16. 19, 23, 29.
17. 2, 5, 7, 11, 13.
18. $3^2, 5, 7^2, 13$.
19. 2, 5, 7, 11, 41.

Art. 170.

2. 14.
3. 32.
4. 5.
5. 18.
6. 144.
7. 22.
8. 42.
9. 24.

Art. 171.

2. 4.
3. 7.
4. 27.
5. 2.
6. 1.
7. 13.
8. 13.
9. 113.
10. 17; 87.
11. 124; 2.
12. 12 ft.
13. 3 bu.
14. 4329 bags.
15. 5; 9; 11 hr.
16. 8162 rails.

Art. 177.

2. 2856.
3. 120.
4. 450.
5. 30030.
6. 13860.
7. 1680.
8. 5280.

Art. 178.

2. 4896.
3. 16800.
4. 51282.
5. 1560.
6. 7200.
7. 3060.
8. 1680.
9. 315.
10. 240.
11. 180 ft.
12. \$60.
13. 384.
14. 63.
15. \$4536.
16. 720 bu.

Art. 182.

3. 13.
4. 4.
5. 14.
6. 130.
7. 33.
8. 61.
9. 14839.
10. 16.
11. $9\frac{1}{7}$.
12. $1\frac{1}{98}$.
13. 32.
14. 403.
15. $1\frac{3}{7}$.
16. $41\frac{1}{7}$.
17. 10 tons.
18. 98 bbl.
19. $8\frac{1}{8}$ tubs.
20. \$2.00.
21. \$.50.
22. \$.36.
23. 120 bu.
24. 44 yd., 1st.
22 yd., 2d.
25. \$.77.
26. 144 bu.
27. 4 chests.
28. 48 days.

Art. 209.

13. $\frac{4}{5}$.
14. $\frac{43}{47}$.
15. $\frac{4}{7}$.
16. $\frac{21}{22}$.
17. $\frac{71}{193}$.
18. $\frac{3}{13}$.
19. $\frac{71}{198}$.
20. $\frac{7}{8}$.
21. $\frac{19}{20}$.
22. $\frac{117}{265}$.
23. $\frac{1106}{1665}$.
24. $\frac{23}{794}$.

Art. 211.

3. $\frac{1244}{24}$.
4. $\frac{1248}{16}$.
5. $\frac{200}{7}$.
6. $\frac{337}{20}$; $\frac{438}{20}$.
7. $\frac{2105}{15}$.
8. $\frac{2459}{18}$.
9. $\frac{18879}{26}$.
10. $\frac{21739}{40}$.
11. $\frac{17512}{95}$.
12. $\frac{169227}{84}$.
13. $\frac{4907}{24}$ days.
14. $\frac{22212}{136}$.
15. $\frac{115887}{54}$.
16. $\frac{90557}{90}$.

Art. 213.

5. $17\frac{13}{38}$.
6. $28\frac{3}{5}$.
7. $30\frac{3}{4}$.
8. $18\frac{3}{8}$.
9. $1018\frac{15}{17}$.
10. $50\frac{5}{29}$.
11. $60\frac{1}{3}$.
12. $98\frac{899}{1046}$.
13. $1029\frac{183}{617}$.

Art. 218.

2. $\frac{27}{63}$; $\frac{35}{63}$.
3. $\frac{35}{60}$; $\frac{48}{60}$.
4. $\frac{72}{168}$; $\frac{105}{168}$; $\frac{112}{168}$.
5. $\frac{21}{48}$; $\frac{32}{48}$; $\frac{18}{48}$.
6. $\frac{63}{70}$; $\frac{50}{70}$; $\frac{35}{70}$.
7. $\frac{39}{234}$; $\frac{52}{234}$; $\frac{36}{234}$.
9. $\frac{36}{60}$; $\frac{35}{60}$; $\frac{44}{60}$.
10. $\frac{42}{48}$; $\frac{33}{48}$; $\frac{34}{48}$.
11. $\frac{24}{78}$; $\frac{45}{78}$; $\frac{14}{78}$.
12. $\frac{160}{168}$; $\frac{28}{168}$; $\frac{102}{168}$.
13. $\frac{32}{48}$; $\frac{42}{48}$; $\frac{33}{48}$; $\frac{34}{48}$.
14. $\frac{105}{140}$; $\frac{380}{140}$; $\frac{84}{140}$; $\frac{238}{140}$.
15. $\frac{125}{20}$; $\frac{7}{20}$; $\frac{140}{20}$; $\frac{30}{20}$.
16. $\frac{360}{960}$; $\frac{200}{960}$; $\frac{210}{960}$; $\frac{2688}{960}$.
17. $\frac{114}{168}$; $\frac{67}{168}$; $\frac{238}{168}$; $\frac{24}{168}$.

Art. 221.

3. $1\frac{13}{30}$.
4. $5\frac{23}{40}$.
5. $82\frac{29}{48}$.
6. $262\frac{6}{35}$.
7. $2\frac{5}{1332}$.
8. $9\frac{27}{28}$.
9. $22\frac{17}{70}$.
10. $106\frac{1}{24}$.
11. $69\frac{5}{27}$.
12. $490\frac{13}{18}$.
13. $251\frac{7}{24}$ yd.
14. $9\frac{5}{18}$ yd.
15. $201\frac{17}{48}$.
16. $88\frac{1}{8}$.
17. $191\frac{20}{21}$.

Art. 224.

3. $\frac{7}{18}$.
4. $\frac{13}{28}$.
5. $\frac{331}{840}$.
6. $26\frac{3}{14}$.
7. $56\frac{7}{108}$.
8. $164\frac{29}{48}$.
9. $\frac{31}{88}$.
10. $\frac{11}{13}$.
11. $1\frac{13}{15}$.
12. $12\frac{9}{11}$.
13. $7\frac{25}{72}$.
14. $10\frac{7}{12}$.
15. $\frac{1}{960}$.
16. $16\frac{46}{375}$.
17. $183\frac{17}{37}$.
18. $248\frac{1}{132}$.
19. $115\frac{18}{25}$.

Art. 226.

1. $88\frac{5}{52}$, the greater.
2. $69\frac{20}{63}$.
3. $182\frac{41}{56}$.
4. $1014\frac{44}{65}$.
5. $\frac{5}{48}$.
6. $\frac{23}{63}$.
7. $149\frac{1}{48}$.
8. $\$1\frac{37}{48}$.
9. $1\frac{17}{168}$.
10. $6\frac{4}{27}$.
11. $34\frac{13}{14}$.
12. $68\frac{11}{30}$.
13. $158\frac{17}{27}$.
14. $328\frac{63}{65}$.
15 $265\frac{3}{10}$.
16. $699\frac{9}{20}$.

Art. 229.

2. $7\frac{5}{7}$.
3. $2\frac{3}{7}$.
4. $11\frac{8}{17}$.
5. $1\frac{2}{5}$.
6. $13\frac{1}{3}$.
7. $6\frac{3}{8}$.
8. $\frac{3}{5}$.
9. $16\frac{5}{6}$.
10. 24.
12. 15.
13. $2\frac{2}{5}$.
14. 20.
15. 126.
16. $128\frac{4}{7}$.
17. 255.
18. $5\frac{13}{47}$.
19. 72.
20. $119\frac{1}{3}$.
22. 1532.
23. 1287.
24. 5386.
25. $3949\frac{1}{3}$.
26. 15099.
27. $12756\frac{3}{25}$.
29. 1212.
30. 3624.
31. 7429.
32. 3729.
33. 13272.
34. $10200\frac{3}{8}$.
35. 23586.
 $208993\frac{1}{2}$.
 322.
36. \$580.
37. $\$15\frac{1}{5}$.
38. $\$1769\frac{1}{16}$.
39. $\$940\frac{3}{4}$.
40. $\$7\frac{1}{2}$; $\$22\frac{1}{2}$; \$60; $\$112\frac{1}{2}$.

Art. 232.

2. $\frac{11}{36}$.
3. $\frac{4}{9}$.
4. $\frac{17}{72}$.
5. $\frac{3}{8}$.
6. $\frac{1}{344}$.
7. $\frac{16}{119}$.
9. $9\frac{131}{333}$.
10. 512.
11. 32.
12. 1530.
13. $62\frac{4}{7}$.
14. 5278.
15. $18\frac{1}{7}$.
16. $\frac{99}{190}$.
17. $\$69\frac{3}{8}$.
18. $\$21\frac{21}{32}$.
19. \$345.
20. $\$12\frac{3}{20}$.
21. $\$63\frac{9}{80}$.
22. $\$145\frac{3}{8}$.
23. $\$23\frac{1}{8}$.
24. $\$11.65\frac{1}{4}$.
25. $\$2293\frac{3}{4}$.
26. \$7196.
27. \$5734.
28. $\$47\frac{17}{24}$.
29. $\$\frac{13}{32}$.
30. $\$28\frac{1}{3}$.
31. $\$199\frac{33}{80}$.
32. $\$73\frac{7}{10}$.
33. $\$110\frac{23}{24}$.
34. $424\frac{49}{81}$.

Art. 235.

2. $\frac{4}{37}$.
3. $\frac{5}{316}$.
4. $\frac{13}{306}$.
5. $\frac{5}{311}$.
6. $\frac{1}{512}$.
7. $\frac{1}{42}$.
9. $6\frac{1}{8}$.
10. $16\frac{11}{160}$.
11. $4\frac{43}{756}$.
12. $5\frac{25}{144}$.
13. $12\frac{77}{300}$.
14. $17\frac{1}{36}$.
15. $\$\frac{1}{12}$.
16. $5\frac{17}{24}$.
17. $\frac{7}{180}$ A.
18. $\$10\frac{19}{48}$.
19. $14\frac{7}{15}$.
20. $\$85\frac{3}{8}$.
21. $39\frac{7}{256}$.
22. $6\frac{27}{32}$ lb.
23. $176\frac{1}{12}$ lb.

Art. 238.

2. 117.
3. 126.
4. $205\frac{5}{7}$.
5. $408\frac{13}{25}$.
6. $877\frac{7}{9}$.
7. $1486\frac{10}{13}$.
8. \$147.
9. $13\frac{8}{17}$.
10. $20\frac{35}{143}$.
11. $29\frac{3}{5}$
12. $67\frac{7}{19}$.
13. $\$3\frac{1}{5}$.
14 6 sons.
17. $\frac{6}{7}$.
18. $\frac{3}{4}$.
19. $\frac{56}{81}$.
20. $3\frac{3}{8}$.
21. $1\frac{5}{8}$.
22. $\frac{3}{7}$.
23. 36.
24 $\frac{8}{675}$.
25. $1\frac{5}{7}$.
26. $\frac{23}{36}$.
27. $5\frac{23}{33}$.
28. $17\frac{25}{31}$.
29. $1\frac{1}{4}$.
30. $1\frac{107}{325}$.
31. $1\frac{5}{7}$.
32. $1521\frac{9}{16}$.
33. 63.
34. $8\frac{23}{39}$.
35. \$10450.
36. $12\frac{1}{4}$ tons.
37. $\$\frac{43}{148}$.
38. $\$2.18\frac{1}{4}$.
40. $1\frac{1}{6}$.
41. $1\frac{23}{135}$.
42. $6\frac{23}{31}$.

43. $1\frac{17}{18}$.
44. 2.
45. $\frac{18}{85}$.
46. $6\frac{1}{4}$ mo.
47. $1\frac{139}{350}$.
48. $2\frac{49}{68}$.
49. $\frac{35}{96}$.
50. $15\frac{37}{146}$.
51. $5\frac{73}{135}$.
52. $\frac{747}{3848}$.

Art. 242.

1. $\frac{3}{4}$.
2. $\frac{1}{64}$.
3. $\frac{3}{4}$.
4. $\frac{6}{7}$.
5. $\frac{3}{10}$.
6. $\frac{5}{36}$.
7. $\frac{1}{15}$.
8. $\frac{4}{25}$.
9. $\frac{1}{12}$.
10. $\frac{1}{26}$.
11. $\frac{1}{6}$.
12. $\frac{1}{5}$.
13. $\frac{1}{3}$.
14. $\frac{22}{35}$.
15. $\frac{23}{150}$.
16. $\frac{1}{20}$.

Art. 245.

1. $\frac{20}{72}$, $\frac{48}{72}$, $\frac{63}{72}$, $\frac{56}{72}$.
2. 84.
3. $1677\frac{7}{40}$.
4. $3\frac{17}{80}$.
5. $3043\frac{1}{8}$.
6. 9072.
7. $4612.
8. 10588\frac{4}{17}$.
9. $7\frac{7}{13}$.
10. $10946.
11. $4.
12. $.75.
13. 4577\frac{3}{80}$.
14. 410\frac{13}{18}$.
15. $4\frac{7}{8}$ tons.
16. $146\frac{2}{25}$ miles.
17. $3\frac{3}{7}$.
18. $13\frac{1}{3}$ days.
19. 192\frac{13}{16}$.
20. 4\frac{31}{152}$.
21. $5625.
22. Inc'd $\frac{1}{80}$.
23. Dim'd $\frac{8}{35}$.
24. $9\frac{43}{48}$ bu.
25. $\frac{25}{84}$.
26. $$\frac{1}{3}$.
27. 378 bbl.
28. 3329\frac{1}{6}$; 47 acres.
29. $108.
30. $14\frac{7}{10}$ cords.
31. 20 bbl.
32. 1\frac{3}{4}$.
33. $1840.
34. $152\frac{1}{2}$ ft.
35. $1\frac{7}{10}$; $2\frac{1}{2}$.
36. $3224 cotton.
$2418 sugar.
$1488 mol'es.
$9672 total.
37. $6\frac{169}{1050}$.
38. $\frac{35}{88}$.
39. $5\frac{50}{81}$.
40. $1\frac{16}{75}$.
41. $\frac{611}{860}$.
42. $2\frac{1}{3}$.
43. $22\frac{1}{15}$.
44. $\frac{817}{1628}$.
45. $\frac{1}{2}$.

Art. 267.

18. .596.
19. .0625.
20. .0012.
21. .000074.
22. .0000105.
23. .000099010.
24. .437549.
25. .3040127.
26. .600.00000-024.
27. 495705000.-0043075.
28. 4735000.-00903624.

Art. 283.

15. $$\frac{3}{8}$.
16. $$\frac{5}{8}$.
17. $$\frac{1}{12}$.
18 $\frac{1}{16}$.
19. $\frac{7}{12}$.
20. $\frac{15}{16}$.
21. $$\frac{1}{8}$.
22. $$\frac{3}{8}$.
23. $$\frac{1}{6}$.
24. $\frac{7}{36}$.
25. $\frac{4}{9}$.
26. $\frac{33}{40000}$.
27. 15\frac{2}{5}$.
28. 36\frac{3}{4}$.
29. 9\frac{5}{8}$.
30. 27\frac{3}{8}$.
31. $24\frac{4}{15}$.
32. $84\frac{1}{18}$.
33. $38\frac{5}{12}$.
34. $104\frac{1}{125}$.

Art. 285.

3. $.75.
4. $.875.
5. .56.
6. .9375.
7. $.8.
8. $.495.
9. .024.
10. .8125.
11. .83333+.
12. .25925+.
13. .76785+.
14. .24666+.
15. .60625.
16. .05078+.
17. .003125.
18. .005625.
19. .7.
20. .032.
21. $1.875.
22. $.066.
23. 101.75.
24. 225.625.
25. 11.125.
26. 8.6625.
27. $.934375.
28. $4.008.
29. 12.69.

Art. 288.

2. 1.703326.
3. 599.007.
4. $206.874.
5. .058815.
6. 51.180606.
7. $275.215.
8. 150.0660325.
9. 79.9992.
10. 111.233 A.
11. $70.03.
12. 1.5547+.
13. $7062.15.
14. 387.33 rods.
15. $5984.80.

Art. 290.

2. 253.86319.
3. $533.06.
4. $26.6875.
5. .376118.
6. $161.085.
7. 1.99655.
8. 10.040174.
9. 103.5.
10. 4.9999875.
11. $.25.
12. $.0625.
13. 6.3045.
14. .238517+.
15. 1.873125.
16. $129.0625.
17. $.35\frac{2}{3}$.
18. .57675.
19. .09.
20. 2194.85 A.
21. 6.458\frac{1}{3}$.
22. $411.58.
23. 1.6625.
24. 4.1375.
25. $95.
26. $47.07.

Art. 293.

2. .33615.
3. 14.21628.
4. .00087.
5. 24.5470625.
6. $105.138.
7. $36.0062.
8. 572.8.
9. 620.7.
10. 1.375.
11. 676.
12. 20.496.
13. .04765625.
14. .0431388.
15. 7.03125.
16. 15.015.
17. .0084375.
18. 1252.6875.
19. $53.5.
20. 114.75.
21. .0615.
22. $155.8475.
23. 556.718bu. +
24. $446.25.
25. $438.
26. $14891.925.
27. $53.696 +.
28. $12300.75.
29. $113.235 +.
30. $101.175.
31. $389.49.
32. $242.937 +.
33. 402.788976.
34. 4.437.
35. 1.69064.
36. 7.03175.

Art. 296.

2. 22.66⅔.
3. 4500.
4. .2.
5. 1.25.
6. 36.4.
7. 4602.
8. 73.73;
24.5766 +;
5898.4;
85.0730 +.
9. $95;
$15.125.
10. 14⅓ times;
80 "
6.42 + "
11. 8⅔; 6.66⅔;
.075.
12. 3.13133⅔;
313.133⅔;
3131.33⅔;
31313.3⅔;
313133⅔.
13. 387.5;
38.75;
3.875;
.3875.
14. .6455.
15. 50000 times.
16. $8000.
17. .4.
18. .16.
19. .1344.
20. .0175.
21. .00734.
22. $.72.
23. .00001.
24. 100000.
25. 121.875.
26. .0033⅓; 10.
27. 23 tons.
28. 12 coats.
29. 17 horses.
30. 136 bbl.
31. $65.406 +.
32. 550 lb.
33. $6.25.
34. 135 lb.
35. .831.
36. 1554.
37. 688.
38. 2.07887 +.
39. 1744.0598 +.
40. $295.

Art. 305.

1. .4375.
2. .8; 875; .36;
.8125; .575;
.088; .385.
4. $.\dot{8}5714\dot{2}$;
$\dot{7}$; $.\dot{8}\dot{1}$; $.\dot{3}2\dot{4}$;
$.\dot{4}7619\dot{0}$;
$.\dot{1}707\dot{3}$.
6. $.41\dot{6}$; $.5\dot{3}$;
$.5\dot{9}\dot{0}$; $.3\dot{6}$;
$.31\dot{3}$.
7. .12; .125;
$\dot{2}941176470$-
$58823\dot{5}$;
484375; $.\dot{6}$;
28125; .088;
$\dot{2}3809\dot{5}$;
.2288.

Art. 306.

2. $\frac{5}{11}$.
3. $\frac{2}{3}$.
4. $\frac{11}{37}$.
5. $\frac{25}{37}$.
6. $\frac{13}{37}$.
7. $\frac{42}{101}$.
8. $\frac{81}{111}$.
9. $\frac{107}{333}$.
10. $\frac{65}{101}$.
11. $\frac{97}{909}$.
12. $\frac{39}{41}$.
13. $\frac{12}{13}$.
14. $\frac{85}{37}$.
15. $\frac{447}{37}$.

Art. 307.

2. $\frac{26}{45}$; $\frac{11}{225}$;
$\frac{113}{1125}$; $\frac{173}{375}$.
3. $7\frac{263}{495}$; $2\frac{559}{990}$;
$7\frac{5}{396}$.
4. $\frac{43}{925}$.
5. $\frac{523}{666}$.
6. $\frac{13}{14}$.
7. $\frac{13}{37}$.
8. $\frac{5}{396}$.
9. $\frac{25}{18}$.

Art. 313.

4. $172.
5. $19.83⅓.
7. $856.
8. $384.

Art. 315.

2. $157.875.
3. $3986.722.
4. $44.83⅓.
5. $4696.30.
6. $65.875.
7. $438.75.
8. $9.1875.
9. $40.176.
10. $17.71.
11. $325.80 +.
12. $183.15.
13. $212.75.
14. $85.93 +.
15. $23.96 +.

Art. 316.

2. $6.66 +.
3. $30.34 +.
4. $630.70 +.
5. $39.65.
6. $20.173 +.

Art. 320.

1. Dr. $812.72.
2. Cr. $21788.16.

Art. 327.

1. $448.07.
2. $1489.46.
3. $1489.84.
4. $6053.50.
5. $81.80.
6. $258.85.
7. Cr. Bal., $169.675.
8. Note to Bal., $176.16.

Art. 328.

1. $60.
2. $59.57.
3. $21.375.
4. $7.50.
5. $3228.34.
6. $3.40.
7. $.50.
8. $1165.
9. $83.531¼.
10. $191.10.
11. $4.63¾.
12. $122.50.
13. $.33⅓.
14. $.15.
15. $176.475.
16. $104.10.
17. 23 bu.
18. $21.125.
19. 134¾ tons.
20. 21557.47343.
21. 48 lb. each.
22. $158¼, gain.
23. $450.
24. $5.25.
25. $1.25 per C.
26. .15.
27. $4.50.
28. $5.06+.
29. $196.21+.
30. 1100 lb.
31. 80 bu.
32. 1.69+.
33. $3.40+.
34. 138 bu.
35. $74, cost. $59, selling price.
36. $232.745 g'n.

Art. 425.

2. 45515 gr.
3. 105948 oz.
4. 910 in.
5. 68245 min.
6. 63964 ft.
7. 2046 in.
8. 222 eighths.
9. 43695 sq. ft.
10. 224800 P.
11. 8960 A.
12. 29106 l.
13. 9696 cu. ft.
14. 6216 pt.
15. 792 qt.
16. ʒ 4800.
17. 1008 gi.
18. 14918 lb.
19. 5480 pwt.
20. ℈ 4785.
21. 525600 min.
22. 7948800 sec.
23. 8784 hr.
24. 19325.
25. 200 quires.
26. 864 doz.
27. 78360 d.
28. 2650 ct.
29. 8280 d.
30. $1045.50.
31. 960 rd.
32. 563 bbl.
33. 80 boxes.
34. $29.25.
35. 13440 times.
36. 1440 min.
37. 2160 sheets.
38. $432.
39. 2419200.
40. 11082240.
41. 414.96 st. mi.
42. 36 of each.
43. 876576 hr.
44. 1485 vols.
45. 256 pp.
46. 3625 lb.
47. 8344 lb.
48. 2950 lb.
49. 32620.
50. 6325 lb.
51. 8150 lb. N.Y.
52. 7545 " "
53. 461824 "
54. 1400 lb.
55. 1800 lb.
56. $136.262.
57. $124.095+.
58. $4.825.
59. $10.13⅝.

Art. 428.

2. 15 w. 4 da. 9 hr. 40 min.
3. 10 mi. 8 ch. 20 l.
4. 2031 lb. 9 oz. 10 pwt.
5. 50 mi.
6. 1605 A.
7. 1 sq. mi.
8. 125 cu. ft. 840 cu. in.
9. 297 C. 26 cu. ft.
10. 15 hhd. 19 gal. 3 qt. 1 pt.
11. 846 bu.
12. 264 bbl. 26 gal. 3 qt.
13. 12965 gal.
14. Cong. 63, O. 2, ℥ 10.
15. 14 lb. 10 oz. 18 pwt. 22 gr.
16. 25 T. 15 cwt. 70 lb.
17. 25 cwt. 37 lb. 15 oz.
18. 12 lb. 6 oz.
19. 201 bu.
20. 15½ bbl.
21. 203 bu.
22. 31.72 quin.
23. 5 w. 1 da. 1 hr. 1 min. 1 sec.
24. 191 mo. 8 da. 11 hr. 40 min.
25. 557° 33′ 20″.
26. 87 deg. 50 naut. mi.
27. 836 gro. 1 doz. 4 pens.
28. 227⅚ doz
29. 251 sc.
30. 22 Rm. 7 Qu. 10 sh.
31. 151 Bund. 8 Qu.
32. 411 Cr. 2s.
33. 2038 fl.
34. 80 half-sov.
35. £44 2s. 2d. 2 far.
36. 450 fr.
37. 46 sov. 6s. 3.9+d.
38. 200 marks.
39. $1689600.
40. 4725 lb.
41. $32.55 N. Y.
42. 7° 3′.
43. 456 da. 12 hr. 45 min.
44. $1.87½.
45. $48.
46. $56.16.

Art. 431.

2. $\frac{1}{8}$ ft.
3. $\frac{1}{8}$ gr.
4. $\frac{3}{4}$ pt.
5. .24s.
6. $\frac{3}{80}$ oz.
7. $\frac{7}{9}$ sq. rd.
8. .32 pt.
9. $1\frac{5}{7}$ yd.
10. .33 ft.
11. $\frac{8}{9}$ oz.
12. $\frac{5}{6}$ yd.
13. $\frac{5}{9}$ l.
14. .252 min.
15. $\frac{54}{325}$ sq. rd.

Art. 433.

2. 10s. 10d.
3. ℥4 ʒ1 ℈1 gr. 16.
4. 85 rd. 5 ft. 6 in.
5. 9 oz.
6. 3 ft. 9 in.
7. 8.8 oz.
8. 17 da. $3\frac{3}{7}$ hr.
9. 11s. 1.2d.
10. 86 P. 4 sq. yd. 5 sq. ft. $127\frac{5}{13}$ sq. in.
11. $6\frac{2}{5}$ oz. Avoir.
12. 14 cu. ft. $691\frac{1}{5}$ cu. in.
13. 5° 48′ 7.2″.
14. fʒ5 ♏36.
15. 18 cwt. 96 lb. 14 oz.
16. 55 gal. 1 pt.
17. 16 sq. yd. 7 sq. ft. 36 sq. in.
18. ℥8 ʒ1 ℈1 gr. $7\frac{3}{11}$
19. 6 gro. $10\frac{2}{7}$ doz.
20. 2 mi. 101 rd. 6 ft. $6\frac{1}{4}$ in.
21. 3 gal. 3 qt. 1 pt. 2 gi.
22. 2 pk. 2 qt. 1 pt.
23. 212 rd.
24. 4 T. 5 cwt. $55\frac{5}{9}$ lb.
25. 1 A. 60 P.
26. 2 Cd. 89.6 cu. ft.
27. 57 rd. 9 ft. $10\frac{1}{5}$ in.
28. 6 Qu. 6 sheets.
29. $\$54.16\frac{2}{3}$.

Art. 435.

2. $\frac{1}{44}$ gal.
3. £$\frac{1}{90}$.
4. .01 bu.
5. .0001 lb.
6. .0004 ton.
   .018 ton.
7. $\frac{3}{98}$ cord.
8. .00045 oz.
9. $\frac{1}{2800}$ ton.
10. $\frac{1}{640}$ da.; .005 da.
11. .02 rd.
12. $\frac{12}{35}$ pt. less.
13. $\frac{7}{1440}$ A.

Art. 437.

2. $\frac{1}{8}$ bbl.
3. .4375 Cd.
4. .22+ hhd.
5. $\frac{563}{640}$ lb.
6. $\frac{1}{8}$.
7. .092.
8. ʒ.7.
9. .005489 Tp.
10. .3.
11. $\frac{23}{42}$.
12. .581 lea.
13. $\frac{3}{16}$.
14. .001625.
15. .09.
16. $\frac{25}{36}$.
17. £$1\frac{2}{13}$.

Art. 438.

1. 44352 steps.
2. 18 h. 45 min.
3. $53.1665125.
4. $199.25.
5. $111.94.
6. £60.
7. $\$128.23\frac{1}{3}$.
8. $34.574.
9. $708\frac{2}{13}$ bu., Ill.
   $657\frac{4}{7}$ bu., La.
   $634\frac{26}{29}$ bu., N. Y.
10. $5714\frac{2}{7}$ bu., Ct.
    $5333\frac{1}{3}$ bu., N. J.
11. $332.679.
12. 15 carats.
13. 424.98775 A.
14. $90.
15. $13457.46+.
16. $3.525.
17. $87.815.
18. $2.54+.
19. $\frac{7}{9}$.
20. $138.95.
21. 3125 bu.
22. 720 centals.
23. 5 bbl. 152 lb.
24. $21.988+.
25. $\frac{1728}{4784}$.
26. 3.02.
27. $487\frac{1}{7}$ Rm.
28. $307\frac{1}{57}$ Rm.

Art. 440.

3. 22 yd. 2 ft. 10 in.
4. 19 Cd. 3 cd. ft. 13 cu. ft.
5. 2 hhd. 17 gal. 2 qt. 3 gi.
6. 15 h. 28 min.
7. 60 gal. 1 qt.
8. 22 cwt. 84 lb. $14\frac{2}{7}$ oz.
9. 10 Pch. $9\frac{29}{40}$ cu. ft.
10. ℔11 ʒ2 ℈2 gr. 5.
11. $133.24.
12. $61.50.
13. 31 yr. 11 mo. 3 da.

Art. 441.

3. 25 Cd. 6 cd. ft. 4 cu. ft.
4. 239 rd. 11 ft.
5. 8 cwt. 41 lb. 10 oz.
6. 10s. $7\frac{1}{5}$d.
7. 5 lb. 3 oz. 10 pwt.
8. 1 w. 6 da. 5 hr. 17 min. 16.8 sec.
9. 29 gal. 1 qt. 1 gi.
10. 1 mi. 193.7 rd.

11. $6\frac{5}{6}$ doz.
12. ℥3 ʒ5 ℈2.
13. 20 h. 24 min.
14. 65.44 P.
15. 8 gal. 3 qt.
16. $7306.71 +.
17. 13 cwt. 38 lb.
18. 30 Cd. 5 cd. ft. 14 cu. ft.

Art. 442.

2. 7 yr. 9 mo. 1 da.
3. 3 yr. 11 mo. 28 da.
4. 2 yr. 5 mo. 24 da.
5. 258 da.
6. 1 yr. 10 mo. 12 da. 6 h.
8. 204 da.
9. 157 da. 21 h.
10. 2 yr. 5 mo. 8 da. 9 h. 22 min.
11. 2 yr. 8 mo. 10 da. 4 hr. 14 min. 59 sec.

Art. 443.

2. 26 bu. 1 pk. 6 qt.
3. 39 Cd. 3 cd. ft.
4. 13 hhd. 42 gal. 3 qt.
5. 2715 bu.
6. Cong. 25 O. 6 ℥11 ʒ5 ♏36.
7. £120 18s. 6d.
8. 189 A. 40 P. 16 sq. yd. 6 sq. ft.
10. 89 T. 11 cwt. 1 qr. 19 lb. 14 oz.
11. 557 yd. 2 ft. $11\frac{1}{2}$ in.
12. 13 T. 3 cwt. 67.85 lb.
13. $196.796.
14. $125.25.

Art. 444.

2. 51 A. 31 P. 8 sq. ft.
3. £7 1s. 11d.
£5 1s. $4\frac{3}{7}$d.
£4 8s. $8\frac{3}{4}$d.
4. 31 bu. 1 pk. 5 qt. 1 pt.
28 bu. 1 pk. 1.9 pt.
23 bu. 2 pk. 2 qt. $\frac{1}{4}$ pt.
5. 12 yd. $5\frac{55}{63}$ in.
6 yd. $2\frac{59}{63}$ in.
6. 2 Cd. 5 cd. ft. $13\frac{1}{3}$ cu. ft.
7. 17160 rails.
8. 1 sq. mi. 42 A. 112 P. 26 sq. yd. 8 sq. ft.
9. 337 yd. 1 ft. $7\frac{1}{4}$ in.
10. 70 times.
11. 243 boxes.
13. 9 cwt. 42 lb.
14. 165 A. 25 P. 24.4 sq. yd,, nearly.

Art. 448.

2. 107° 19′ $48\frac{3}{4}$″.
3. 122° 26′ 45″ W.
4. 71° 12′ 15″ W.
5. 90th W.; 90th E.; 180th E.

Art. 450.

2. 50 min. $21\frac{1}{3}$ sec.
3. 1 h. 17 min. 24 sec. A.M.,or next day.
4. 5 hr. 57 min. 49 sec.
5. 5 h. 59 min. 51 sec.
6. 1 hr. 3 min. 58 sec.
7. 1 h. 13 min. $32\frac{2}{3}$ sec.
8. 51 min. 18 sec.
9. 1 hr. 40 min. 8 sec.
10. 6 hr. 28 min. 27 sec.
11. 1 h. 33 min. 27 sec.
12. 5 hr. 6 min. $15\frac{14}{15}$ sec. A.M. at Cinn.
4 hr. 53 min. 43 sec. A.M. at Chi.
4 h. 43 min. 18 sec. A.M. at St. Louis.
13. 12 h. 7 min. 41 sec., at night, B.
4 h. 53 min. $46\frac{1}{3}$ sec. P.M., St. P.
2 h. 58 min. 6 sec P.M., Ast. Or.
14. 5 h. 46 min., later Rome.
5 h. 5 min. 32 sec., later, Paris.
15. 10 h. 58 min. 37 sec., gains.

Art. 454.

1. 52 ft. 9′.
2. 319 ft. 4′ 3″.
3. 23 ft. 10′ 9″.

Art. 456.

2. 68 ft.
3. 55 ft. 10′ 3″ 2‴ 8⁗.
4. 240 ft. 9′ 4″.
5. 50 ft. 9′ 10″ 6‴.

Art. 465.

1. 63 sq. yd.
2. $7\frac{1}{2}$ ft. wide.
3. 61 ft. long.
4. 152 sq. yd. 1 sq. ft.
5. 348 sq. yd. 4 sq. ft.
6. $379\frac{1}{4}$ sq. rd.
7. 32 sq. ch. 2 P.
8. $427\frac{1}{3}$ sq. ft.
9. 7 sq. rd. 1 sq. yd. 6 sq. ft. 88 sq. in.
10. 18 ft. 3 in. width.
11. 7 ch. 25 l. length
12. 18 yd. 2 ft.
13. $58\frac{7}{18}$ sq. yd.
14. 48. planks.
15. 44 yd.
16. 260 yd.
17. $88\frac{2}{3}$ yd.
18. $81\frac{3}{4}$ yd.

19. \$98.54+.
20. \$106.48.
21. \$81.87.
22. \$146.40.
23. \$60.95.
24. 1080 tiles.
25. \$608.40.
26. \$65.475.
27. $21.29\frac{11}{12}$ squares.
28. \$139.57.
29. \$198.
30. \$27.378.
31. 840 sods.
32. $28\frac{1}{4}$ yd.
33. $13\frac{3}{5}$ rolls.
34. \$71.60.
35. \$13.525+.
36. 11316 shingles.
37. \$25.55.
38. \$447.989+.

Art. 467.

1. 90 A.
2. 32 rd. wide.
3. $190\frac{2}{3}$ farms.
4. .625 A.
5. 264 rd.
6. \$7504.80+.
7. $\frac{40}{49}$.
8. 25 rd.
9. \$220 less.
10. \$4000 gain.

Art. 468.

1. 80 A.; $\frac{1}{8}$ Sec.
2. 5760 rails;
\$230.40.
3. \$340 gain.
4. 240 A.; $\frac{3}{8}$ Sec.
5. 120 A. left;
\$27.20 gain.
6. 420 A. left;
\$635 gain.

Art. 474.

1. 96 cu. ft.
2. 108 cu. ft.
3. $8\frac{1}{3}$ ft.
4. 221 cu. ft.
5. 208 cu. yd.
6. 8 cu. yd. 26 cu. ft.
297 cu. in.
7. 7 cu. yd. 11 cu. ft.
200 cu. in.
8. 5 cu. yd. 25 cu. ft.
9. $1\frac{1}{8}$ in., height.
10. 8 in., height.
11. 9 ft. 2 in., length.
12. 4840 cu. ft.
13. $12\frac{3}{16}$ Cd.
14. 8 ft.
15. \$13.182+.
16. \$166.60.
17. 8 ft.
18. 80 cans.
19. \$410.156+.
20. 24 ft.

Art. 477.

2. $31278\frac{6}{11}$ bricks.
3. 60 Pch.
4. $49\frac{1}{11}$ Pch.
5. 62006+ bricks.
6. \$1607.82+.
7. $\$471.66\frac{2}{3}$.
8. $667\frac{7}{11}$.
9. \$3276.
10. \$423.53.
11. 2142;
\$333.20.

Art. 481.

3. $53\frac{1}{3}$. *12.* \$6.14+
4. $93\frac{1}{2}$. *13.* \$1.064.
6. \$4.20. *15.* $13\frac{1}{3}$ ft.
7. \$15.75. *16.* $62\frac{2}{3}$ ft.
9. 6 ft. *17.* \$9.90.
11. 16 in. *18.* \$9.425.
19. 315 board ft;
$26\frac{1}{4}$ cu. ft.
20. \$159.365.
21. \$90.
22. 396 posts;
11880 ft. lumber.
\$274.824, cost.

Art. 485.

1. 96 bu.
2. 160 cu. ft.
3. $138\frac{2}{3}$ bu.
5. 15 ft.
6. $9\frac{3}{8}$ ft.
7. 3 ft.
8. $108\frac{3}{8}$ bu.
9. 192 bu. oats;
$153\frac{3}{5}$ bu. potatoes
10. $57\frac{3}{5}$ bu. apples;
72 bu. barley.
11. \$1920.
12. \$173.25.
13. \$205.056.
14. \$259.072.
15. $39\frac{3}{47}$ bbl.
16. $697\frac{1}{2}$ tons.
17. \$861.13+.
18. $8\frac{1}{2}$ tons.
19. \$27.
20. 270 tons.
\$1485.
21. \$14.75.

Art. 490.

2. $149\frac{47}{77}$ gal.
3. 6 bbl.
4. $404\frac{1}{4}$ cu. ft.
5. $54\frac{6}{7}$ hhd.
6. $1968\frac{3}{4}$ lb.
7. $3500\frac{68}{77}$ gal.
8. 5 ft. $7\frac{5}{8}$ in.
9. 7163.1 cu. in.
10. 7.4805+ gal.
11. $680\frac{8}{11}$ gal.
$5687\frac{1}{5}$ lb.
12. 604.8 cu. in.
13. \$43.09+ gain.
14. $4\frac{6}{77}$ gal.
15. $537\frac{7}{64}$ bu.
16. 23625 lb.
17. \$22.797+.
18. $314\frac{1}{3}$ cu. ft.
$2352\frac{48}{77}$ gal.
19. 469.39+ Im. Gal.

Art. 493.

3. 11 lb. 8 oz. 7 pwt.
7 gr.
4. 13 lb. 12+ oz.
5. \$75.46875.
6. $87\frac{1}{2}$ oz.
7. \$57.986+.
8. 52545 gr.

Art. 512.

2. 243¼ lb.
3. $6321.
4. £263 2s. 6d.
5. 2912 bu.
6. $175.
7. $205.49.
8. 14.076 rd.
9. $6014.40.
10. $3180.01.
11. 2 mi. 277 rd. 5½ ft.
12. 386⅔ ft.
13 21⅗ bu.
14. 437½ lb.
15. 123 men.
16. $\frac{1}{180}$ yr.
17. .004 hhd.
18. 264⅗ lb.
19. $9896.25.
20. $677.33⅓ Ex. Savings, $922.66⅔.
21. .52 ; $45760.
22. $3902.40.

Art. 515.

2. 25%.
3. 25%.
4. 108%.
5. 5%.
6. $14\frac{2}{7}$%.
7. 5%.
8. 5½%.
9. 62½%.
10. $73\frac{4}{15}$%.
11. 12½%.
12. 7½%.
13. 75%.
14. 8%.
15. 37½%.
16. 6%.
17. 112½%.
18. 20%.
19. 12½%.
20. 50%.
21. 65%.

Art. 518.

2. $360.
3. $750.
4. 91.2 A.
5. 528 lb.
6. 690.
7. 5800.
8. .6.
9. 100.
10. $750.
11. $5450.
12. 600 bu.
13. 10080 bbl.
14. 8000 bu.
15 4500 bu.
16. $3000.
17. $78133.33⅓.
18. $922.25.

Art. 520.

2. 2500.
3. $6000.
5. $1250.
6. $7400.
7. $3392.86.
8. 36000.
9. $2275.
10. 900 bu.
11. 800.
12. 325 A.
13. $2480.
14. $375.40.
15. $31 pr. A.
16. $45 pr. bale.
17. $4398.55.
18. $8750.
19. $3400,1st yr. $3570,2d yr.
20. $208.33⅓.

Art. 530.

2. $349.
3. $842.40.
4. $636.375.
5. $204.86.
6. $253.75.
7. $1437.60.
8. $306.67.
9. $144.32.
10. $11016.
11. $300.

Art. 531.

3. $208.125.
4. $11.31¼.
5. $ 17.
6. $6.22½.
7. $4.375 ; $2.80.
8. $.53⅛. $1.06¼.
9. $.11⅝ per.lb.

Art. 532.

3. 18¾% gain.
4. 12½% loss.
5. 20%.
6. 28%.
7. $14\frac{2}{7}$%.
8. 24%.
9. 66⅔%.
10. 23%.
11. 50%.
12. 37½%.
13. 60%.

Art. 533.

3. $9375.
4. $8.80.
5. $1.50.
6. $14.14.
7. $16666.66⅔.
8. A. $16000 ; B. $10000.

Art. 534.

2. 6.86.
3. .75.
4. $4.91.
5. $.20.
6. $244.094.
7. $183.33⅓.
8. $586.66⅔.
9. $6553.60.

Art. 535.

2. $1.47.
3. $150.
4. $1.06⅔.
5. $96.

Art. 547.

2. $378.125.
3. $82.11.
4. $379.40.
5. $285.19.
6. $20.18.
7. $584.17½.
8. $96.90.

Art. 548.

2. 3½%.
3. 5%.
4. 1⅓%.
5. 2¾%.
6. 5%.
7. 6¼%.

Art. 549.

2. $2784.
3. $3500.
4. $9600.
5. $9000.
6. $960.40.

Art. 550.

2. $3750.
3. $583.33⅓.
4. $25372.

Art. 551.

2. $4696.65.
3. $3182.55.
4. $1500.
5. $10648.
6. $6400.76 Inv.; $320.04 Com.

[illegible] int.;
[illegible] amt.;
[illegible] int.;
[illegible] amt.
[illegible]

Art. 574.

2. $120.
3. $.04.
4. $10.58.
5. $82.36.
6. $10.96.

Art. 575.

1. $58.93.
2. [illegible].40.
3. [illegible]7.67.
4. $159.745.
5. [illegible]
6. [illegible].11.
7. [illegible]
8. [illegible]7.51.
9. $115.20.
10. [illegible]
11. [illegible]

Art. 577.

[illegible]

Art. 581.

2. 7%.
3. 7%.
4. $8\frac{1}{5}\%$
5. 6%.
6. 2% a month.
7. $10\frac{5}{13}\%$.
8. 25%; $16\frac{2}{3}\%$; $12\frac{1}{2}\%$; 10%.
9. 100%; 40%; $28\frac{4}{7}\%$; $16\frac{2}{3}\%$; 10%.
10. $7\frac{1}{5}\%$.
11. The 2d is $1\frac{17}{80}\%$ better.

Art. 583.

2. 7 mo. 10 d.
3. 6 yr. 8 mo.
4. 7 mo. 6 da.
5. 3 yr. 4 mo. 24 da.
6. $33\frac{1}{3}$; 20; $16\frac{2}{3}$; $13\frac{1}{3}$; 10 yr.
7. 50; 40; $28\frac{4}{7}$; 25; 16 yr.
8. $12\frac{1}{2}$; $6\frac{1}{4}$; 25 yr.

Art. 586.

2. $428.76.
3. $189.15.
4. $1176.14.
5. $100.32.
6. $41.99+.
7. $1495.77.
8. $53.38.
10. $1525.64.
11. $1540.79.
12. $987.23.
13. $1934.84.
14. $18142.81.

Art. 589.

2. $464.10.
3. $7308.
4. $11.30.
5. $1161.04.
6. $1047.52.

Art. 597.

3. $659.94.
4. $30.14.
5. $162.25.

Art. 598.

2. $312.47.
3. $355.16.

Art. 603.

2. $281.83.
3. $102.90.
4. $1137.61.
5. $43.65 in favor of dis.
6. $931.20.
7. $338.26.
8. $45\frac{63}{100}\%$.
9. $931.88.
10. $.05 per bbl. more profitable to buy at $8.75 on 6 mo.
11. $3677.75.

Art. 615.

2. $6.27 Bk. dis. $591.23 proceeds.
3. $1614.48.
4. $10839.83.
5. Mat. Oct. 30; 81 days term of dis.; $940.38 proceeds.
6. Mat. April 8; 46 days term of dis.; $917.37 proceeds.
7. Mat. Aug. 2; 79 days term of dis.; $1295.82 proceeds.
8. Mat. Dec. 15; 30 da. term of dis. $1281.77 proceeds.

Art. 617.

2. $1434.20.
3. $719.61.
4. $1951.03.
5. $2291.44.
6. $321.46.
7. $659.88.
8. $368.25.

Art. 619.

2. $188.43 bal. July 1st.
3. $4.90.
4. $369.36.
5. $327.927.

Art. 648.

2. $34256.25.
3. $16856.25.
4. $15842.75.

Art. 649.

2. 250 shares.
3. 220 "
5. 220 "
6. 480 "
7. 200 "

Art. 650.

2. $25500.
3. $21100.
4. $6930.

Art. 651.

2. $8\frac{8}{9}\%$.
3. 8% bonds at 110 $\frac{20}{33}\%$ better.
4. 6% bonds at 84 $\frac{10}{81}\%$ better.
5. $9\frac{23}{53}\%$.
6. $5\frac{15}{17}\%$.
7. $3\frac{31}{33}\%$.

Art. 652.

2. $62\frac{1}{2}$.
3. $33\frac{1}{3}\%$.
4. $71\frac{3}{7}$.
5. $40.
6. 75; $66\frac{2}{3}$.

Art. 653.

2. $5466.28.
3. $268.20.
4. $262.66 better to pay in currency.

Art. 654.

2. $4000; $4035.87; $4109.59.
3. $74000.
4. $1755800.
5. Dim. $26.25.
6. $113 per annum.
7. Stock invest. is $50 better, or $\frac{20}{57}$% yearly.
8. $21384 in N. Y. S. 6's; $42768 U. S. 5's of 81.
9. $792.

Art. 664.

2. $42.75.
3. $24.06.
4. $187.50.
5. $156.25.

Art. 665.

2. $1\frac{3}{4}$%.
3. $\frac{1}{2}$%.
4. $\frac{3}{8}$%.

Art. 666.

2. $13600.
3. $8960.
5. $22220.77.
6. $49147.91.
7. $24500.
8. $24766.58.
9. $9.90.

Art. 675.

2. $284.78.
3. $1055.30.
5. $527.65.
6. $5888.57.
7. $4416.57.
8. $3263.93.
9. $1131.12 loss.
10. $7200.

Art. 685.

2. $11350.
3. $19072.16.
4. $401920.
7. $25.09.
8. $87.38.
9. $112.50.
10. $226.50.
11. .0228 tax rate. $214.65.
12. $410.95.
13. $224.37.
14. $178.13.
15. $420000.

Art. 700.

2. $1566.15.
4. $4764.84.
5. $5153.24.
6. $6388 80.
7. $5632.20.

Art. 701.

2. $787.46.
3 $720.
4. $316.45.
5. 451 shares.
6. $97\frac{7}{8}$%.
7. $20108.35.

Art. 706.

2. $2303.25.
3. $3317.63.
5. $134.78.
6. $352.67.
8. $421.09.
9. $566.50.
11. $801.94.
12. $4621.16.
13. $5243.80.
14. $3500.40.

Art. 707.

2. £1543 4s. 2d.
4. 2318.84 marks.
5. 1664.13 marks.
7. 31888.83 francs.
8. 12918.75 francs.

Art. 711.

2. $179.21.
5. 5.31 francs.
6. $4 987.
7. £1055 12s. 4d.; £21 9s. 9.7d.
8. $32.78 ind. ex.
9. 696.374 guild. loss.
10. $12617.08.

Art. 726.

2. $437.50.
3. $1706.25.
4. $1843.75.
5. $1234.38.
6. $63.18.
7. $5775.
8. $2376.28 duty. $6815.75 cost in currency.
9. $1755.89.
10. $987.08.

Art. 733.

2. 3 mo. 25 da.
3. 6 mo. 26 da. time of Cr.; June 27, '77 Eq. time.
4. May 5, 1875.
5. 5 yr. 20 da. from date of last paym't.
7. Nov. 26, Eq. time.
8. 73 da. term of Cr.: Feb 26, Eq. time
9. Mar. 7, Eq. time.; $1178.01 cash value.

Art. 734.

2. Aug. 19, 1875, Eq. time.
3. June 7, 1876.

4. June 27, 1874; Dis. $149.28.
5. Apr. 23, 1874.
6. $2337.02.
7. May 20, 1875.

Art. 737.

2. Dec. 13, Eq. time.
3. Dec. 19.
4. Jan. 24, 1879.

Art. 738.

2. May 18; $1486.17 due.
3. Dec. 5, 1875.
4. $2069.59.
5. Oct. 27; $2102.58.
6. $1272.33.

Art. 739.

2. $2331.65 Sales; $762.83 Charges; $1568.82 Net proceeds; Bal. due, Dec. 27.
3. $3966.25 Sales; $412.98 Charges; $3553.27 Net proceeds; Eq. time Apr. 14, 1875.

Art. 767.

2. 60 bu.
3. $100.
4. $4.05.
5. 44$\frac{4}{9}$ bbl.

Art. 770.

3 9 horses.
4. 100 yd.
5. 16 men.
6. 96 sheep.
7. $5355.
8. 7 hr. 13$\frac{1}{3}$ min.
9. 355 bu.
10. 112$\frac{1}{2}$ mi.
11. 59$\frac{3}{8}$ da.
12. $7320.
13. 9 yd.
14. 1\frac{41}{84}$.
15. 46 A. 134 P.
16. $63.
17. $10958.90.
18. $3.25.
19. $89.60.
20. $120.
21. 2 yr. 6 mo.

Art. 772.

2. 43$\frac{1}{6}$ tons.
3. 5$\frac{1}{8}$ weeks.
4. 432 mi.
5. 15 da.

Art. 774.

2. $498.08.
4. 1120 bu.
5. $6428.57.
6. 114$\frac{3}{25}$ ream.
7. 220$\frac{4}{5}$ Cd.
8. $52.79.
9. 9 men.
10. 546 bbl.
11. 2080 lb.
12. $100.
13. 236605$\frac{5}{7}$ brick.
14. $236.25.
15. 694$\frac{2}{5}$ yd.
16. $1728.
17. 5 da.
18. 150 yd.
19. 3 yr. 4 mo. 24 da.
20. 11.66\frac{2}{3}$.
21. 9 men.
22. 8.116 ft.
23. $48.
24. $53.08.
25. 1.6 mo. +

Art. 782.

3. A's share $320.
B's " $216.
C's " $184.
4. A. $303.45.
B. $337.17.
C. $404.61.
D. $682.77.
5. A. $1710.
B. $870.20.
6. A. $6000.
B. $8402.25.
C. $5055.75.
D. $3042.
7. $5785.20, the first; $5142.40, the second.
8. $3516.80 A's gain;
5861.33\frac{1}{3}$ B's "
8205.86\frac{2}{3}$ C's "
9. $269559.55 Resources;
$26434.55 Liabilities;
$243125 Stock;
$125000 Original capital;
$118125 net gain;
$56700 Ames' share;
$37800 Lyon's share;
$23625 Clark's share.

Art. 783.

2. $2400 Barr;
2666.66\frac{2}{3}$ Banks;
2933.33\frac{1}{3}$ Butts.
3. $388.704 + A.;
$249.169 + B.;
$112.122 C.
4. $1344.164 A.;
$2027.836 B.
5. $5700 A.;
$3760 B.;
$1340 C.
6. $1688.434 Crane;
$3868.863 Childs;
$2012.703 Coe.

Art. 787.

2. \$.32.
3. \$.30 per bushel.
4. \$6 gain.
5. \$6.16.

Art. 788.

3. 2 lb. of first;
2 lb. of second;
3 lb. of third.
4. 1 at \$4;
5 at \$5;
3 at \$6;
1 at \$8.
5. 3 bbl. at \$$5\frac{1}{4}$;
3 bbl. at \$6;
2 bbl. at \$$7\frac{3}{4}$.
6. 3 gal. at \$1.20;
3 gal. at \$1.80;
15 gal. at \$2.30;
8 gal. water.

Art. 789.

2. 10 cows at \$32;
10 cows at \$36;
60 cows at \$48.
3. 10 lb. at \$.80;
10 lb. at \$1.20;
70 lb. at \$1.80.
4. 12 yd. at \$$3\frac{1}{3}$;
16 yd. at \$$1\frac{1}{2}$.
5. 150 acres.

Art. 790.

2. 30 men, 5 women, 20 boys.
3. $33\frac{1}{3}$ gal. water.
4. 16, 24, 4, and 12 da. respectively.

Art. 792.

1. 72 and 48.
2. D's age 16;
E's age 24;
F's age 84.
3. 15 bu.
4. 18 da.
5. $8\frac{3}{4}$ da.
6. Starch \$2 a box;
Soap \$3.
7. $8\frac{8}{9}$ da.;
First in $26\frac{2}{3}$ da.;
Second in 40 da.;
Third in 20 da.;
\$180 share of 1st;
\$120 share of 2d;
\$240 share of 3d.
8. 14 bbl. at \$10;
6 bbl. at \$7.
9. 16 min. $21\frac{9}{11}$ sec. past 3 o'clock.
10. Wheat \$$1.33\frac{1}{3}$ per bu.;
Oats \$.50 per. bu.
11. 8 da.
12. \$347.71.
13. 50 bu.
14. $\frac{4}{5}$.
15. 27% nearly.
16. \$$7384\frac{8}{13}$ younger;
\$$11076\frac{12}{13}$ elder.
17. $146\frac{2}{7}$ ft.
18. \$800.
19. \$960 first;
\$720 second;
\$840 third.
20. \$1570.31.
21. 506 lb.
22. Oct. 26, 1875.
23. \$37439.998;
\$33345;
\$27359.999;
\$25106.82.
24. \$1.60.
25. 42 geese;
58 turkeys.
26. \$5700.
27. \$282.24 Sim. Int.;
\$2202.24 Amt.;
\$295.56 Com. Int.;
\$2215.56 " Amt.;
\$1673.93 + Present Worth;
\$246.07 True Dis.;
\$283.20 Bk. Dis.;
\$1636.80 Proc'ds;
\$2252.199 Face.
28. \$315.79.
\$473 69.
\$710.52.
29. \$900, July 28.
30. \$$.97\frac{7}{8}$.
31. \$10665.80 in U. S 6's, 5-20.
\$21331.60 in U. S. 5's of '81.
32. A. 3600 bu.;
B. 1200 bu.;
C. 1200 bu.
33. \$1.72.
34. $\frac{2}{13}$.
35. \$5614.27 Net Proceeds.
July 10, Eq. time.
36. \$6400 M.'s Cap.;
15 mo. N.'s time.
37. \$2023.22; Apr. 24.
38. \$2244.66.

Art. 802.

2. 1369; 1764;
3136; 5625.
3. 3375; 5832;
74088; 157464.
4. 3969; 110592;
1048576; 248832.
6. $\frac{49}{256}$; $\frac{1728}{29791}$.
7. $\frac{6561}{38416}$; $\frac{3125}{32}$.
8. 645.16.
9. 1191016.
10. $1958\frac{1}{16}$.
11. $\frac{14641}{50625}$.
12. .00116964.
13. .015625.
14. 46733.803208.
15. .065528814274496.
16. $33169\frac{33}{64}$.
17. $16.6056\frac{1}{4}$.
18. 24.76099.
19. .000000250047.
20. 1520875.
21. $2023\frac{97}{256}$.
22. 5.887.
23. 640000.
24. 2540.0390625.
25. 125. *26.* 1200.

Art. 803.

3. 1764.
4. 2304.
5. 3136.
6. 9604.
7. 15625.
8. 11025.
9. 50625.
10. 38809.
11. 116964.

Art. 804.

2. 39304.
4. 110592.
5. 262144.
6. 857375.
7. 1953125.

Art. 810.

2. 8; 16; 24; 81.
3. 9; 14; 21; 15.

Art. 813.

2. 85.
4. 242.
5. 98.
6. 115.
7. 109.
8. 997.
9. 1432.
10. 5464.
12. $\frac{13}{14}$.
13. $\frac{25}{81}$.
14. $\frac{24}{185}$.
15. .035.
16. 14.0048+.
17. 1.5005+.
18. 7.625.
19. 4.213+.
20. 103.9.
21. 59049.
22. 3.0000165+.
23. 5.656854+.
24. 1.5411.
25. .91287+.
26. .04419.
27. 36.37.
28. 1.50748+.
29. 64.
30. $\frac{1}{18}$.
31. 1.
32. 1.78+.
33. 72.
34. 90.
35. 480.8827.

Art. 815.

1. 1008 ft.
2. 240.33 rd.
3. 52 rd.
4. 200.56 rd.
5. 145½ rd.
6. $187.20.

Art. 819.

3. 25.
4. 55.
5. 101.
6. 165.
7. 1015.
8. 1598.
10. $\frac{10}{11}$.
11. $\frac{24}{35}$.
12. 1.42+.
13. 34.
14. .45.
15. 2.34.
16. 4624.
17. .0809.
18. .7936.
19. 5.73+.
20. $\frac{11}{24}$.
21. .5569.
22. 1.
23. 14.75.
24. 60.8.

Art. 821.

1. 3 ft.
2. 8 ft.
3. 2 ft.
4. 12150 sq. ft.
5. 5 ft. 8+ in.
6. 9 ft. 5.3+in.
7. 8 ft. 1.4 in.

Art. 822.

2. 274.
3. 32.
4. 543.
5. 1.05+.

Art. 829.

3. 8.
4. 17.
5. 33.
6. 149.
7. 16.
8. $7\frac{11}{24}$.

Art. 830.

2. 2.
3. 2.
4. $\frac{2}{3}$.
5. 4.
6. 7%.
7. $\frac{5}{24}$.

Art. 831.

2. 9.
3. 15.
4. 4.
5. 27.
6. 11 yr.

Art. 832.

2. 600.
3. 154.
4. 125000.
5. 78.
6. 57900 ft.

Art. 840.

3. $\frac{1}{256}$.
4. 6144.
5. 3.
6. $524288.
7. $315.619+.
8. $10485.76.

Art. 841.

2. ½.
3. 5.
4. 5.
5. 3.

Art. 842.

2. 9.
3. 7.
4. 8.

Art. 843.

3. 765.
4. $7\frac{77}{108}$.
5. 16.
6. $1\frac{1}{3}$.
7. 2.
8. 280.
9. $1023.
10. $5314.40.

Art. 853.

3. $3819.75.
4. $1292.31.
5. $3625.
6. 6 yr.
7. 7%.
8. $375.30.

Art. 854.

3. $300.
4. $3907.665+.
5. $1182.05+.
6. $3725.87+.
7. $629.426+.

Art. 882.

2. 600 sq. ft.
3. $42\frac{3}{16}$ sq. ft.
4. 22 A. 6 sq. ch. 13.45 P.
5. $449.07.
6. $147.
7. 210 sq. ft.

Art. 883.

2. 4½ ft.
3. 13 in.
4. 28 rd.
5. 672 rd. $5\frac{4}{11}$ yd.
6. 8½ ch.
7. 50 rd.

Art. 884.

2. 111.80 sq. ft.
3. 3 sq. ft. 1.7 sq. in.
4. 13 A. 41.76 P.
5. 349.07 sq. ft.

Art. 886.

2. 39 ft.
3. 25 ft. 7.34 in.
4. 33.97 ch.
5. 28 ft. 3.36 in.

Art. 887.

2. 45 yd.
3. 19 ft. 2.5 in.
4. 360 ft. $6\frac{2}{3}$ in.
5. 20 ft.

Art. 898.

2. 84 sq. ft.
3. $5\frac{1}{4}$ A.

Art. 899.

2. 11178 sq. ft.
3. $28\frac{2}{3}$ sq. ft.
4. 2 A.

Art. 900.

2. 213 sq. ft.
3. 17 A. 8 ch. 3.4 P.

Art. 904.

3. 15 ft. 10.98 in.
4. 5 ft. 10.67 in.
5. 5 ft.
6. 7 ft. 3.96 in.

Art. 905.

4. 318.3 A. +
5. 114.59 A.

Art. 906.

3. 7 rd.
4. 19.098 ft. Diam.
   59.998 ft. Circum.

Art. 907.

2. 141.42 ft.
3. 23.4 yd. +
4. 7.07 ft. +

Art. 908.

2. 32.98 sq. ft. +
3. 796.39 sq. ft.
4. 1 A. 75.62 P. land.
   78.54 P. water.

Art. 909.

2. 84.
3. 28.
4. $\frac{4}{11}$.
5. 32 lb. 13.7 oz.

Art. 910.

5. 369 rd. L.;
   123 rd. W.
6. 3.5 in.
7. 221; 238; and 255 ft.
8. $75.
9. 126.78 rd.

Art. 911.

1. $185.53.
2. 35.35 ft. +
3. 403.7 rd. +
4. $5812.50.
5. $32.40.
6. 28.66 P. +
7. 5 A.; or twice as large.
8. $724.75.
9. 20 ft.
10. 98 A. 28 P.
11. 14.645 ft.
12. 294 rd.; 45.36 rd.
13. 14 A. 150.04 P.
14. 6 in.

Art. 918.

4. 207.34 sq. ft.
5. $168\frac{7}{8}$ sq. ft.
6. 263.89 sq. ft.
7. 301.177 sq. ft.

Art. 919.

3. $274\frac{5}{8}$ cu. ft.
4. $27.
5. 73.63 cu. ft.
6. $53.63.

Art. 925.

2. 824 67 sq. ft.
3. $429\frac{1}{4}$ sq. ft.
4. 512.9 sq. ft.
5. $25.

Art. 926.

3. 39.27 cu. ft.
4. $29.23.
5. 192000 cu. ft. vol
   22284.6 sq. ft. surface.

Art. 927.

2. 345 sq. ft.
3. $256\frac{3}{8}$ sq. yd.

Art. 928.

2. 58.1196 cu. ft.
3. $38\frac{1}{8}$ cu. ft.
4. 64.99 cu. ft.

Art. 932.

2. 28.27 sq. ft.
   12.57 sq. ft.

Art. 933.

2. 8 cu. ft. 313.2 cu. in.
   523.6 cu. yd.

Art. 934.

2. 10 ft.; 15 ft.; and 20 ft.
3. 24 ft.; 32 ft.; and 40 ft.

Art. 936.

1. 13.228 ft. edge.
   2315.03 cu. ft. vol.
2. 11 ft. 7 in.
3. 1494.257 gal.
4. $5.46.
5. 576 ft.
6. 17.32 in.
7. 40 sq. ft. $7\frac{3}{8}'$.
8. 1 cu. ft. vol. of cube
   1 cu. ft. 659 cu. in. vol. of sphere.
9. 9 lb.
10. 5 hr. 26.4 min.
11. 12 ft. 6.79 in.
12. 53.855 bu.

Art. 937.

2. 99.144 gal.
3. 120.09 gal.

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